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OPTICAL PULSE COLLAPSE IN DEFOCUSING ACTIVE MEDIUM

L.Kramer, E.A.Kuznetsov[+]*, S.Popp, S.K.Turitsyn*[*]

Physikalisches Institut der Universität Bayreuth
Lehrstuhl für Theoretische Physik II
W-8580 Bayreuth, Germany

[+]*Landau Institute for Theoretical Physics*
117334 Moscow, Russia

[*]*Institute of Automation and Electrometry*
630090 Novosibirsk, Russia

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We study a phase-gradient mechanism of a pulse compression in the Kerr-lens mode locking laser systems that can be described by the complex Ginzburg-Landau equation. We have found that a pulse collapse occurs in a *defocusing* gain medium contrary to the collapse arrest in a focusing active media.

1. The self-focusing of a powerful light beam propagating through a nonlinear Kerr medium is a classical example of the wave collapse in a distributed Hamiltonian system (see for a review [1] and references therein). The possibility of collapse (or, sometimes, it is called blow-up) in such systems significantly depends on the problem dimension. By contrast the singularity formation in a finite time due to nonconservative mechanisms demonstrates rather different behavior. First of all, the collapse in this case depends slightly on the dimensionality factor and therefore for its study it is enough to consider low-dimensional systems. In the present paper we treat this problem for the one-dimensional complex Ginzdurg–Landau (GL) equation when the nonlinearity does not saturate linear exponential growth but, on the contrary, leads to the explosive growth of the amplitude for the homogeneous states. Such situation, in particular, is found in Rayleigh–Benard convection in binary mixtures [2–4] and in laser mode-locking laser systems [5–8]. In spite of the existence of such tendency for the homogeneous generation, the pulse onset and its dynamics, as we shall show, represent a result of a specific interference of the Hamiltonian and non-Hamiltonian terms, that is a *phase-gradient mechanism.* This mechanism was suggested at first in Refs. [3, 4] for the explanation of the observed dynamics of coherent structures in convection of binary mixtures.

In this Letter we study a pulse compression due to the phase-gradient mechanism in passively mode-locking laser systems. We demonstrate that a collapse takes place in a *defocusing* gain medium in contrast to the collapse arrest in a focusing active media. Basing on the GL model for the passive mode-locking laser systems, we explain how a quasi-stable pulse can be formed without limiting action of the fast saturable absorber (quintic term in the GL equation). We also show that the sign of the pulse frequency chirp (i.e., the increase or the decrease of the local pulse frequency from the forward pulse front to the backward one) is one-to-one correlated with the sign of the Kerr nonlinearity, i.e., it being focusing or defocusing.

2. A theory of passive mode-locking developed in [5, 6] has highlighted the generic nature of many mode-locking systems. A large variety of mode-locking laser systems exploiting rather different physical effects can be described by the same model which is close to the complex GL equation. We consider here systems exploiting additive pulse mode-locking (APM) or Kerr lens mode locking (KLM) [6, 7]. In KLM, the pulse-shortening mechanism is provided by the saturable absorber. The saturable-absorber action is based on the self-phase modulation effect occurring due to the Kerr-type nonlinearity. A similar mechanism is realized in APM, where a pulse narrowing results from the coherent addition of self-phase-modulated pulses.

If the relative changes in pulse parameters during the passage through the system elements are small, the pulse evolution can be considered as a propagation through some equivalent medium that is composed of a nonlinear dispersive part and a gain medium. In this case the governing equation takes the form of the complex GL equation:

$$\frac{\partial E}{\partial z} = E + (1 + iC_1)\frac{\partial^2 E}{\partial t^2} + (1 - iC_2)|E|^2 E. \qquad (1)$$

Here $E(z,t)$ is the normalized complex envelope of the electromagnetic wave; z indicates the number of passes through the laser system; the maximal dimensionless amplification coefficient (before E on the right-hand-side of the equation) is chosen to be equal to 1; the coefficient $C_1 = D/g_2$ where D and g_2 account for the group-velocity dispersion and the gain dispersion, respectively; the parameter $C_2 = \gamma/\delta$ where γ is a nonlinear Kerr coefficient, and $\delta > 0$ describes the nonlinear action of the fast saturable absorber. The gain contour is approximated by a parabola near the pulse carrier frequency. This equation also can be applied to the long amplifier when z has a meaning of a physical coordinate of an amplified pulse.

One should note that an amplitude equation in the form of the GL equation (1) can be derived for many systems by means of a standard method expanding the basic equations near the threshold of the oscillatory instability (for details see, for instance, the review [9]). In the limit $C_1, C_2 \to \infty$ Eq. (1) tends to the nonlinear Schrodinger equation. One can see that the case $C_1C_2 < 0$ corresponds to the focusing medium and $C_1C_2 > 0$ to the defocusing one. Since Eq. (1) is invariant under a simultaneous sign change of C_1 and C_2 and $E \to E^*$, we may restrict considering only the case $C_2 \geq 0$.

Obviously, a vacuum solution ($E = 0$) of Eq. (1) is unstable. Any initial noise will be amplified. When the amplitude of solution becomes sufficiently large, the nonlinear term comes into play causing an explosive growth of the amplitude and E may develop a singularity at some finite z_0.

To demonstrate the explosive action of the nonlinear term in Eq. (1), consider a time-independent solution ($\frac{\partial}{\partial t} = 0$). In this case a simple integration of equation

(1) gives:

$$|E(z)|^2 = \frac{1}{\exp\left[-2(z-z_0)\right]-1}$$

where $z_0 = \frac{1}{2}\ln(1+I_0)I_0^{-1}$ and $I_0 = |E(z=0)|^2$.

Hence one can see that at the point $z = z_0$ the amplitude becomes infinite. Close to this point $|E(z)|^2 \sim (z_0 - z)^{-1}$.

Simultaneously one has a rapid phase rotation accelerating when approaching z_0, $\phi \sim -C_2/2\log(z_0 - z)$. This means that the nonlinear amplification term leads to the explosive growth of the amplitude. A similar effect takes place for a pulse evolution in some region of the (C_1, C_2) plane. In real systems, of course, this growth is limited, because additional effects that are not included in the model equation (1) become significant when amplitudes increase. Below it will be shown that in some region of parameters (C_1, C_2) the phase-gradient mechanism can provide a generation of pulses with finite amplitude, without account of the higher-order saturation terms, i.e. pulse formation can be described in the framework of Eq.(1).

3. The phase-gradient effect manifests itself as a fast phase rotation resulting from explosive pulse amplitude increase. As we show, a pulse collapse initiated by the nonlinear saturable absorber action (cubic term $|E|^2E$ in Eq.(1)) can be stopped by a combined action of the Kerr nonlinearity and the gain dispersion. This effect, in contrast to the limiting action of the quintic term, may be considered as an *intrinsic* stabilizing mechanism. To understand this effect it is convenient to rewrite Eq. (1) in the hydrodynamic form,

$$\partial_z I + \frac{\partial}{\partial t} I v = 2(1 + I - \Omega^2)I + 2\sqrt{I}\frac{\partial^2}{\partial t^2}\sqrt{I}\,, \tag{2}$$

$$\partial_z \Omega + v\frac{\partial}{\partial t}\Omega - C_2\frac{\partial}{\partial t}I = C_1\frac{\partial}{\partial t}\frac{(\sqrt{I})_{tt}}{\sqrt{I}} + \frac{\partial}{\partial t}\frac{1}{I}\frac{\partial}{\partial t}\Omega I. \tag{3}$$

Here $E(z,t) = \sqrt{I}\exp(i\phi)$, $\Omega = \partial_t\phi$ and "velocity" $v = 2C_1\Omega$.

For the sake of simplicity let us consider first the limit $C_1 \ll 1$ and $C_2 \gg 1$. Then the influence of the Kerr-term is amplified by additional large factor C_2. Therefore it is natural to assume the frequency Ω to be large, that is equivalent to the usual semiclassical approach (the exact criterion of the applicability of this approximation is: $|\Omega|^2 \gg |E|_{tt}/|E|, |\Omega_{tt}/\Omega|$). In this limit Eqs.(2, 3) can be reduced to the form

$$\partial_z I = 2I + 2(I - \Omega^2)I, \tag{4}$$

$$\partial_z \Omega = C_2\frac{\partial I}{\partial t}. \tag{5}$$

For this system it is easy to represent the solution qualitatively. Consider an initial pulse, both weak and broad, more specifically, a pulse having a wide plateau with sufficiently small amplitude and decaying away from the plateau. Let $\Omega = 0$ initially. At the linear stage of the instability and then in the forthcoming blow-up regime the amplitude will be approximately constant inside the plateau region. At the boundary a sharp gradient of the amplitude will be formed. This gradient, in turn, will act as a source for the generation of a *phase-gradient*, i.e., Ω in this narrow region. On the other hand, the large value of Ω will saturate at first the blow-up. As a result, two counter-propagating waves, representing the moving plateau boundaries generating the large wave number Ω, will be formed. The wave propagation speed will not be constant but will grow especially at the blow-up stage. Pulse will be "eaten" by these waves moving from the edges to the center. This should finally result in a compression of the pulse up to the stage when the effects neglected before come into play. The typical evolution of the pulse dynamics in the framework of the reduced system (4), (5) are presented on Fig.1 and Fig.2. Results of the numerical integration of the system show evidently a pulse compression due to the phase-gradient effect. Starting from the initial conditions in the form of a pulse with plateau one can see how at first two counter-propagating phase-gradient waves are formed and how then they serve as the main factor of pulse death.

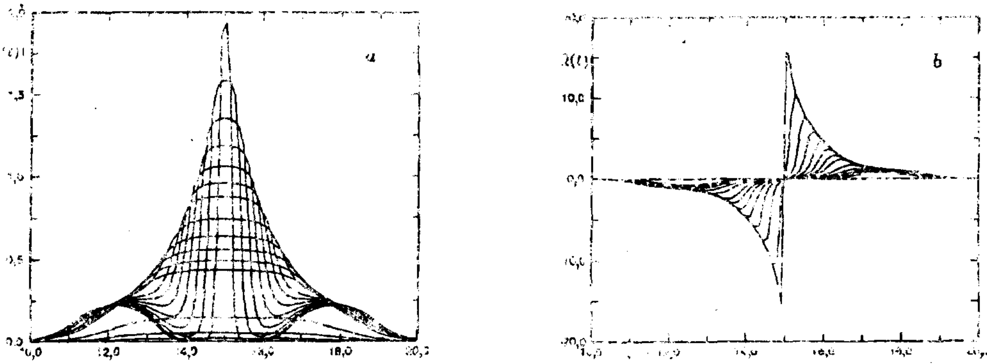

Fig.1. z-evolution of pulse amplitude $|E(t)|$ (a) and of local frequency $\Omega(t)$ (b) for the reduced system (4), (5) with $C_1 = 0$ and $C_2 = 15$. Initial distribution is $E(0,t) = 0.01[\tanh(t-16) - \tanh(t-24)]$. The lower curves illustrate a motion of two counter-propagating waves of the phase-gradient towards the pulse center, and the upper curves demonstrate the simultaneous process of an amplitude growth till some "moment" z and a pulse compression.

4. From the above qualitative consideration it becomes clear that any gradient or *inhomogeneity* in the amplitude distribution will lead to the generation of phase gradient and, as a consequence, to a suppression of the explosive growth of the amplitude. Therefore, any mechanism smoothening the pulse amplitude distribution works in favor of collapse (in this case a system will be 'close' to the homogeneous state) and any effects leading to the sharpening of the gradients effectively work against collapse. Numerically this question was treated at first in paper [10] . In particular, the boundary in the (C_1, C_2)-plane between collapsing and non-collapsing regions was determined. The collapsing region occupies two sectors in this plane for both focusing $(C_1C_2 < 0)$ and defocusing $(C_1C_2 > 0)$ media. The non-collapsing sector represents the region located around the C_2 -axis with the dominant part in the focusing quadrant. The hydrodynamic form of the Eq. (1) can give an idea why a focusing medium can prevent collapse more than a defocusing nonlinearity. Let $C_2C_1 < 0$, then any gradient in the I -distribution leads to the generation of an Ω that can suppress collapse. But in this case the "gas" velocity $v = 2C_1\Omega$ is directed to the center of a pulse which means that a pulse becomes narrower with z, gradients increase and the amplitude growth is saturated. In the opposite case $C_1C_2 > 0$ the generation of Ω is accompanied by a "gas" motion away from the pulse center leading to the effective smoothening

of the solution. The closer the solution becomes to the uniform distribution, the smaller is the effect of pulse amplitude growth suppression by Ω. The numerical integration of the complete equation (1) as well as the results of the paper [10] confirm that collapse is arrested in focusing medium if $C_2 \leq -4C_1$. For large ratio $|C_2/C_1| >> 1$ a typical behavior of a pulse is similar to that for the reduced system.

In Fig.3 we show the evolution of the initial pulse in the case ($C_1 = 6$ and $C_2 > 0$) when collapse takes place. If the maximal amplitude near singularity behaves with a good accuracy as $\sim (z_0 - z)^\alpha$ with $\alpha \approx -0.50$, the exponent β of the pulse width d ($d \sim (z_0 - z)^\beta$) occurs at the collapse stage near 0.3885. It is important to emphasize that the pulse compression takes place in the collapse regime (compare with [9]) as well as due to the phase-gradient mechanism. The main difference between two regimes is that, unlike the collapse, the phase-gradient mechanism can provide a finite amplitude pulse together with a pulse compression.

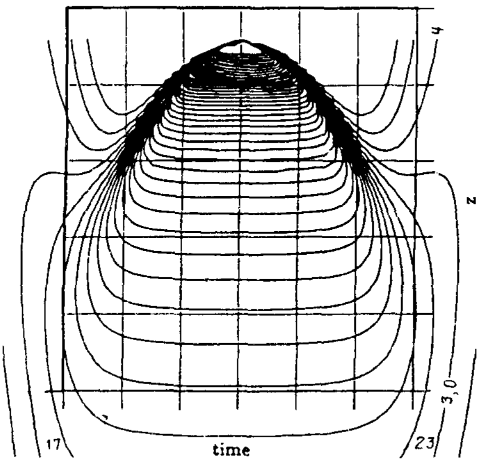

Fig.2.

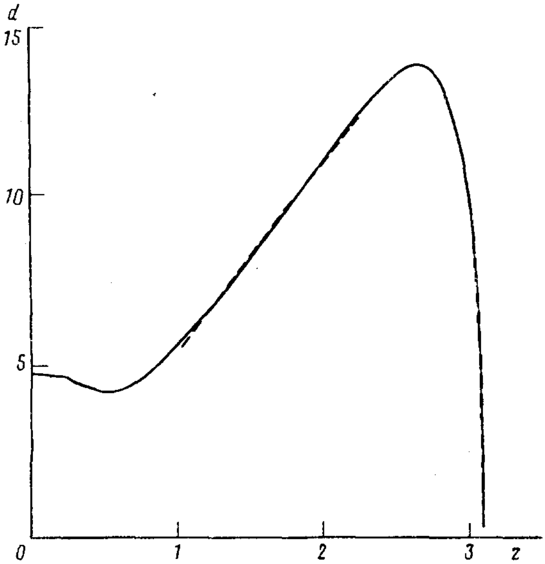

Fig.3.

Fig.2. The level lines of the amplitude for the reduced system on $(t, z)-$ plane. One can see that a pulse is almost reduced for $z > 3.9$. Initial conditions are the same as for Fig.1

Fig.3. The dependence of pulsewidth d on distance z due to the collapse mechanism at $C_1 = 6, C_2 = 15$. One can distinguish two regimes (dashed lines): linear pulse growth when pulsewidth $d = 0.0034 + 5.45z$ and then collapse regime, $d = 26.69(3.09 - z)^{0.3885}$

For the mode-locking laser systems this mechanism of stabilization may be described as follows. At the first step, nonlinear action of the saturable absorber dominates, and a pulse approaches collapse, i.e. its amplitude is growing up with increase of number of passes z through laser system. Sharp increase of the pulse amplitude leads to the generation of a large phase-gradient located at the pulse boundaries that results there in a decrease of the amplitude growth with z and finally stopping of the collapse. At the final stage the pulse will be 'eaten' almost completely. As a result, the output signal will be represented as a pulse train of finite length. At the beginning of this train there will be the initial pulse, then the pulse duration will decrease and simultaneously its amplitude will increase. For pulses of later generations, near the end of this train, the pulse amplitude will at first reach its maximum and will then sharply will decrease with simultaneous vanishing of the pulse width. The total energy of the last pulses as well as its intensity will vanish which corresponds to the complete pulse reducing. Each pulse in this train will have the frequency chirp $\partial\Omega/\partial t$, its sign will be opposite to the sign of the Kerr constant C_2. In a case of the long amplifier the output pulse, in dependence of the amplifier size Z, will correspond to a pulse of the $Z-$th generation in a case of mode-locking lasers.

5. In conclusion, we have studied analytically and numerically the model describing a pulse generation in the complex GL equation describing mode-locking laser systems and other active gain media. It has been shown that the phase-gradient mechanism can be used to compress pulses in such systems. We have demonstrated that collapse takes place in defocusing gain medium and is stopped in a focusing amplifying medium in sufficiently larger region of parameters than in a defocusing case. In any regime the pulse has a chirp and its sign correlates with the sign of the constant C_2.

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