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PHONON CONTRIBUTION TO THE NOISE OF ONE-CHANNEL LANDAUER RESISTOR

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We work out a theory of shot noise in semiconductor nanowires under the condition of phonon-assisted quasiballistic transport. A general expression for the noise spectral density is derived and used to investigate particular cases of interest. For low temperatures, a remarkable threshold effect for the noise is predicted.

Recently a general relation between the shot noise spectral density P and the transmission matrix of the mesoscopic conductor has been derived for elastic electron scattering [1 – 3]. For a one-channel case the shot noise is $P = 2e|V|(e^2/h)T_0(1-T_0)$, where T_0 is the channel transmission. The shot noise is finite if and only if the transmission coefficient is neither 0 nor 1. The shot noise due to the inelastic electron-phonon scattering has been pointed out by Kulik et al.[4] for a classical 3D ballistic point contact. The purpose of the present paper is to study effects of electron-phonon scattering on the noise of a Landauer resistor where the resistance is strongly quantized.

We consider a wire of length L along x-axis. In the spirit of the Landauer approach we assume the wire to be connected with the reservoirs which we call "left" (L) and "right" (R), each of them being in equilibrium with itself. If the wire is long enough the electron motion along x axis may be treated classically and one can use a semiclassical kinetic theory to treat the electron transport in this direction (see also Ref.[5]). The introduction of the phonons into this picture can be done along the lines worked out in Refs. [6, 7].

Using an approach described in Refs.[8, 9], we will study the time evolution equation for $< \delta F \delta F >$ along with the equation for $\bar{F}$, which both have the form of quasiclassical Boltzmann equations. We recall an equation for the average distribution function $\bar{F}(x,p,t)$:

$$\partial\bar{F}(x,p,t)/\partial t + v\,\partial\bar{F}(x,p,t)/\partial x = I\{\bar{F}(x,p,t)\}, \tag{1}$$

where p and v are the x-components of the electron quasimomentum and velocity. $I\{\bar{F}(x,p,t)\}$ is the electron-phonon collision integral, which, as usual, is a difference of "in" and "out" terms:

$$I^{(\mathrm{out})}\{F(p,x)\} = F(p,x)\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}[1-F(p',x)]\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^d}\left|\langle 0|\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)|0\rangle\right|^2 W_\mathbf{q}\times$$
$$\times\left[N_\mathbf{q}\delta(\epsilon'-\epsilon-\hbar\omega_\mathbf{q}) + (N_\mathbf{q}+1)\delta(\epsilon'-\epsilon+\hbar\omega_\mathbf{q})\right], \tag{2}$$
$$I^{(\mathrm{in})}\{F(p,x)\} = [1-F(p,x)]\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}F(p',x)\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^d}\left|\langle 0|\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)|0\rangle\right|^2 W_\mathbf{q}\times$$
$$\times\left[(N_\mathbf{q}+1)\delta(\epsilon'-\epsilon-\hbar\omega_\mathbf{q}) + N_\mathbf{q}\delta(\epsilon'-\epsilon+\hbar\omega_\mathbf{q})\right], \tag{3}$$

where $|0>$ is the ground state wave function of transverse quantization, and $\epsilon = p^2/2m$, and $\epsilon' = p'^2/2m$. In the isotropic approximation for the scattering by acoustical phonons $W_\mathbf{q} = (\pi\Lambda^2 q^2/\rho\omega_\mathbf{q})$, where Λ is the deformation potential constant for the longitudinal phonons, and ρ is the mass density. We integrate in Eqs. (2), (3) over the three components of the phonon wave vector. $\mathbf{q}_\perp$ indicates the two transverse wave vector components. The third component is given by $q_x = \pm(p-p')/\hbar$. Therefore the third integration is equivalent to the integration over the electron quasimomentum, p', because of the conservation of quasimomentum.

The boundary conditions for Eq. (1) are:

$$\bar{F}(p>0, x=-L/2) = f_L(p) \equiv \frac{1}{\exp[(\varepsilon_p - \mu_L)/k_B T]+1}, \tag{4}$$

while for $p<0$ we have the same condition with the replacements $(-L/2)\to(L/2)$, $f_L \to f_R$, and $\mu_L \to \mu_R$. Here T is the temperature, μ_L and μ_R are the chemical potentials of the reservoirs, and $eV = \mu_L - \mu_R$ is the voltage bias across the conductor.

In the absence of collisions and under stationary conditions the solution of Eq.(1) is $\bar{F}^{(0)}(x,p) = \theta(p)f_L(p) + \theta(-p)f_R(p)$, where $\theta(p)$ is the step function. This solution corresponds to a purely ballistic motion. Adding a weak electron-phonon interaction, we have $\bar{F} = \bar{F}^{(0)} + \overline{\Delta F}$ with $\overline{\Delta F}$ satisfying the first iteration of the transport equation $v\partial\overline{\Delta F}/\partial x = I\{\bar{F}^{(0)}\}$. Taking into account the boundary conditions, Eq.(4), and assuming the zero of the coordinate system at the midpoint of the wire we arrive at the solution of this equation in the form [6]:

$$\overline{\Delta F(x,p)} = (1/|v|)[x + (L/2)\,\mathrm{sign}\,p]I\{\bar{F}^{(0)}(p)\}. \tag{5}$$

We assume that the phonons are in equilibrium and hence $N_\mathbf{q}$ is the Bose function. Detailed balance guarantees a vanishing collision term for the equilibrium distribution function and constant temperature and chemical potential. This means that the distribution function $F^{(0)}$ gives finite contribution to the collision term if and only if p and p' are of opposite sign, so that their chemical potentials are different. In other words, only those phonons contribute that can *backscatter* the electrons — see Ref.[6].

This gives the *averaged* electron distribution function along the wire in the presense of a weak inelastic scattering. As for the *fluctuating part* of the distribution function, δF, according to Ref.[8] the correlation function $< \delta F(x', p', t')\delta F(x, p, t) >$ satisfies for $t' > t$ the Boltzmann equation, Eq.(1), in the first set of variables with the initial condition

$$< \delta F(x', p', t)\delta F(x, p, t) >= h\delta(x - x')\delta(p - p')\bar{F}(x, p, t)[1 - \bar{F}(x, p, t)]. \qquad (6)$$

The current through a cross section of the wire, which we choose to be near the right reservoir, is:

$$J(t) = \frac{2e}{h}\int_{-\infty}^{\infty} dp\, v\, F(x = L/2, p, t). \qquad (7)$$

The current has the average value, $\bar{J}$, and the fluctuating part, $\delta J(t) = J(t) - \bar{J}$. The current noise spectral density in the limit of zero frequency is given by:

$$P = 4\int_0^{\infty} dt < \delta J(t)\delta J(0) >= 4\left(\frac{e}{h}\right)^2 \int_0^{\infty} dt \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dp'\, vv'\, \chi, \qquad (8)$$

where $\chi =< \delta F(L/2, p', t)\delta F(L/2, p, 0) >$. Here we have made use of the conservation of the current which permits one to choose any cross section to calculate the current fluctuations for $\omega \to 0$.

Electrons with $p > 0$ reach right reservoir without further scattering. Therefore, χ contains only terms proportional to $\delta(t)$:

$$\chi(p > 0) = \frac{h}{|v'|}\delta(t)\delta(p' - p)\bar{F}(L/2, p)[1 - \bar{F}(L/2, p)], \qquad (9)$$

where $\bar{F}(L/2, p)$ is given by a sum of $F^{(0)}$ and Eq. (5), taken at $x = L/2$ and $p > 0$:

$$\bar{F}(L/2, p > 0) = f_L(p) + \frac{L}{|v|}I^{(\text{in})}\{f_L(p)\} - \frac{L}{|v|}I^{(\text{out})}\{f_L(p)\}. \qquad (10)$$

Electrons with $p < 0$ can be, in fact, backscattered within the wire and cross the cross-section $x = L/2$ again. In order to take into account all backscattering trajectories we, at first, divide the wire into small pieces $[x_1, x_2]$, $[x_2, x_3]$, ... $[x_i, x_{i+1}]$,... Then we introduce the probability for the electron from the right reservoir to be backscattered per unit of time, $\mathcal{U}(p < 0 \to p_k > 0)$ and sum over contributions from all the pieces. As a result, we obtain:

$$\begin{aligned}\chi(p < 0) \;\; = \;\; & \frac{h}{|v'|}\left[\delta(t)\delta(p' - p) + \sum_{ik} h\frac{\Delta x_i}{|v|}\mathcal{U}(p < 0 \to p_k > 0)\delta(t - t_i)\delta(p' - p_k)\right] \times \\ \times \;\; & \bar{F}(L/2, p)[1 - \bar{F}(L/2, p)]. \qquad (11)\end{aligned}$$

Here p_k is the final electron state after backscattering, t_i is the time spent in the wire, and $\Delta x_i \equiv x_{i+1} - x_i$. Distribution function $\bar{F}(L/2, p)$ for electrons with $p < 0$ is exactly $f_R(p)$, as it is seen from Eq. (5).

Substitution of Eqs. (9) and (11) into the expression for the shot noise spectral density, Eq. (8), and integration over t and p' finally gives the shot noise power:

$$P = 4\left(\frac{e}{h}\right)^2 \int_{-\infty}^{\infty} dp\, |v| \left\{\theta(p)\,\frac{1}{2}\,\left(f_L(p) + \frac{L}{|v|}I^{(\text{in})}\{f_L(p)\} - \frac{L}{|v|}I^{(\text{out})}\{f_L(p)\}\right) \times\right.$$

$$\times \left(1 - f_L(p) - \frac{L}{|v|} I^{(\mathrm{in})}\{f_L(p)\} + \frac{L}{|v|} I^{(\mathrm{out})}\{f_L(p)\}\right) +$$

$$+ \theta(-p) \left[\frac{1}{2} - \frac{L}{|v|} \frac{\delta I^{(\mathrm{out})}\{f_R(p)\}}{\delta f_R(p)}\right] f_R(p)[1 - f_R(p)]\Bigg\}. \tag{12}$$

Here $\delta I^{(\mathrm{out})}\{f_R(p)\}/\delta f_R(p) = \sum_{p'} \mathcal{U}(p < 0 \rightarrow p' > 0)$ is the variational derivative of the collisional integral. Eq.(12) should be understood in the following way. It contains terms of the zeroth, first and second order in the small parameter $(L/|v|)I^{(\mathrm{in,out})}\{f_{L,R}(p)\}$. In general, it is not always permissible to retain the terms of the second order. However, we have not discarded them because they are meaningful in some cases (see below).

This formula allows us to study various cases of interest. In particular, we will be interested in 2 regimes: $eV \gg k_B T$, and $eV \ll k_B T$.

Let us first assume the voltage across the conductor, $eV = \mu_L - \mu_R$, to be finite, while $T = 0$. Then $N_q = 0$ and only processes of phonon emission are possible. The distribution functions for electrons in the leads are the step functions: $f_{L,R}(E) = \theta(\mu_{L,R} - E)$. This leads to a substantial simplification of the problem. In particular, $I^{(\mathrm{in})}\{f_L(p)\}$ and $\delta I^{(\mathrm{out})}\{f_R(p)\}/\delta f_R(p)$ become zero, while $(L/|v|)I^{(\mathrm{out})}\{f_L(p > 0)\} = f_L(p)\mathcal{R}(\varepsilon_p)$, where:

$$\mathcal{R}(\varepsilon_p) =$$

$$= \frac{L}{|v|} \int_{-\infty}^{0} \frac{dp'}{2\pi\hbar} \int \frac{d^2 q_\perp}{(2\pi)^d} \theta([\varepsilon_p - \mu_R] - \hbar\omega_q) |\langle 0| \exp(i q_\perp \cdot r_\perp)|0\rangle|^2 W_q \delta(\varepsilon_p - \hbar\omega_q - \varepsilon_{p'}). \tag{13}$$

Here we introduced the effective "inelastic reflection" coefficient $\mathcal{R}(\varepsilon_p)$, which describes efficiency of the electron backscattering in the course of phonon emission.

For the shot noise spectral density we have now, making use of Eq.(12):

$$P = \frac{2e^2}{h} \int_{\mu_R}^{\mu_R + e|V|} dE\, \mathcal{R}(E)[1 - \mathcal{R}(E)]. \tag{14}$$

Therefore, the shot noise power is determined by the energy dependence of "inelastic reflection" coefficient. We wish to emphasize that here we retain the terms quadratic in the reflection coefficient (along with terms linear in $\mathcal{R}$). At the same time we do not take into account the quadratic terms while calculating the corrections due to the electron scattering to the distribution function itself (see Eq.(5)). This is permissible, because in the case under discussion ($eV \gg k_B T$) the latter are of *the third* order in $\mathcal{R}$. Indeed, the second-order corrections to ΔF could appear after taking into account the second scattering event for an electron which has already experienced a scattering. However, the probability of this event is proportional to the number of empty states $(1 - F(p))$, which in its turn is proportional to $\mathcal{R}$.

We consider electron backscattering. Therefore, there should be some minimal wave vector for acoustic phonon to be emitted. Namely, $q_{min} = 2p_F/\hbar$, where p_F is the Fermi momentum. This leads to a *threshold* in the "inelastic reflection" coefficient energy dependence, and, furthermore, in $P(V)$.

Indeed, $\mathcal{R}(E) = 0$ for $E < 2p_F s \equiv E_{th}$, where we introduce the notation $E = \varepsilon_p - \mu_R$. One can show that for energies near the threshold, when $(E - E_{th})/E_{th} \ll 1$, in the first approximation in this small parameter the coefficient of inelastic reflection is:

$$\mathcal{R}(E) = \frac{E - E_{th}}{E_{th}} R_0, \tag{15}$$

where

$$R_0 = \frac{L}{|v_F|} \frac{mW_{2p_F/\hbar} p_F}{\hbar^4 \pi^2}. \tag{16}$$

Here we have taken into account that in real structures $E_{th} = 2p_F s$ is of the same order as $\hbar s/d$. The shot noise power in this limit is $P(V) = (e^2/h) R_0 [(e|V| - -E_{th})^2/E_{th}]$.

Well above the threshold, when $E/E_{th} \gg 1$, the matrix element $< 0|\exp(i\mathbf{q}_\perp \mathbf{r}_\perp)|0 > \propto (q_\perp d)^{-2}$, where d is the thickness of the nanowire. Then $R(E) = R_0$, while the shot noise power becomes proportional to the applied voltage

$$P(V) = \frac{2e^2}{h} e|V| R_0. \tag{17}$$

One sees, that the voltage dependence of the nonequilibrium noise differs from a simple linear law, which is due to the inelastic electron backscattering within the wire.

It is worthwhile to mention the case of inelastic optical phonon emission. Optical phonon is a simplest example of phonon mode with non-vanishing minimal frequency. We denote the optical phonon energy $\hbar\omega_0$ and assume no dispersion at all. Under this condition the contribution of optical phonon emission processes to $\mathcal{R}$ vanishes for electron energies smaller than $\hbar\omega_0$, while in the vicinity of $\hbar\omega_0$ the reflection coefficient exhibits a jump. Therefore, the shot noise, $P(V)$, has a bend when V crosses $\hbar\omega_0/e$, which can be observed in experiment.

For small applied voltage $eV = \mu_L - \mu_R \ll k_B T$, expansion of Eq. (12) in powers of eV gives a voltage-dependent correction, $P_1(V)$, to the thermal noise:

$$P_1(V) = \frac{1}{3} \frac{(eV)^2}{k_B T} \frac{e^2}{h} R_0, \tag{18}$$

see Eq. (16).

To summarize, we have developed a theory of the shot noise in quantum quasiballistic channels under the condition of phonon-assisted transport. A formula for the shot noise caused by weak electron-phonon scattering in otherwise purely ballistic quantum channels is derived. It is used to work out expressions for some particular cases of interest. We have studied the cases of small, $k_B T \ll eV$, and large, $k_B T \gg eV$, temperatures. For small temperatures a remarkable threshold effect for the shot noise is predicted.

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НОВЫЕ ИЗМЕРЕНИЯ МАССЫ ИЗОТОПА ^{4}H В РЕАКЦИЯХ С РАДИОАКТИВНЫМ ПУЧКОМ ^{6}He И ИОНАМИ ^{6}Li.

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Впервые измерены сечения реакции D(^{6}He,α) при энергии $E_{^6\mathrm{He}}$= 19,3 МэВ. В спектре α-частиц обнаружены два резонанса, которые соответствуют основному и возбужденному состояниям ядернонестабильного изотопа ^{4}H, лежащие выше порога диссоциации $^4\mathrm{H}\to t+n$ на 2,0 $\pm$ 0,3МэВ и 5,2 $\pm$ 0,5МэВ. Сделано предположение, что возбужденное состояние ^{4}H имеет конфигурацию $(d+{}^2n)$ и квантовые характеристики 1^+. Факт ядерной неустойчивости ^{4}H (2,3 $\pm$ 0,3МэВ) подтвержден измерениями реакции ^{6}Li(^{6}Li,^{8}B) при $E_{^6\mathrm{Li}}$= 85 и 93МэВ. В результате получено новое значение дефекта массы ядра ^{4}H, равное 25,3 $\pm$ 0,3МэВ.

Более четверти века назад был установлен факт существования ядернонестабильного изотопа ^{4}H, являющегося самым легким нейтронноизбыточным ядром, неустойчивым относительно распада на $t+n$. Тем не менее, "мера" его нестабильности до сих пор не определена однозначно. В работе [1] показано, что ^{4}H недосвязан на 2,3–2,6 МэВ, тогда как в последующих работах [2–7] утверждается, что распад $^4\mathrm{H}\to t+n$ идет с выделением энергии 3,1–3,8МэВ. Общеприняты в настоящее время данные, полученные из фазового анализа упругого рассеяния нейтронов на тритии [8], которые объясняют поляризацию и сечения взаимодействия $n+T$ наличием широких р-одночастичных резонансов, лежащих при энергиях 3,4 МэВ (2^-) и 5,1 МэВ (1^-) над порогом диссоциации $^4\mathrm{H}\to t+n$. Однако авторы работы [9] после тщательно проведенного исследования функции возбуждения $n+T$ и $p+{}^3$He показали, что