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SHOT NOISE IN THE MESOSCOPIC DIFFUSIVE CONDUCTORS IN THE ELECTRON TEMPERATURE LIMIT

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The shot noise power of a diffusive conductor in a presence of strong inelastic electron-electron scattering is calculated. It is shown, that electron-electron inelastic scattering does not suppress shot noise strongly and the noise power is $(\sqrt{3}/2)e|\bar{I}|$.

Electric current noise in small devices has attracted recently a substantial attention. Contribution to this noise proportional to the current itself and persisting down to zero temperatures is known as a shot noise. In the absence of correlations between the carriers, as it is, for instance, for classical tunnel junction, the noise power has a form analogous to the Poisson process ("full shot noise") $P_{tun} = 2e|V|G = 2e\bar{I}$, where $\bar{I} = GV$ is the average current and G the conductance. All correlations reduce the shot noise, so that P remains below the full shot noise level. In perfect ballistic metallic systems the shot noise is completely suppressed due to Fermi correlation [1]. This clearly does not hold for not-perfectly transmitting systems, for which general expression for the shot noise power (current-noise spectral density) P in the case of full quantum coherence [2] was recently derived. In particular, the shot noise power in a disordered diffusive conductor with purely elastic electron scattering was shown [3] to be reduced with respect to the "full shot noise" level with the reduction factor $\gamma = 1/3$. All these papers mainly dealt with the situation of *elastic* electron scattering and it is generally believed that all *inelastic* scattering leads to strong suppression of the shot noise.

In the present work we show that electron-electron inelastic scattering, even being very effective, does not suppress shot noise strongly and leads only to some change in its magnitude. As we will show, the results obtained are in excellent agreement with experiment [4], where a shot noise of 2D diffusive mesoscopic conductor in the limit of strong electron-electron interaction was studied.

Let us consider a diffusive conductor with a length L. In order to find an expression for the shot noise in this system we will use the semiclassical kinetic theory due to Gantsevich, Gurevich and Katilius [5], which allows to express the correlation function of the fluctuating distribution function $F(\mathrm{r}, \mathrm{p}, t)$ in terms of the average over the temporal fluctuations $\langle F \rangle \equiv \bar{F}(\mathrm{r}, \mathrm{p}, t)$. This average is the solution of the Boltzmann equation, which we, for simplicity, take in 1D form assuming a homogeneity in the other two directions:

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{F}(x, \mathrm{p}, t) = -\hat{I}^{col}_{\mathrm{p}}\{\bar{F}(x, \mathrm{p}, t)\}, \tag{1}$$

where the derivative $(\mathrm{d}/\mathrm{d}t) \equiv (\partial/\partial t) + v_x(\partial/\partial x) + e\mathrm{E}(\partial/\partial \mathrm{p})$ accounts for semiclassical motion of an electron with momentum $\mathrm{p} = mv$ in the electric field E. $\hat{I}^{col}_{\mathrm{p}} =$

$\hat{I}^{im}_{\mathrm{p}} + \hat{I}^{ep}_{\mathrm{p}} + \hat{I}^{ee}_{\mathrm{p}}$ is the collisional operator, which includes terms responsible for impurity scattering (im), electron-phonon (ep), and electron-electron (ee) interaction. We assume stationary Fermi distribution f_F in the leads of the conductor as the boundary conditions for Eq. (1):

$$\bar{F}(\pm(L/2), \mathrm{p}, t) = f_F(\varepsilon_{\mathrm{p}} - \mu_\pm, T_{bath}), \;\; \mu_\pm = \mu_0 \pm \frac{eV}{2}, \tag{2}$$

where μ_0 is the chemical potential of the electron gas in the absence of the bias, T_{bath} is the thermal bath temperature. Note that the boundary conditions of such form imply that the channel length L is much large than the channel width.

It is a rather hard task to solve Eq. (1). In order to simplify the problem, we consider the situation, when electron-electron scattering is effective enough to make description in terms of *electron temperature* possible. Also we assume that diffusion of particle is governed by impurities scattering mainly, which is the case if effective time of electron-impurity collisions τ_p is much smaller than that of electron-electron ones τ_{ee}. In this approximation even with respect to $\mathrm{p} \to -\mathrm{p}$ transformation part of the electron distribution has the Fermi form along the whole sample:

$$\bar{F}(x, \varepsilon_{\mathrm{p}}) = f_F(\varepsilon_{\mathrm{p}} - \mu(x), T_{el}(x)) \tag{3}$$

with chemical potential $\mu(x)$ and temperature $T_{el}(x)$, which depend on coordinate and on the voltage applied, while odd with respect to this transformation part is considered as small. The task reduces in this case to calculation of electron temperature profile $T_{el}(x)$ along the sample. The most direct way to do this is to derive the equation for the total energy of quasiparticle gas, which is uniquely related to T_{el}.

Let us first neglect electron-phonon interaction. Making use of the relaxation time approximation for the impurity scattering $\hat{I}^{im}\{F\} = F/\tau_p$, we can obtain the equation for $\bar{F}(x, \varepsilon_p)$. Then we multiply all terms in this equation by quasiparticle energy $\varepsilon_p - \mu$ and sum over all ε_p. The final equation for the obtained this way total energy of quasiparticle gas takes the form:

$$D\frac{\partial^2}{\partial x^2}\sum_{\varepsilon_p}(\varepsilon_p - \mu)\bar{F}(x, \varepsilon_p) + (eE)^2 D\sum_{\varepsilon_p}(\varepsilon_p - \mu)\frac{\partial^2 \bar{F}(x, \varepsilon_p)}{\partial \varepsilon_p^2} = 0, \tag{4}$$

where $D = v_F^2\tau_p/3$ is a diffusion coefficient. Both terms of Eq. (4) have a straightforward physical meaning. First one is a diffusion term, while the second is a Joule source of heat. Deriving Eq. (4) we have taken into account that electron-electron collisions conserve the total energy of quasiparticle gas and thus $\sum_{\varepsilon_p}(\varepsilon_p - \mu)\hat{I}^{ee}_{\mathrm{p}}\bar{F}(x, \varepsilon_p) = 0$. Summation in Eq. (4) results in the equation for $T_{el}(x)$:

$$\frac{\pi^2}{6}\frac{\partial^2}{\partial x^2}\left[T_{el}(x)\right]^2 + (eE)^2 = 0. \tag{5}$$

Making use of the boundary conditions (compare with Eq. (2)) $T_{el}(x = \pm L/2) = T_{bath}$ and $E = V/L$ we finally obtain the expression, which describes the electron temperature profile along the sample. In the most interesting case $eV \gg T_{bath}$ this takes the form:

$$T_{el}(x) = \frac{\sqrt{3}}{2\pi}e|V|\sqrt{1 - \left(\frac{2x}{L}\right)^2}. \tag{6}$$

At this point we have found the explicit form of the nonequilibrium distribution function. Now we will calculate the shot noise power in the conductor in a way similar to that used by Nagaev [3]. Current density in the point r of the conductor is $\mathbf{j}(\mathbf{r},t) = (e/\Omega_0)\sum_{\mathbf{p}} \mathbf{v}F(\mathbf{r},\mathbf{p},t)$, where $F(\mathbf{r},\mathbf{p},t)$ is a non-averaged over temporal fluctuations electron distribution function, $\Omega_0 = SL$ total volume of the sample (S is the sample cross-section). Total current through the conductor is $I(t) = (S/\Omega_0)\int d^3\mathbf{r}\, j_x(\mathbf{r},t)$. Therefore, zero-frequency shot noise power equals to

$$\begin{aligned} P &= 4\int_0^\infty dt\ \langle \delta I(t)\,\delta I(0)\rangle = \\ &= 4\frac{e^2}{S^2L^4}\int d^3\mathbf{r}\,d^3\mathbf{r}'\sum_{\mathbf{p}\mathbf{p}'} v_x v_x' \int_0^\infty dt\ \langle \delta F(\mathbf{r},\mathbf{p},t)\,\delta F(\mathbf{r}',\mathbf{p}',0)\rangle\,, \end{aligned} \tag{7}$$

where $\delta F = F - \bar{F}$. According to the kinetic theory [5] and within the electron temperature approximation we use throughout this paper

$$\int_0^\infty dt\ \langle \delta F(\mathbf{r},\mathbf{p},t)\,\delta F(\mathbf{r}',\mathbf{p}',0)\rangle = \Omega_0\,\delta(\mathbf{r}-\mathbf{r}')\,\tau_p \bar{F}(\mathbf{r},\mathbf{p})\,[1-\bar{F}(\mathbf{r}',\mathbf{p}')]\,\delta_{\mathbf{p}\mathbf{p}'}. \tag{8}$$

After integration in Eq. (7) with account of Eq. (3) we arrive at the result:

$$P = \frac{4G}{L}\int_{-L/2}^{L/2} dx \int_0^\infty d\varepsilon_p\ \bar{F}(x,\varepsilon_p)\,[1-\bar{F}(x,\varepsilon_p)] = \frac{4G}{L}\int_{-L/2}^{L/2} dx T_{el}(x). \tag{9}$$

Here $G = e^2 D(\nu_F/\Omega_0)(S/L)$ is the total conductance, ν_F density of electron states at the Fermi level. Taking into account the obtained earlier profile of $T_{el}(x)$, Eq. (6), we arrive at the final expression for the shot noise power in the diffusive conductor in the presence of strong electron-electron scattering:

$$P_{ee} = \frac{\sqrt{3}}{2}\,e|V|\,G. \tag{10}$$

The obtained value is only $4/\sqrt{3}$ times smaller than the "full shot noise" level P_{tun}, which corresponds to the reduction factor $\gamma = 0.43$. Therefore, electron-electron inelastic scattering, even being very effective, does not suppress shot noise strongly and lead only to not too large change in its magnitude. This result is in excellent agreement with recent experiment by Liefrink et al., Ref. [4], where $\gamma \approx 0.45$ was found in the limit of weak electron-phonon scattering.

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