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METAL-INSULATOR TRANSITION IN A TWO-DIMENSIONAL HOLE GAS OF Si/SiGe HETEROSTRUCTURES. QUANTUM OR CLASSICAL PERCOLATION?

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We study current-voltage characteristics in the magnetic-field-induced insulating state of a 2D hole gas in low-mobility Si/SiGe heterostructures. The observed behaviour of $I-V$ curves is explained by the breakdown of the insulating phase in the classical percolation model. Analysis of all experimental results obtained on different 2D systems shows that none of them is in obvious contradiction with classical percolation.

The metal-insulator transition is an important topic in the physics of 2D systems subjected to a magnetic field. In the 2D case, dependent on the value of conductivity σ_{xy}, one can distinguish two types of insulating phase. (i) The Fermi level lies in localised Landau-states and extended states below the Fermi level are available (as temperature tends to zero, $\sigma_{xx} \to 0$ and σ_{xy} is quantized). Traditionally this is called the quantum-Hall-effect phase. (ii) The Fermi level is in the lowest band of localised states (both σ_{xx} and σ_{xy} vanish with decreasing temperature). These two might, in principle, be of different origin since the ground state of an ideal 2D electron system in the extreme quantum limit is expected to be a Wigner crystal.

Intensive studies of the scaling behaviour of the peaks in resistivity ρ_{xx} on relatively low-mobility heterostructures [1-4] created the impression that it is possible to determine the critical exponent s for the localisation length L near the metal-insulator transition in the quantum-Hall-effect phase. The unexpected result $s \approx 2.3$ (the classical percolation model predicts $s = 4/3$ [5]) caused the appearance of a number of theoretical works taking account of quantum tunneling and interference effects in the percolation picture. Among these, some numerical investigations [6-9] were performed for microscopic samples which are several orders of magnitude smaller than those used in experimental studies. Therefore, despite the coincidence between the calculated and experimental values of critical exponent, their relationship would be tenuous if it were not for the analytical results [10-12] substantiating the quantum percolation argument.

Recently, an approach has been formulated for determining the localisation length in a 2D electron system, based on the analysis of nonlinear current-voltage characteristics in an insulating phase [13-15]. The experimental results for a high-mobility 2D electron gas in both Si MOSFET's and AlGaAs/GaAs heterostructures are interpreted within the classical percolation model, in contrast to the conclusions of previous publications. The properties of all insulating phases, including the zero-Hall-conductivity one, have been found to be very similar. The authors also argue that the scaling behaviour is explained by the thermal broadening of conductivity

peaks. In recent investigations of the metal-insulator transition in high-mobility Si MOSFET's [16], it has been found that the set of temperature dependences of the longitudinal resistance near the phase boundary in both the metallic and insulating phase can be scaled into one curve by using a single parameter $T_0 = e^2/\varepsilon L$. The localisation length L proves to diverge with the critical index $s \approx 1.6$, which is close to that expected for classical percolation. Thus, the results obtained by these alternative techniques are in complete disagreement with the interpretation of scaling measurements. This discrepancy might be due to the different quality of samples used in different experiments.

Here we employ a method of current-voltage characterisation to investigate the metal-insulator transition in the insulating phase with $\sigma_{xy} = 0$ of low-mobility Si/SiGe heterostructures. We find that also this transition is described in terms of the model of classical percolation which seems adequate for the currently available 2D layers in MOSFET's and heterostructures.

The samples used are grown by solid source molecular beam epitaxy: detailed information on the growth method is given elsewhere [17]. The 2D holes are confined in an approximately triangular potential well at the surface of a $Si_{0.8}Ge_{0.2}$ layer and have mobility ~ 2000 cm^2/Vs at $T = 4.2$ K and density 2.5×10^{11} cm^{-2}. Both the size quantisation and the strain present in the SiGe layer are responsible for the splitting of light- and heavy-hole bands. The 2D hole gas (2DHG) originates from heavy holes with angular momentum projection $m_J = \pm 3/2$. The sample geometry is a standard Hall bar with dimensions 4.7×1.0 mm^2; the distance between potential probes is equal to 1.7 mm. At liquid helium temperatures there is no sign of parallel conduction in the samples. The experiments are performed in a ^{3}He/^{4}He dilution refrigerator with a base temperature ≈ 25 mK. We employ a four-terminal dc technique with a Keithley 617 electrometer as high-input-resistance amplifier. Currents through the sample do not exceed 1 nA; in this range hole heating effects are found to be negligible.

The magnetoresistance ρ_{xx} shows Shubnikov – de Haas oscillations with periodic minima in reciprocal magnetic field. Following the standard procedure, we determine the values of both the filling factor corresponding to each oscillation minimum and the hole density. The oscillation numbers turn out to be odd, which can be interpreted as the coincidence of the cyclotron energy and spin splitting. As compared to conventional quantum oscillations, there is another distinction in the oscillations observed here: an enormous peak in the magnetoresistance between the filling factors 1 and 3. At minimum temperatures the typical values of ρ_{xx} in this region of magnetic field far exceed the Hall resistance ρ_{xy}, so that both the conductivities $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$ and $\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2)$ tend to zero with decreasing temperature. This means that in the vicinity of the ρ_{xx} peak the 2DHG is in an insulating phase; i.e., with increasing magnetic field there occurs a metal-insulator-metal transition. This is very similar to the re-entrant behaviour of the metallic and insulating phases observed in Si MOSFET's and AlGaAs/GaAs heterostructures.

In a tilted magnetic field, the peak in magnetoresistance increases strongly, which implies that the metal-insulator phase boundary is pushed to higher hole densities. As a result, in this case one can move more deeply into the insulating phase when sweeping magnetic field. In the present paper, the measurements are carried out at the angle $\Theta = 60°$ between the direction of magnetic field and the normal to the interface.

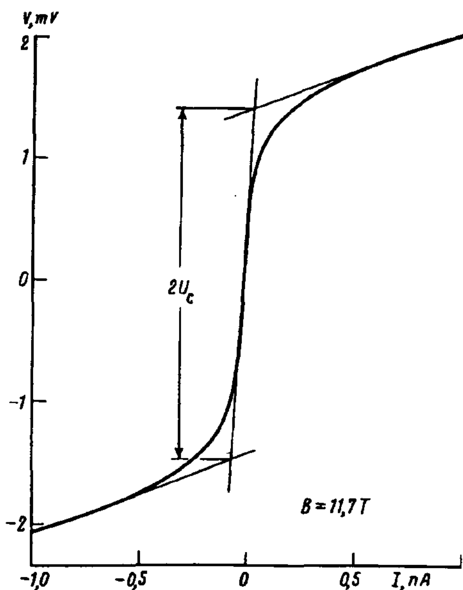

Fig.1

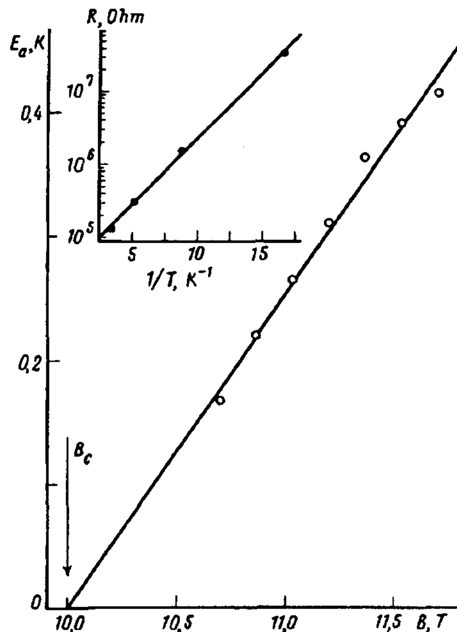

Fig.2

Fig.1. Typical current-voltage curve in an insulating phase. Temperature $T = 60$ mK is chosen to make observable the linear interval of $I - V$ characteristics. Procedure for determining the critical voltage is shown

Fig.2. Dependence of the activation energy on magnetic field. Metal-insulator transition point is indicated by the arrow. Inset: Arrhenius plot of the resistance in the linear interval of $I - V$ characteristics

In the insulating phase we observe strongly nonlinear current-voltage characteristics (Fig.1) while in the metallic phase these are linear, at least for the range of currents used. Two quantities determine the shape of the $I - V$ characteristics: the critical voltage, U_c, corresponding to the onset of nonlinearities and the resistance in the linear regime. We establish the type of metal-insulator transition by studying the behaviour of $I - V$ curves in the insulating phase near the metal-insulator phase boundary.

The resistance in the linear interval of $I - V$ curves shows an activated temperature dependence. The activation energy, E_a, is determined from an Arrhenius plot (inset in Fig.2). Similar to the case of high-mobility Si MOSFET's, the value of the pre-exponential factor is close to 100 kΩ. As seen from Fig.2, the activation energy changes linearly with magnetic field. The slope of this linear dependence is inversely proportional to the density of states at a metal-insulator transition point B_c which can be found by extrapolating the straight line to zero activation energy (Fig.2).

The critical voltage proves to be roughly a parabolic function of magnetic field (Fig.3). This dependence runs to zero at the same magnetic field B_c within experimental uncertainty. Thus, in a similar way to high-mobility Si MOSFET's and AlGaAs/GaAs heterostructures, both the nonlinearity of $I - V$ characteristics

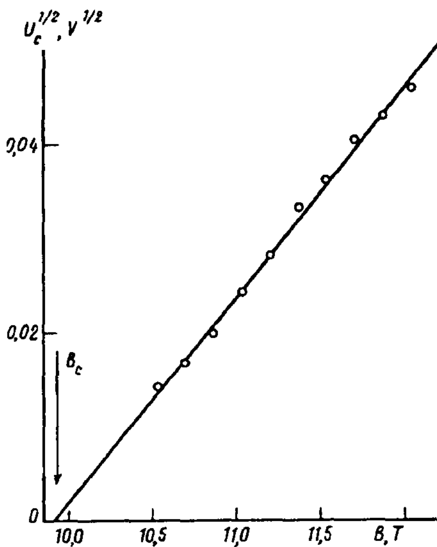

Fig.3. Square root of the critical voltage as a function of magnetic field at $T \approx 25$ mK. The value of magnetic field B_c corresponding to metal-insulator transition coincides with that in Fig.2 within experimental uncertainty

and the activated behaviour of resistance in the linear regime disappear at the same value of magnetic field, which is considered to be a point of metal-insulator transition.

These results are now explained in the framework of classical percolation. In this picture, metallic clusters in a 2DHG are separated at saddle points by potential barriers with height equal to an activation energy E_a that is defined as the difference between the percolation threshold E_c and the Fermi level. In sufficiently small electric field F, the clusters are equipotential regions and so the barrier height decreases by eLF, where L is the cluster dimension (similar ideas for doped semiconductors were developed in Ref. [18]). If the energy eLF becomes equal to the activation energy (i.e., the barrier height vanishes) localisation in the 2D system breaks down and the conductivity grows abruptly, leading to nonlinear $I-V$ curves [13−15]. Since near the threshold E_c the cluster dimension diverges as $L(E) \propto \mid E - E_c \mid^{-s}$, the critical electric field for the breakdown is written $F_c \propto E_a^{s+1} \propto \mid B - B_c \mid^{s+1}$. By comparing the latter expression with the experimental data we obtain $s \approx 1$, in satisfactory agreement with the theoretical value $s = 4/3$. Thus, in low-mobility Si/SiGe heterostructures as well, the breakdown of an insulating phase can be described by the model of classical percolation.

The data obtained here, alongside the conclusions of Refs. [13−15], allow us to suggest that classical percolation should be universal for 2D systems. Obviously, this is in contradiction with both the results of scaling measurements [1−4] and the theoretical statements [6−12]. To understand the reasons for the discrepancy let us first consider the theories [10−12] in more detail. The idea used there is to modify the classical percolation picture by making allowance for quantum tunneling at the saddle points of a random potential. The tunneling gives rise to increasing the cluster dimension and, as a result, to a change in the critical exponent. We note that the phase coherence, which is necessary in the quantum percolation model, takes place only if the tunneling processes are dominant. This implies, at very least, temperatures close to zero and a temperature-independent

conductivity. Otherwise, if the temperature is not sufficiently low either the variable range hopping at a saddle point or thermoactivation to the percolation threshold is expected to be more preferable than tunneling. Scattering events break the phase coherence, thereby leading back to the classical picture. It is easy to check experimentally which mechanism dominates: the results indicate that, at minimum available temperatures, the conductivity in an insulating phase is always due to variable range hopping. Therefore the theories [10–12] and also numerical calculations [6–9] describe the limiting case that currently is not achievable in experiments.

The majority of experimental papers on the scaling behaviour deal with measurements of the half-width $\Delta\nu$ of peaks in ρ_{xx} as a function of temperature. The data is described by the power-law temperature dependence $\Delta\nu \propto T^{\kappa}$; however the exponent obtained is not universal with $\kappa = 0.21$ [4] to ≈ 1 [2]. Moreover, when plotted on a linear scale the same data follow the linear temperature dependence with a finite peak width at zero temperature, which is consistent with the thermal broadening of σ_{xx} (or ρ_{xx}) peaks in the classical percolation picture (see Ref. [14]). In these studies there is also an inherent disadvantage: the authors perform scaling measurements on Hall bar samples and neglect influence of edge channels on the peaks in ρ_{xx}. This results in weakening of the temperature dependence of $\Delta\nu$ and an underestimate of the value of the exponent κ; however, it is difficult to make strict evaluations of the effect. Similar problems exist with the interpretation of the measurements of $\Delta\nu$ on the samples of different dimensions [3]. Hence, for the time being there are no experimental results which would strongly disagree with the model of classical percolation.

In summary, we have investigated the metal-insulator transition in the zero-Hall-conductivity insulating phase of a 2D hole system in Si/SiGe heterostructures. The behaviour of nonlinear $I-V$ characteristics in the insulating phase allows us to establish the percolation nature of metal-insulator transition and to determine the critical exponent $s \approx 1$ for the localisation length. The experimental data is similar to that obtained on high-mobility 2D systems in Si MOSFET's and AlGaAs/GaAs heterostructures and can be explained in the framework of classical percolation. This is in contradiction to the interpretation of scaling measurements.

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CLUSTER ALGORITHM SPECIAL PURPOSE COMPUTER FOR THE 3D ISING MODEL

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We describe the ideas employed in the first special-purpose Wolff cluster algorithm computer able to simulate 3-dimensional Ising models. The computer is fast enough to generate accurate data for lattices containing more than 16 million spins, at and near the Ising critical point. We have used this computer to obtain test results for the 2D and 3D Ising models at criticality. These are in an excellent agreement with exact results and with independent simulations in software.

During the last decades, the Ising model has acquired a reputation as a breeding ground for the development of new approaches to the physics of phase transitions.

The exact solution of the 2D model, first given by Onsager [1], was simplified by a number of authors. Now it is clear that the 2D Ising model is just a system of free fermions. Nevertheless, even in the 2D case intriguing problems still exist. For example, we mention the influence of impurities on critical behavior [2].

In the 3D case no exact solution is available. Theoretical attempts to solve the model persist for decades, and many interesting methods were developed on this way, but the final success has not been reached so far.

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