РОССИЙСКАЯ АКАДЕМИЯ НАУК

П И С Ь М А
В ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНОЙ И ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОСНОВАН В 1965 ГОДУ *ТОМ 62, ВЫПУСК 3*
ВЫХОДИТ 24 РАЗА В ГОД *10 АВГУСТА, 1995*

Журнал поддерживается в 1995 году Российским фондом фундаментальных исследований по проекту № 95-02-91030.

Pis'ma v ZhETF, vol.62, iss.3, pp.177 - 181 © 1995 August 10

$SU(3)$-SKYRMION WITH $B=2$ AND LARGE STRANGENESS CONTENT

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Submitted 15 June 1995

The bound $SU(3)$-skyrmion is found with baryon number $B=2$ and strangeness content close to 0.5. It is of molelular (dipole) type and has the binding energy of several tens of MeV.

The effective theory of chiral fields proposed at first in [1] provides an attractive possibility to describe mesons and baryons starting from few basic ingredients. The existence of bound states of skyrmions – SO(3) [2] and $SU(2)$ [3] – opens wide prospects for application of this theory also in nuclear physics [4–7]. The predictions of the spectrum of strange as well as nonstrange dibaryons obtained mainly by means of collective coordinates quantization procedure allow to check the concept of chiral soliton approach [7, 4].

A question of principle is that about the lowest in energy state in $SU(3)$ configuration space for each baryon number B. Such configuration should be used as a starting one for quantization of zero and nonzero modes to get the observable spectrum of physical states. This question is closely related to the problem of the existence of strange matter fragments investigated previously in the framework of other approaches [8–10].

[1)]The work supported in part by Volkswagenstiftung, FRG and by Russian Foundation for Fundamental Research, grant 95-02-03868a.

Here we investigate this problem in the $SU(3)$-extension of the model for the case of baryon number $B = 2$. The most general configuration can be described by 8 functions of 3 variables (instead of 3 functions in $SU(2)$-case).

The expression for the energy of the soliton can be obtained from well known Skyrme lagrangian [1] extended to $SU(3)$ in terms of 8 functions L_i which determine the left Cartan–Maurer current $U^+ d_i U$ (or. equivalently, R_i): $U^+ d_i U = i\lambda_p L_{p,i}$, $p = 1, ...8$.
The second order (kinetic) term is especially simple:

$$E_{kin} = \frac{F_\pi^2}{8}(L_1^2 + L_2^2 + ... + L_8^2). \tag{1}$$

The Skyrme term equals to

$$\begin{aligned} E_{Sk} = \frac{1}{2e^2}\{&[L_1L_2]^2 + [L_2L_3]^2 + [L_3L_1]^2 + [L_4L_5]^2 + [L_6L_7]^2 + \\ &+\frac{3}{4}([L_4L_8]^2 + [L_5L_8]^2 + [L_6L_8]^2 + [L_7L_8]^2) + \\ &+([L_4L_6]^2 + [L_4L_7]^2 + [L_5L_6]^2 + [L_5L_7]^2 + [L_1L_4]^2 + [L_1L_5]^2 + [L_1L_6]^2 + [L_1L_7]^2 + \\ &+[L_2L_4]^2 + [L_2L_5]^2 + [L_2L_6]^2 + [L_2L_7]^2 + [L_3L_4]^2 + [L_3L_5]^2 + [L_3L_6]^2 + [L_3L_7]^2)/4 + \\ &+\frac{\sqrt{3}}{2}([L_8L_4]([L_1L_6] + [L_3L_4] - [L_2L_7]) + [L_8L_5]([L_1L_7] + [L_2L_6] + [L_3L_5])) + \\ &+\frac{3}{2}([L_1L_2]([L_4L_5] + [L_7L_6]) + [L_2L_3]([L_4L_7] + [L_6L_5]) + \\ &+[L_1L_3]([L_6L_4] + [L_7L_5]) + [L_4L_5][L_6L_7])\}. \end{aligned} \tag{2}$$

In $SU(2)$-case there remain only the first 3 terms in the expression for E_{Sk}. The baryon (winding) number density in terms of L_i is:

$$\begin{aligned} B = \frac{1}{2\pi^2}\epsilon_{ijk}\int d^3r\{&L_{1i}L_{2j}L_{3k} + \frac{1}{2}[L_{1i}(L_{4j}L_{7k} - L_{5j}L_{6k}) + \\ &+L_{2i}(L_{4j}L_{6k} + L_{5j}L_{7k}) + L_{3i}(L_{4j}L_{5k} - l_{6j}L_{7k})] + \frac{\sqrt{3}}{2}L_{8i}(L_{4j}L_{5k} + L_{6j}L_{7k})\}. \end{aligned} \tag{3}$$

The explicit expressions for L_i depend on the ansatz for $SU(3)$-matrix U. At first we have used the ansatz similar to that which is used often for the description of zero modes $SU(3)$-rotations in the procedure of quantization of rotated skyrmions, see e.g. [7]:

$$U = U_L U_4 U_8 U_R \tag{4}$$

where U_L and U_R describe $SU(2)$-skyrmions embedded into $SU(3)$. The left and right baryon numbers B_L and B_R can be arbitrary in general case; $U_4 = \exp(-i\nu\lambda_4)$, $U_8 = \exp(-i\rho\lambda_8/\sqrt{3})$. In this case we have, after some chiral rotation which has influence on chiral symmetry breaking mass terms only:

$$L_{1i} = c_\nu \lambda_{1i} + r_{1i}, \quad \text{etc.} \tag{5}$$

Here $l_{k,i}$ and $r_{k,i}$ are left and right $SU(2)$ Cartan–Maurer currents connected with U_L and U_R.

The following step after the choice of the ansatz is the choice of the boundary conditions on the functions which enter it. The choice $\nu_0 = 0$ corresponds to the incident strangeness content $SC = 0$. The interference between left and right $SU(2)$-currents is especially strong in this case.

For baryon number $B = 2$ concentrated only in U_L or U_R we have found that $SU(2)$ torus is a local minimum in $SU(3)$. For $B_L = B_R = 1$ and for topological centers located in different points direct calculation shows that the point $\nu = 0$ also is a local minimum of $SU(3)$ relative to the rotations into "strange" direction.

The other parametrization of interest is

$$U = U_L(u,s)U(u,d)U_R(d,s) \tag{6}$$

with $U-s$ on the right-hand side located in different $SU(2)$ subgroups of $SU(3)$.One of $SU(2)$-matrices, e.g. $U(u,d)$ depends on two functions, a and b:

$$U(u,d) = \exp(ia\lambda_2)\exp(ib\lambda_3). \tag{7}$$

The total number of independent functions equals to 8 again. For this case we have, after some chiral rotation:

$$\begin{gathered} L_{1i} = s_a c_a l_{3i}, \quad L_{2i} = d_i a, \quad L_{3i} = (c_{2a} l_{3i} - r_{3i})/2 + d_i b, \\ L_{4i} = l_{1i} c_a, \quad L_{5i} = c_a l_{2i}, \quad L_{6i} = l_{1i} s_a + r_{1i}(b), \\ L_{7i} = s_a l_{2i} + r_{2i}(b), \quad L_{8i} = \sqrt{3}(l_{3i} + r_{3i})/2; \end{gathered} \tag{8}$$

l_i and r_i are connected with $U(u,s)$ and $U(d,s)$.

The kinetic term equals to:

$$\begin{gathered} E_{kin} = \frac{F_\pi^2}{8}\{\mathrm{l}_i^2 + \mathrm{r}_i^2 + (1 + s_a^2) l_{3i} r_{3i} + 2 s_a (l_{1i} r_{1i}(b) + l_{2i} r_{2i}(b)) + \\ + (d_i a)^2 + (d_i b)^2 + d_i b (c_{2a} r_{3i} - l_{3i})\}; \\ r_1(b) = r_1 c_b - r_2 s_b, \quad r_2(b) = r_2 c_b + r_1 s_b. \end{gathered} \tag{9}$$

The baryon number density can be rewritten in terms of $SU(2)$-currents and functions a and b also. The additivity of $SU(2)$-baryon numbers density is clear then from this expression.

We start from the product ansatz with (u,s) and (d,s) hedgehogs in most attractive orientation, similar to $(u,d) - SU(2)$ case.The procedure of minimization with the help of "hat" method applied previously for the case of $SU(2)$-skyrmions [3, 7] allows to obtain some configurations which are local minima, at least relative to local deformations of all functions which enter the energy functional, and global rotations also.

The equal mass density lines for the $B = 2$ configuration which obviously possesses dipole-type form are shown in Fig.1. for the values of model parameters $F_\pi = 186$ MeV and $e = 4.12$, in flavor symmetric case. The binding energy of this configuration representing the new local minimum equals to 73 Mev, its strangeness content $SC = 0.494$ and the distance between topological centers of incident $SU(2)$-skyrmions $d = 1.06 Fm$, along z-direction. When two hedgehogs are in different $SU(2)$-subgroups of $SU(3)$ and interact in one common degree of

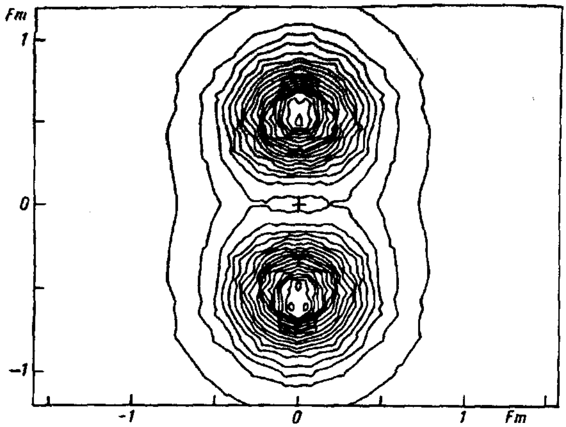

Fig.1. Equal mass density lines for flavor symmetric (FS) case and distance between topological centers of solitons $d = 1.06$ Fm; $F_\pi = 186$ MeV, $e = 4.12$

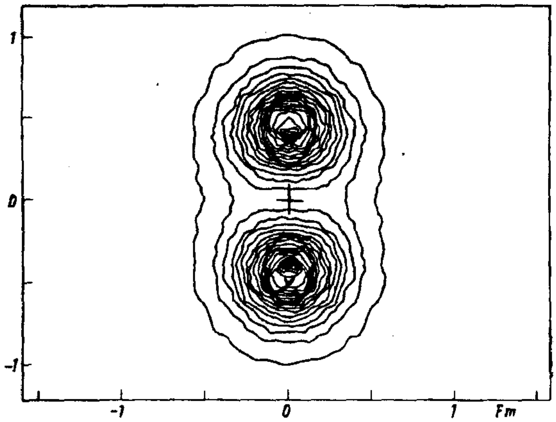

Fig.2. Equal mass density lines for the case of FSB, $d = 0.86$ Fm on the start

freedom the attraction between them is not sufficient to form the torus-like bound state [3] where their individuality is lost completely.

The flavor symmetry breaking (FSB) in the mass term can be taken into account easily in terms of real parts of diagonal matrix elements of matrix U, v_1, v_2, v_3 and kaon and pion masses:

$$M_{m.t.} = \frac{1}{8}F_\pi^2 m_\pi^2 (2 - v_1 - v_2) + \frac{1}{8}F_K^2 m_K^2 (1 - v_3). \tag{10}$$

The real part of (3,3) diagonal matrix element of matrix U which defines the strangeness content of the configuration equals to:

$$v_3 = f_1 q_1 - f_4 q_4 + s_a(s_b(f_2 q_3 - f_3 q_2) - c_b(f_3 q_3 + f_2 q_2)), \tag{11}$$

where $\bar{U}_L = f_1 + i(\bar{\tau}_1 f_2 + \bar{\tau}_2 f_3 + \bar{\tau}_3 f_4)$, $\bar{U}_R = q_1 + i(\tilde{\tau}_1 q_2 + \tilde{\tau}_2 q_3 + \tilde{\tau}_3 q_4)$, $\bar{U}_L$ and $\bar{U}_R$ are $SU(2)$ parts of U_L and $U_R, \bar{\tau}_i$ and $\tilde{\tau}_i$ are corresponding Pauli matrices.

$$SC = (1 - v_3)/(3 - v_1 - v_2 - v_3). \tag{12}$$

The equal mass density lines are shown in the Fig.2. for the case of flavor symmetry breaking. This configuration has the binding energy about 60 MeV for

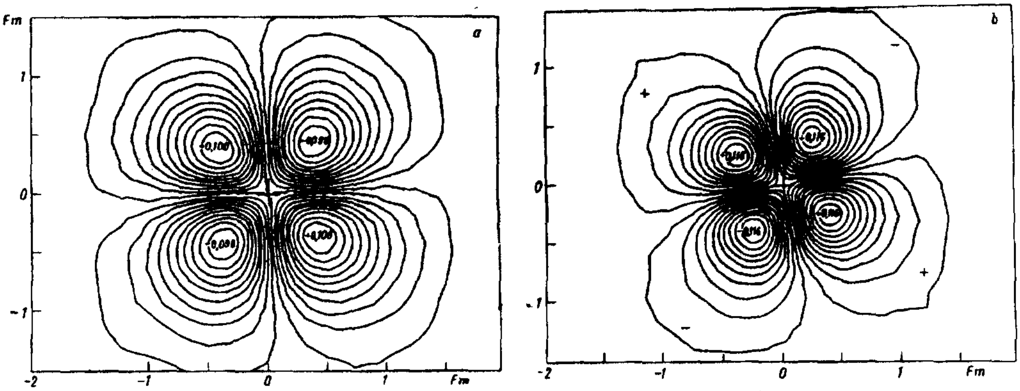

Fig.3. The equal values lines for the function a defining relative local orientation of both deformed strange hedgehogs in isospace in the plane $z = 0$ *a*) FS, *b*) FSB cases

the same parameters of the model, but smaller dimensions which is quite natural: the distance between maxima of mass density is about 0.8 Fm. Strangeness content of this configuration is close to 0.5 also: $SC = 0.499$. So, the values of binding do not differ considerably from the flavor symmetric case, within accuracy of calculation, about 5-10 Mev. The strangeness content of the configuration does not drop for flavor symmetry breaking case in comparison with flavor symmetric case. Note, that for SO(3)-hedgehog [2] $SC = 1/3$ as it should be also for the strange matter.

The function a defining local relative orientation in isospace of two (u,s) and $(d,s)B = 1$ solitons is shown in Fig 3*a* and 3*b* for FS and FSB -cases, in the plane $z = 0$. The decrease of dimensions of the configuration with inclusion of FSB terms is well illustrated.

The quantization of the dipole-type configuration should be performed, similar to the quantization of the torus-like configurations, leading to the other branch of predictions for the spectrum of strange dibaryons, additional to [7]. The correct quantum numbers of states should be established after this procedure, e.g. the electric dipole moment of physical states should be equal to zero.

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