

## PHASE DIAGRAM FOR THE SUPERFLUID FERMI-GAS IN A STRONG MAGNETIC FIELD

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We calculate the strong-coupling corrections to the Ginzburg-Landau free-energy functional in a strong magnetic field and analyze local and global minima of this functional. We show that the global minima in strong magnetic field correspond in the case of Fermi-gas with repulsion only to  $A_1$  and  $A_2$  phases discovered by Anderson, Brinkman and Morel [5].

1. In our previous papers [1,2] we calculated from the first principles the strong-coupling corrections to the Ginzburg-Landau free-energy functional in a superfluid Fermi-gas with repulsion. These calculations were performed in the absence of magnetic field up to the third order in gas parameter  $\lambda = \frac{2\alpha p_F}{\pi}$ , where  $\alpha$  is the scattering length and  $p_F$  is a Fermi momentum. They yield the following expressions for the standard  $\beta_1 \dots \beta_5$  coefficients [3]:

$$\begin{aligned} \beta_1 &= |\beta_1^{WC}| \left\{ -1 + \frac{T_C}{2\varepsilon_F} (-76.1\lambda^2 + 286.5\lambda^3) \right\}, \\ \beta_2 &= |\beta_1^{WC}| \left\{ 2 + \frac{T_C}{2\varepsilon_F} (-7.20\lambda^2 + 126.1\lambda^3) \right\}, \\ \beta_3 &= |\beta_1^{WC}| \left\{ 2 + \frac{T_C}{2\varepsilon_F} (-6.40\lambda^2 - 16.30\lambda^3) \right\}, \\ \beta_4 &= |\beta_1^{WC}| \left\{ 2 + \frac{T_C}{2\varepsilon_F} (-48.4\lambda^2 - 233.0\lambda^3) \right\}, \\ \beta_5 &= |\beta_1^{WC}| \left\{ -2 + \frac{T_C}{2\varepsilon_F} (-110\lambda^2 - 277.5\lambda^3) \right\}, \end{aligned} \quad (1)$$

where  $T_C \simeq \varepsilon_F \exp \left\{ -\frac{128}{(\pi\lambda)^2} \right\}$  is a superfluid transition temperature of a Fermi-gas with repulsion in a triplet  $p$ -wave state [4],  $\varepsilon_F$  is a Fermi energy,  $\beta_1^{WC} = -\frac{N(0)}{T_C^2} \frac{\zeta(3)}{240\pi^2}$  is the Ginzburg-Landau coefficient  $\beta_1$  in a weak-coupling approximation [3],  $\zeta(z)$  is the Riemann zeta-function ( $\zeta(3) = 1.202$ ) and  $N(0) = \frac{m p_F}{\pi^2}$  is the density of states at the Fermi surface. The analysis of the phase diagram performed in paper [2] with the coefficients  $\beta_i$  from [5] shows the tendency of shifting of the global minima from the  $B$  to  $A$  phase when  $\lambda$  - increases approaching unity. (Formal application of [5] gives the transition from  $B$  to  $A$  phase for  $\lambda = 1.26$ .) All the other phases of a triplet superfluid either lie above the  $B$  and  $A$  phases on the energy scale or do not even correspond to the local minima of the energy.

Note that the results of the exact calculations of  $\beta_1 \dots \beta_5$  differ rather significantly in third order in  $\lambda$  from the results of standard  $s-p$  approximation but lead to qualitatively similar phase diagram. Note also that large values of numerical coefficients ( $\sim 200$ ) near  $\lambda^3$  terms in  $\beta_1 \dots \beta_5$  is due to the tensor structure of the order parameter and to the big number of diagrams.

2. In the present letter we generalize the result of papers [1,2] on the case of a strong magnetic field. To be specific we consider magnetic fields larger then paramagnetic limit for  $B$  phase:  $H > H_p = \frac{T_C}{\mu_B}$ . In this case  $S_z = 0$  - projection of the spin  $S = 1$  of the triplet Cooper pair is totally suppressed. In other words  $\Delta_{\uparrow\downarrow}$  component of  $2 \times 2$  order parameter matrix  $\Delta_{\alpha\beta}$  is zero and there are only two critical temperatures  $T_C^{\uparrow\uparrow}$  and  $T_C^{\downarrow\downarrow}$  corresponding to  $S_z = 1$  ( $\Delta_{\uparrow\uparrow}$ ) and  $S_z = -1$  ( $\Delta_{\downarrow\downarrow}$ ). Exact analytical expressions for  $T_C^{\uparrow\uparrow}$  and  $T_C^{\downarrow\downarrow}$  as a functional of the gas parameter and magnetic field (or spin polarisation  $\eta = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$ ) were obtained in [6]. The results show strongly nonmonotonic behavior of  $T_C^{\uparrow\uparrow}$  with a large and broad maximum at  $\eta = 0.48$ .  $T_C^{\downarrow\downarrow}$  is a monotonically decreasing function of  $\eta$ . Note that for real  $^3\text{He}$   $T_C \sim 1 \text{ mK}$  and hence  $H_p \sim 1 \text{ T}$  corresponds to  $\eta_p \approx \frac{2}{3} \frac{\mu_B H_p}{\epsilon_F} \approx 0.004$ .

The Ginzburg-Landau free-energy functional in strong magnetic fields can be rewritten in the following convenient form:

$$\begin{aligned} \Delta F \equiv F_S - F_N &= a_{\uparrow} \Delta_{0\uparrow\uparrow}^2 M_+ + b_{\uparrow} \Delta_{0\uparrow\uparrow}^4 (2M_+^2 + |N_+|^2) + \\ &+ a_{\downarrow} \Delta_{0\downarrow\downarrow}^2 M_- + b_{\downarrow} \Delta_{0\downarrow\downarrow}^4 (2M_-^2 + |N_-|^2) + \\ &+ \Delta_{0\uparrow\uparrow}^2 \Delta_{0\downarrow\downarrow}^2 (\alpha M_+ M_- + \beta |R|^2 + \gamma |P|^2), \end{aligned} \quad (2)$$

where  $M_+ = A_{+k}^* A_{+k}$ ,  $M_- = A_{-k}^* A_{-k}$ ,  $N_+ = A_{+k} A_{+k}$ ,  $N_- = A_{-k} A_{-k}$ ,  $R = A_{+k} A_{-k}$ ,  $P = A_{+k} A_{-k}^*$ . In formula (2):

$$A_{+k} = iA_{2k} - A_{1k}; \quad A_{-k} = iA_{2k} + A_{1k};$$

$$a_{\uparrow} = \frac{1}{3} N(0) \ln \frac{T}{T_C^{\uparrow\uparrow}}; \quad a_{\downarrow} = \frac{1}{3} N(0) \ln \frac{T}{T_C^{\downarrow\downarrow}}; \quad b_{\uparrow} = b_{\downarrow} = \frac{N(0)}{15T_C^2} \left[ 1 + O\left(\lambda^2 \frac{T_C}{\epsilon_F}\right) \right];$$

$\alpha = 2\beta_2 + 2\beta_5$ ,  $\beta = 4\beta_1 + 2\beta_3$  and  $\gamma = 2\beta_4 + 2\beta_5$  are the coefficients in front of the strong-coupling invariants with the structure  $\Delta_{0\uparrow\uparrow}^2 \Delta_{0\downarrow\downarrow}^2$ . Note that  $3 \times 3$  matrix  $A_{ik}$  in (2) is connected with the  $2 \times 2$  matrix  $\Delta_{\alpha\beta}$  via the standard formula [3,7]:

$$\Delta_{\alpha\beta} = \Delta_{0\alpha\beta}(T) i(\sigma_2 \sigma_i)_{\alpha\beta} A_{ik} n_k \quad \text{and} \quad A_{3k} = 0 \quad \text{for} \quad H > H_p.$$

Let us consider first the case of weak-coupling. In this case  $\alpha = \beta = \gamma = 0$  and Ginzburg-Landau free-energy functional is reduced to the two independent superfluids with  $S_z = 1$  ( $\Delta_{\uparrow\uparrow}$ ) and  $S_z = -1$  ( $\Delta_{\downarrow\downarrow}$ ). The direct minimization of the free energy  $\frac{\delta \Delta F}{\delta A_{\pm k}} = \frac{\delta \Delta F}{\delta A_{\pm k}^*} = 0$  yields two minima of the free energy.

For  $T < T_C^{\downarrow\downarrow} < T_C^{\uparrow\uparrow}$  the first one corresponds to

$$\Delta_{0\uparrow\uparrow}^2 M_+ = -\frac{a_{\uparrow}}{4b_{\uparrow}}; \quad \Delta_{0\downarrow\downarrow}^2 M_- = -\frac{a_{\downarrow}}{4b_{\downarrow}}; \quad N_+ = N_- = 0;$$

while the second one corresponds to

$$\Delta_{0\uparrow\uparrow}^2 M_+ = -\frac{a_{\uparrow}}{6b_{\uparrow}}; \quad \Delta_{0\downarrow\downarrow}^2 M_- = -\frac{a_{\downarrow}}{6b_{\downarrow}}; \quad |N_+| = M_+; \quad |N_-| = M_-.$$

The Ginzburg-Landau free energy on the first extremum is given by

$$\Delta F = -\frac{a_{\uparrow}^2}{8b_{\uparrow}} - \frac{a_{\downarrow}^2}{8b_{\downarrow}} \quad (3)$$

while on the second one we have

$$\Delta F = -\frac{a_{\uparrow}^2}{12b_{\uparrow}} - \frac{a_{\downarrow}^2}{12b_{\downarrow}}. \quad (4)$$

Expressions (3), (4) show that the global minima corresponds to the first case with  $N_+ = N_- = 0$ . For  $T_C^{\uparrow\downarrow} < T < T_C^{\uparrow\uparrow}$  the global extremum conditions are satisfied for three degenerate phases:

$$A1 = \frac{1}{2} \begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad E = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \sigma_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

(Note that  $E$  phase coincides with the well-known planar phase.)

For  $T < T_C^{\uparrow\downarrow}$  the global extremum corresponds again to  $E$  and  $\sigma_3$  phases together with the  $A2$  phase. The last one has the form

$$A2 = \frac{1}{\sqrt{2(1-\delta^2)}} \begin{pmatrix} 1 & i & 0 \\ i\delta & -\delta & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \delta = \frac{\Delta_{0\uparrow\uparrow} - \Delta_{0\downarrow\downarrow}}{\Delta_{0\uparrow\uparrow} + \Delta_{0\downarrow\downarrow}}.$$

The appearance of the strong-coupling term proportional to  $\Delta_{0\uparrow\uparrow}^2 \Delta_{0\downarrow\downarrow}^2$  mixes  $S_z = 1$  and  $S_z = -1$  superfluids below the lower critical temperature  $T_C^{\uparrow\downarrow}$ . There are two consequences of this fact: the first one is the increase of  $T_C^{\uparrow\downarrow}$  due to the interaction between up-up and down-down bose-condensates. Of course  $\Delta_{0\uparrow\uparrow}$  is also renormalized below  $T_C^{\uparrow\downarrow}$ . The second one is lifting the degeneracy between the phases which correspond to the global minima. Indeed, for  $A2$  phase we have

$$N_+ = N_- = 0; \quad R = 0; \quad |P|^2 = M_+ M_-, \quad (5)$$

and the Ginzburg-Landau free energy on this extremum is given by

$$\Delta F_{A2} = -\frac{a_{\uparrow}^2}{8b_{\uparrow}} - \frac{a_{\downarrow}^2}{8b_{\downarrow}} + (\alpha + \gamma) \frac{a_{\uparrow} a_{\downarrow}}{16b_{\uparrow} b_{\downarrow}}. \quad (6)$$

At the same time for the planar and  $\sigma_3$  phases

$$N_+ = N_- = 0; \quad P = 0; \quad |R|^2 = M_+ M_-, \quad (7)$$

and, accordingly, the Ginzburg-Landau free energy reads

$$\Delta F_{E, \sigma_3} = -\frac{a_{\uparrow}^2}{8b_{\uparrow}} - \frac{a_{\downarrow}^2}{8b_{\downarrow}} + (\alpha + \beta) \frac{a_{\uparrow} a_{\downarrow}}{16b_{\uparrow} b_{\downarrow}}. \quad (8)$$

Formulae (6) and (8) show that to answer the question which phase has a lower energy we need to calculate the combinations  $(\alpha + \beta)$  and  $(\alpha + \gamma)$  of the strong-coupling coefficients. These calculations can be performed in the same way

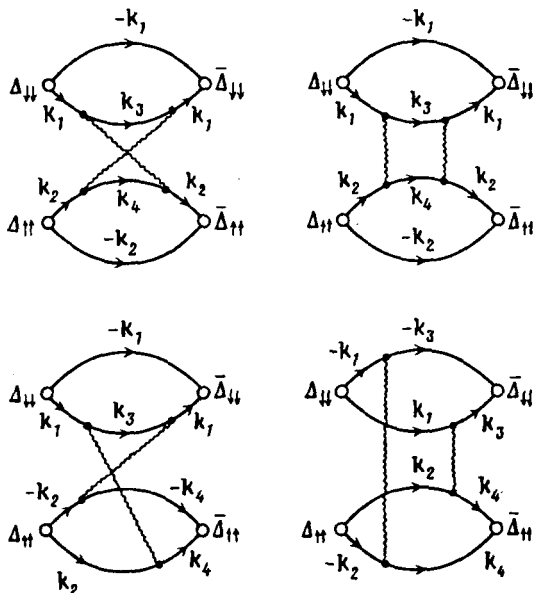


Fig.1. Diagrams which determine strong-coupling coefficients  $\alpha$ ,  $\beta$  and  $\gamma$

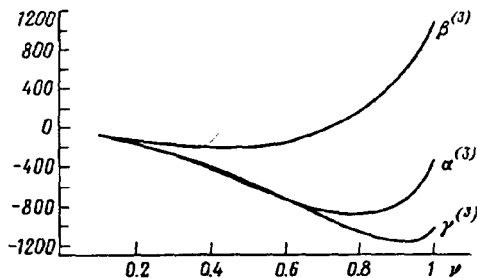


Fig.2. Magnetic field dependence of strong-coupling coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  in third order in gas parameter  $\lambda = \frac{2\omega_F \mu}{\pi}$  (in units  $\frac{T_C}{2\epsilon_F} \lambda^3 |\beta_1^{W.C.}|$ )

and from the same strong-coupling diagrams as in papers [1, 9]. These diagrams are shown on Fig.1. In analytical form they can be written as

$$\begin{aligned}
 & -\frac{T^3}{2} \sum_{\omega_1 \omega_2 \omega_3} \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} |\Gamma_{+-,-}(k_1, k_2; k_3, k_4)|^2 \times \\
 & \times K_{\downarrow}(k_1, \omega_1) K_{\uparrow}(k_4, \omega_4) G_{\downarrow}(k_3, \omega_3) G_{\uparrow}(k_2, \omega_2) |\Delta_{\downarrow\downarrow}(k_1)|^2 |\Delta_{\uparrow\uparrow}(k_4)|^2 - \\
 & -\frac{T^3}{2} \sum_{\omega_1 \omega_2 \omega_3} \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} |\Gamma_{+-,-}(k_1, k_2; k_3, k_4)|^2 \times \\
 & \times K_{\downarrow}(k_1, \omega_1) K_{\uparrow}(k_2, \omega_2) G_{\downarrow}(k_3, \omega_3) G_{\uparrow}(k_4, \omega_4) |\Delta_{\downarrow\downarrow}(k_1)|^2 |\Delta_{\uparrow\uparrow}(k_2)|^2 + \quad (9) \\
 & + 2T^3 \sum_{\omega_1 \omega_2 \omega_3} \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} \Gamma_{+-,-}(k_1, k_2; k_3, k_4) \Gamma_{+-,-}(k_3, -k_2; k_1, -k_4) \times \\
 & \times K_{\downarrow}(k_1, \omega_1) G_{\downarrow}(k_3, \omega_3) F_{\uparrow}(k_2, \omega_2) F_{\uparrow}(k_4, \omega_4) |\Delta_{\downarrow\downarrow}(k_1)|^2 \Delta_{\uparrow\uparrow}^+(k_2) \Delta_{\uparrow\uparrow}^+(k_4) - \\
 & -T^3 \sum_{\omega_1 \omega_2 \omega_3} \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} |\Gamma_{+-,-}(k_1, k_2; k_3, k_4)|^2 F_{\downarrow}(k_1, \omega_1) F_{\downarrow}(k_3, \omega_3) \times \\
 & \times F_{\uparrow}(k_2, \omega_2) F_{\uparrow}(k_4, \omega_4) \Delta_{\downarrow\downarrow}(k_1) \Delta_{\downarrow\downarrow}^+(k_3) \Delta_{\uparrow\uparrow}(k_2) \Delta_{\uparrow\uparrow}^+(k_4),
 \end{aligned}$$

where  $k_4 = -k_3 + k_1 + k_2$ ,  $G_{\uparrow} = (i\omega - \xi_{\uparrow})^{-1}$  and  $G_{\downarrow} = (i\omega - \xi_{\downarrow})^{-1}$  - are the Matsubara Green functions for up and down spins,  $\omega = \pi T(2n + 1)$

is a Matsubara frequency,  $\xi_{\uparrow(\downarrow)} = (p^2 - p_{F\uparrow(\downarrow)}^2)/(2m)$  is a spectrum for up (down) spins,  $K_{\uparrow}(\mathbf{p}, \omega) = G_{\uparrow}^2(\mathbf{p}, \omega)G_{\uparrow}(-\mathbf{p}, -\omega)$ ,  $F_{\downarrow}(\mathbf{p}, \omega) = G_{\downarrow}(-\mathbf{p}, -\omega)G_{\downarrow}(\mathbf{p}, \omega)$ ,  $\Delta_{\uparrow\uparrow}(\mathbf{p}) = \Delta_{0\uparrow\uparrow}(T)A_{+k}n_k$ ,  $\Delta_{\downarrow\downarrow}(\mathbf{p}) = \Delta_{0\downarrow\downarrow}(T)A_{-k}n_k$ ,  $\Gamma_{+,-,+}$  is a total vertex for up and down incoming and outgoing spins. In the first two orders of perturbation theory the total vertex is given by:

$$\Gamma_{+,-,+} \equiv \Gamma_{+-} = g + g^2 C_{+-}(\mathbf{p}_1 + \mathbf{p}_2) + g^2 \Pi_{+-}(\mathbf{p}_1 - \mathbf{p}_4), \quad (10)$$

where  $g = \frac{4\pi a}{m}$  is a coupling constant,  $C_{+-}$  is a Cooper loop formed by up and down spins:

$$C_{+-}(k) = \frac{m}{8\pi^2 k} [Q(p_{F\uparrow}) + Q(p_{F\downarrow})],$$

$$Q(p_{F\uparrow}) = p_{F\uparrow}^2 \ln \frac{(p_{F\uparrow} + \frac{k}{2})^2 - a^2}{(p_{F\uparrow} - \frac{k}{2})^2 - a^2} + \left(\frac{k}{2} + a\right)^2 \ln \left| \frac{p_{F\uparrow} - \frac{k}{2} - a}{p_{F\uparrow} + \frac{k}{2} + a} \right| + \\ + 2kp_{F\uparrow} + \left(\frac{k}{2} - a\right)^2 \ln \left| \frac{p_{F\uparrow} - \frac{k}{2} + a}{p_{F\uparrow} + \frac{k}{2} - a} \right|,$$

$$a^2 = \Delta^2 - \frac{k^2}{4}; \quad \Delta^2 = \frac{p_{F\uparrow}^2 + p_{F\downarrow}^2}{2}; \quad k = |\mathbf{p}_1 + \mathbf{p}_2|,$$

$\Pi_{+-}$  is an exchange contribution which coincides with a polarisation loop for a short-range interaction:

$$\Pi_{+-}(q) = \frac{m}{8\pi^2 q} \left\{ \left[ p_{F\uparrow}^2 - \left( \frac{q^2 + \tilde{\Delta}^2}{2q} \right)^2 \right] \ln \left| \frac{q^2 + \tilde{\Delta}^2 + 2p_{F\uparrow}q}{q^2 + \tilde{\Delta}^2 - 2p_{F\uparrow}q} \right| + \frac{q^2 + \tilde{\Delta}^2}{q} p_{F\uparrow} + \right. \\ \left. + \left[ p_{F\downarrow}^2 - \left( \frac{q^2 - \tilde{\Delta}^2}{2q} \right)^2 \right] \ln \left| \frac{q^2 - \tilde{\Delta}^2 + 2p_{F\downarrow}q}{q^2 - \tilde{\Delta}^2 - 2p_{F\downarrow}q} \right| + \frac{q^2 - \tilde{\Delta}^2}{q} p_{F\downarrow} \right\},$$

$$q = |\mathbf{p}_1 - \mathbf{p}_4|; \quad \tilde{\Delta}^2 = p_{F\uparrow}^2 - p_{F\downarrow}^2.$$

Unfortunately, complete analytical dependence on  $\nu = p_{F\downarrow}/p_{F\uparrow} = \left(\frac{1-\eta}{1+\eta}\right)^{1/3}$  in strong-coupling coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  can be restored only for the leading terms ( $\sim \lambda^2 \frac{T_C}{\epsilon_F} |\beta_1^{WC}|$ ). The results are the following:

$$\alpha^{(2)} = 126.2 \frac{T_C}{2\epsilon_F} \lambda^2 |\beta_1^{WC}| \left( \frac{2\nu^3}{1+\nu^3} \right)^{1/3} \{-0.18 - 1.68\nu^2\}, \\ \beta^{(2)} = 126.2 \frac{T_C}{2\epsilon_F} \lambda^2 |\beta_1^{WC}| \left( \frac{2\nu^3}{1+\nu^3} \right)^{1/3} \{-1.0 - 1.51\nu^2\}, \quad (11) \\ \gamma^{(2)} = 126.2 \frac{T_C}{2\epsilon_F} \lambda^2 |\beta_1^{WC}| \left( \frac{2\nu^3}{1+\nu^3} \right)^{1/3} \{-1.0 - 1.51\nu^2\}.$$

For the maximal magnetic field  $H = 20$  T ( $\nu = 0.95$ ,  $\eta = 0.08$ ) which we can create by brute force  $\alpha$ ,  $\beta$  and  $\gamma$  differ only on 10% from their zero-field values. Coefficients  $\beta$  and  $\gamma$  identically coincide in this order even in nonzero magnetic

field. That is why the  $A2$ -phase, planar phase and  $\sigma_3$  phase are still degenerate. To lift the degeneracy we need to calculate  $\alpha$ ,  $\beta$  and  $\gamma$  in the next order. In zero magnetic field we have from (1):

$$\alpha^{(3)} = -302.9 \frac{T_C}{2\varepsilon_F} \lambda^3 |\beta_1^{WC}|; \quad \beta^{(3)} = 1114 \frac{T_C}{2\varepsilon_F} \lambda^3 |\beta_1^{WC}|; \quad \gamma^{(3)} = -1021 \frac{T_C}{2\varepsilon_F} \lambda^3 |\beta_1^{WC}| < \beta.$$

These values show that  $A2$  is energetically favorable. The complete magnetic field dependence of  $\alpha$ ,  $\beta$  and  $\gamma$  in third order in  $\lambda = 2ap_F/\pi$  is presented in Fig.2. At small spin-polarisations  $\eta \approx \frac{3}{2}(1-\nu)$  all the coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  behave linearly in  $\eta$ . For  $H = 20$  T they again differ only on 10% from their zero-field values. However for spin polarisation  $\eta = 0.3$  ( $\nu = 0.8$ ) which can be created by a rapid melting of  $^3\text{He}$  crystal [10] this difference becomes rather significant. Moreover  $\alpha$  and  $\beta$  have a nonmonotonic dependence from  $\nu$  with a pronounced minimum, while  $\gamma$  is a monotonically decreasing function. For  $\nu \rightarrow 0$  ( $\eta \rightarrow 1$ ) there are no down-spins in the system and that is why  $\alpha$ ,  $\beta$  and  $\gamma$  saturate to the same zero value. The most important thing is that  $\gamma$  is smaller than  $\beta$  for all magnetic fields. Hence  $A2$  phase is a global extremum of the free energy.

3. In conclusion, we have calculated the strong-coupling corrections to the free energy of a triplet superfluid Fermi-gas in a high magnetic fields and found that the phase diagram of the system contains only  $A1$  and  $A2$  phases. Note that our theory is exact for low-density systems (e.g.  $^3\text{He}$ - $^4\text{He}$  mixtures) where  $\lambda < 1$ . In pure  $^3\text{He}$   $\lambda \sim 1$  and our results can be used only as a qualitative estimate. Nevertheless due to the intrinsic character of a superfluid transition in our model the philosophy of the authors is the following: if there are no exotic phases at low densities, there is a small chance to obtain them at higher densities. Hence we consider the present calculations as a strong argument against the possibility of a new phase of  $^3\text{He}$  [11-13] in a high magnetic field. Having in mind our previous results in zero magnetic field (no new phase either) ([1, 2]), we consider any novelty in a phase diagram of superfluid  $^3\text{He}$  to be very unlikely (at least in three dimensions).

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