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THE INTERACTION OF DYONS IN THE $SU(2)$ GAUGE THEORY

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Dyonic classical solutions of $SU(2)$ gluodynamics are considered and their interaction and contribution to the Wilson loop are investigated both analytically and numerically.

1. Introduction The QCD vacuum is known to possess properties of confinement and chiral symmetry breaking (CSB). The first is, for example, characterized by the area law for Wilson loop [1], the second is connected to nonzero values of chiral quark condensate. Both properties were found in lattice calculations [2] and are connected to nonperturbative fluctuations of gluonic field in the vacuum.

By now there is no model of QCD vacuum with properties of confinement and CSB, based directly on the QCD Lagrangian. The most elaborated model is the instanton gas or liquid model [3] which ensures CSB but lacks confinement [4]. Thus it is an urgent need to look for more realistic model of QCD vacuum which obeys both basic properties. The confinement is associated widely with monopole-like degrees of freedom [5] which may be of purely quantum or quasiclassical character. In the latter case one should look for classical solutions of Yang–Mills equations of monopole-like form. These solutions are known for a long time [6, 7]. They have both color-electric and color-magnetic fields and we therefore shall call them dyons [8].

The dilute dyonic gas has been suggested some time ago as a model of QCD vacuum [9] and some simple estimates of Wilson loop have been done for dyons of finite time extension, demonstrating nonzero string tension.

Recently, the interest for the dyons has revived. In particular, lattice studies of a classical and quantum field of a dyon have been done and a qualitative quasiabelian picture of confinement due to dyons was suggested [10].

The purpose of this letter is to make a first step in a more general consideration of dyonic gas. This step is connected to the problem of dyons interaction.

The letter is organized as follows. In section 2 we define a single dyon solution in different gauges. In section 3 we calculate Wilson loop for a single dyon and dyon-antidyon pair and demonstrate the phenomenon of nonabelian screening in the latter case. In sections 4,5 we consider dyonic gas model and the problem of dyons interaction. Section 6 contains concluding remarks and perspectives of future investigations.

2. Dyonic solution. One way to present the (anti)dyonic solution is to consider the case when an infinite number of (anti)instantons (in the 't Hooft Ansatz[11]) are equally spaced along a straight line (time axis).

$$A_i^a = \epsilon_{iab}\partial_b \ln\rho \mp \delta_{ia}\partial_0 \ln\rho$$

$$A_o^a = \pm\partial_a \ln\rho \ ,$$

$$\rho(r,t) = \sum_{n=-\infty}^{\infty} \frac{1}{r^2 + (t-2\pi n)^2} = \tag{1}$$

$$= \frac{1}{2}\frac{\sinh r}{r}\frac{1}{\cosh r - \cos t}$$

(we use here units,where $\gamma = 2\pi/b = 1$, b is the distance between instantons). These solutions are (anti)selfdual. The potentials and fields look like

$$A_i^a = -\epsilon_{iab} n_b f(r,t) \pm \delta_i^a g(r,t) \ ,$$

$$A_0^a = \mp n^a f(r,t) \ ,$$

$$f(r,t) = \frac{1}{r} + \frac{\sinh r}{\cosh r - \cos t} - \frac{\cosh r}{\sinh r} \ , \tag{2}$$

$$g(r,t) = \frac{\sin t}{\cosh r - \cos t} \ ,$$

$$\pm E_i^a = H_i^a = n_i n_a (f' - \frac{f}{r} + f^2) - \delta_i^a (f' + \frac{f}{r} - g^2) \mp \epsilon_{iab} n_b (g' + fg) \ .$$

For large r ($r \gg 1$) the potentials A_i^a, A_0^a go down like $1/r$ and fields E_i^a, H_i^a go down like $1/r^2$. The total action of dyon is proportional to its time extension

$$S = \frac{1}{2g^2}\int d^3r \int_0^T dt((E_i^a)^2 + (H_i^a)^2) = \frac{4\pi}{g^2} T \ . \tag{3}$$

In the future we will refer to this dyonic solution as "dyon in the 't Hooft gauge". In another gauge ("dyon in the Rossi gauge") dyonic solution can be made static[7]. Namely, making the rotation

$$U = \exp(-i\frac{\tau_i}{2} n_i \Theta) \ ,$$

$$\Theta = \tan^{-1}\left(\frac{\sin t \sinh r}{\cosh r \cos t - 1}\right)$$

we find that potentials and fields are

$$A_i^a = \epsilon_{aib} n_b \tilde{f}, \ \tilde{f} = \left(\frac{1}{r} - \frac{1}{\sinh r}\right) \ ,$$

$$A_0^a =_{\mp} n^a \tilde{g}, \ \tilde{g} = \left(\frac{1}{r} - \frac{\cosh r}{\sinh r}\right) \ , \tag{4}$$

$$\pm E_i^a = H_i^a = n_i n_a (\tilde{f}' - \frac{\tilde{f}}{r} + \tilde{f}^2) - \delta_{ia} (\tilde{f}' + \frac{\tilde{f}'}{r}) \ .$$

This means that all gauge invariant quantities are not simply time-periodic but also static. The remarkable feature of dyonic solution in the Rossi gauge is that it looks like t'Hooft–Polyakov monopole in the Bogomol'nyi–Prasad–Sommerfield limit if zero component of dyonic potential is substituted by scalar field of monopole. Thus, (anti) dyon carries, like monopole, magnetic charge. Unlike monopole, it carries electric charge, electric charge being equal to (mines) plus magnetic charge.

3. **Wilson loop for isolated dyon.** We will consider the spacial loop of circular form. When the distances from dyon to all points of the loop are much larger than the size of dyon (the size of dyon equals to $1/\gamma = 1$ in our notations) the calculations are very simple. For large distances from the dyon the fields of dyon can be made purely abelian by appropriate gauge rotation. The integral $i\oint A_i dx_i$ measures then the flow of magnetic field through the loop and is equal to Ω (Ω is the solid angle under which the loop is seen from the point of dyon location). So, we have the answer

$$W = \frac{1}{2}\mathrm{Tr}P\exp(i\oint A_i^a dx_i \frac{\tau_a}{2}) = \frac{1}{2}\mathrm{Tr}\exp(i\Omega\frac{\tau_3}{2}) \ . \quad (5)$$

When the distance from the dyon to some points of the loop is smaller than the size of dyon the finite size effects are important and the result (5) is smoothly modified. Fig.1*a* shows the results of numerical calculations for the loop of the radius equal to 10.

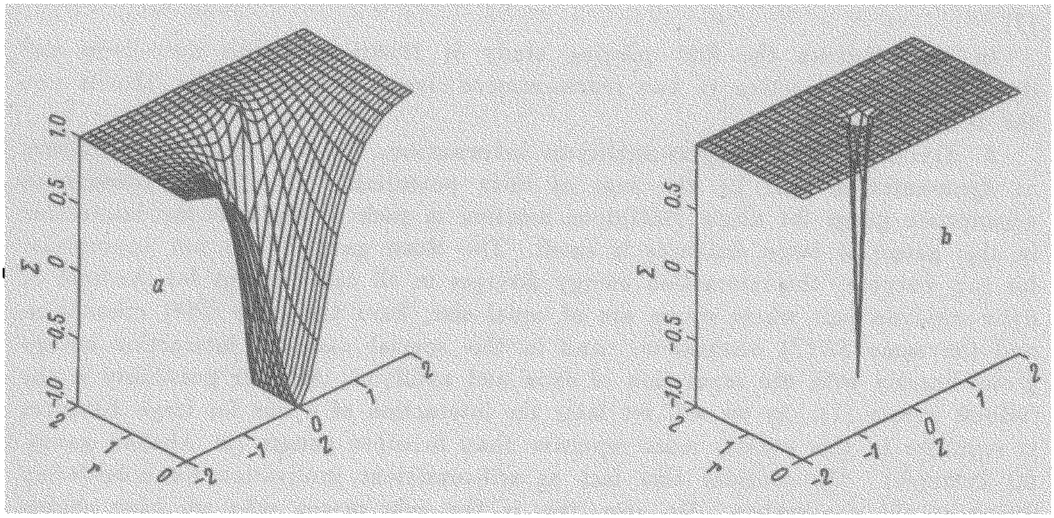

Fig.1. The Wilson loop integral for isolated dyon (a) and for pair of dyons (b). The loop lies in XY plane and has radius equal to 10. The result is presented as function of cylindric coordinates of dyon (a) or pair of dyons (b) (r,z) measured in units of radius of the loop

An interesting feature of nonabelian screening can be seen for two dyons or dyon-antidyon of zero separation. In the Rossi gauge, for example, the spatial potential is then doubled. The magnetic field of such a potential has no Coulombic part and falls off exponentially. The Wilson loop is trivial anywhere except for the boundary of the loop (see Fig.1*b*).

4. Dyonic gas model of the QCD vacuum. We suggest the superposition ansatz for the dyonic gas, similar to instantonic gas [3], namely

$$A_\mu(x) = \Sigma_{i=1}^{N} A_\mu^{(i)}(x) \tag{6}$$

where the individual (anti)dyon field $A_\mu^{(i)}$ depends on the position $R^{(i)}$, $O(4)$ orientation $\omega^{(i)}$ and $SU(2)$ (color) orientation $\Omega^{(i)}$

$$A_\mu^{(i)}(x) = \Omega^{(i)+} A_\mu(x, R^{(i)}, \omega^{(i)})\Omega^{(i)} \tag{7}$$

The resulting potential (6) is no more the solution of Yang-Mills equations, it can be only close to the solution for large separations of dyons (gas approximation). Moreover, the resulting fields depend on the gauge we are using for the (anti)dyon solution (the sum of potentials in one gauge is not connected to that in another gauge by any gauge transformation). To keep the diluteness property of the dyonic system (6) one must choose therefore the gauge of individual solution $A_\mu^{(i)}$ in such a way as to get a minimal attraction. The large distance interaction crucially depends on the asymptotics of $A_\mu^{(i)}$, and the latter is connected to the class of gauges, different classes are connected by singular gauge transformations. In particular, in Rossi gauge the topological charge comes from the infinite point, while in the 'tHooft gauge it comes from the points $r = 0, t = 0, 2\pi, \ldots$. The interaction within the given class depends also on the specific gauge chosen, but that dependence is largely taken into account by the orientation matrices $\Omega^{(i)}$ in (7).

In what follows the first piloting study is reported of the dyon-dyon and dyon-antidyon interaction in two representative classes of gauges: the 'tHooft and the Rossi gauges.

5. Dyon-dyon and dyon-antidyon interaction. We will describe two dyons or dyon-antidyon pair by the sum of their potentials. One should choose an appropriate gauge for single (anti)dyon solution in such a way that the interaction in this gauge at large distances is small. The Rossi gauge [7] is not appropriate for this purpose: the interaction energy diverges in all cases except for the case of dyon-antidyon pair where dyons are of equal size, have the same $O(4)$ orientation and the same $SU(2)$ orientation. And in this special case the interaction energy grows linearly with the separation of dyon and antidyon. Another possibility is the 'tHooft ansatz (2). As we will see later the interaction of dyons for large distances is repulsive in this ansatz, more repulsive than in other gauges (in Abelian gauge, for example). We consider this fact as self-consistent motivation for considering dyons as the gas system. By now, this is the only reason why we have chosen the 'tHooft ansatz for dyon solution. Here we have two peculiarities. Because the potentials of dyons depend on time (they are periodic in time and the period is equal to 2π) the interaction energy is also time-dependent. Therefore, we will average the interaction energy over the period. The interaction energy depends also on the relative shift of dyons in time direction (relative phase). Thus, the interaction potential is the function of separation (r), relative phase (φ), relative $SU(2)$orientation (R), relative velocity (v). Here we will consider only the case when $R = I$ and $v = 0$. The dependence on R, v will be the subject of separate investigation. The functions $V_{dd}(r,\varphi), V_{d\bar{d}}\ (r,\varphi)$ were calculated numerically. They are plotted on Fig.2(a,b). For $r \to 0, \varphi \to 0$ $V_{dd}, V_{d\bar{d}}$ are logarithmically divergent. For large $r (r \approx 10)$ $V_{d\bar{d}}$ is approximately equal to $\frac{0.5}{r}$ and $V_{dd} \approx \frac{20}{r}$, although the $\frac{1}{r}$ dependence is not clearly seen. We can compare this result for V_{dd} and $V_{d\bar{d}}$ with the results that we should expect in the Abelian gauge, where dyon-antidyon have zero interaction (magnetic charges have the same sign and electric charges are of opposite sign) and dyon-dyon interaction is $\frac{2}{r}$ (both electric and magnetic charges are of the same sign).

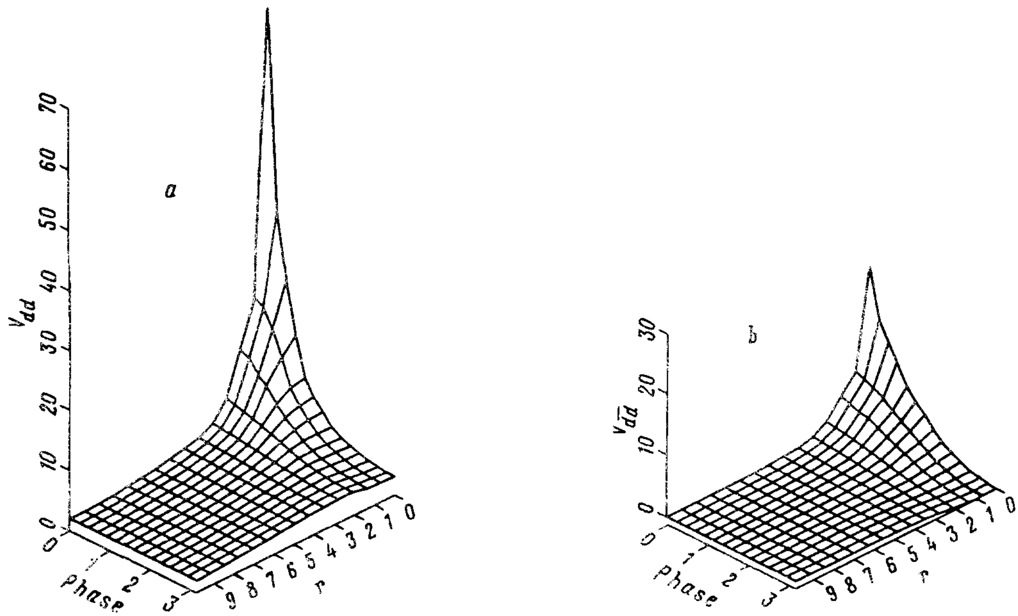

Fig.2. The interaction energy for two dyons (a) and dyon-antidyon pair (b) as function of their separation and relative phase. The interaction energy is measured in units of dyon mass $(\frac{4\pi}{g^2})$

The absolute minimum of V_{dd} is zero $(r \to \infty)$ and the absolute minimum of $V_{d\bar{d}}$ is -1.3 $(\varphi = \pi, r = 0)$. So, dyon-antidyon attract strongly at small distances when the relative phase is π. Dyon-dyon also attract in the region $\varphi = \pi, r = 0$ but in contrast to the dyon-antidyon case the interaction energy is here positive.

6. Concluding remarks. We have calculated the interaction energy for dyon-dyon and dyon-antidyon pairs. In the 'tHooft ansatz for dyonic solution the calculations show that this energy is minimal when dyons have opposite phase. In this case, the dependence of energy on separation between dyons is different in dd and $d\bar{d}$ systems. For large r $V_{dd} \gg V_{d\bar{d}}$ (a factor ≈ 40 for $r = 10$).One should compare our results for V_{dd} with those obtained in the ansatz of Manton [12], where V_{dd} vanishes. This latter property holds for an exact two-dyon solution of general form [13].

As a next step we plan to study the dependence of V_{dd} and $V_{d\bar{d}}$ on relative orientation in color space and relative motion of dyons (we use the word motion in analogy with interaction of particles in Minkovski space). This will be the basis for considering the gas of dyons and calculation of its contribution to the Wilson loop.

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