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SUPERCONDUCTIVITY FROM PURELY REPULSIVE FORCES?

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The old problem raised by Kohn and Luttinger is revisited: Will purely repulsive forces due to screening effects lead to effective attraction in the vicinity of the Fermi surface for finite angular momentum channels? If so, superconductivity could be generated from purely repulsive forces. This problem has been discussed recently by several groups and it was argued that the $l=1$ channel was most favorable. Contrary to these claims it will be argued by this author that the effective interaction remains repulsive: the normal Fermi liquid remains stable against superconductivity!

1. Introduction. The classical theory of superconductivity relied on the electron-phonon interaction to lead to an effective attraction [1] in the vicinity of the Fermi surface. However the original BCS-theory [2] was derived for any type of attractive interaction, irrespective of the microscopic origin, and rather soon after the developement of BCS-theory different mechanisms were proposed to replace the electron-phonon coupling. It was speculated that purely "electronic" mechanisms (even in the absence of electron-phonon coupling) might also lead to superconductivity. The basic idea was that the direct Coulomb repulsion between electrons might be overcompensated through electronic polarization effects, thereby leading to effective attraction. It was even argued that this can result in very high temperature superconductivity, an electronic energy scale replacing the phonon energy scale in determining T_c [3]. In recent years many approaches used in modern theories of high T_c superconductivity are based on similar arguments.

The early paper by Kohn and Luttinger [4], to be abbreviated by KL in the following, was understood by many to answer the question of principle "Can superconductivity be generated by purely repulsive forces?" to the affirmative. Although KL did not claim high transition temperatures (on the contrary: KL themselves estimated the resulting T_c's to be extremely small and to be of theoretical interest only), their paper was of considerable importance, supposedly establishing the principle that pure repulsion will also lead to superconductivity. It is a widespread understanding that the arguments by KL imply that a normal Fermi liquid cannot be the ground state: As a matter of principle at low enough temperature either magnetism or superconductivity supposedly should occur (so it is argued).

These conclusions will be challenged in this paper!

The instabilty of the normal Fermi liquid towards superconductivity in general is signalled by the appearance of a pole in the full two particle vertex function

$$\Gamma_f(p_1, p_3, s) = \Gamma_f(p_1, -p_1 + s; p_3, -p_3 + s). \tag{1}$$

The variables p_i contain momenta $\mathbf{k}_i$ and frequencies ω_i. We follow notation and arguments of Abrikosov, Gor'kov, and Dzyaloshinskii [5], abbreviated by AGD. The pole appears for p_1 and p_3 tending towards the Fermi surface and s tending to zero. As usual we define the irreducible part of Γ_f in the Cooper channel Γ_c:

$$\Gamma_f(p_1,p_3,s) = \Gamma_c(p_1,p_3,s) + \sum_{p_2} \Gamma_c(p_1,p_2,s)D_2(p_2,s)\Gamma_f(p_2,p_3,s) \qquad (2)$$

D_2 is the product of the two intermediate single particle Green functions

$$D_2(p_2,s) = G_1(p_2)G_1(-p_2+s). \qquad (3)$$

Following AGD we can decompose Γ_f and Γ_c into different angular momentum components. For Γ_c we define the Γ_{cl}

$$\Gamma_c = \sum_l \Gamma_{cl} P_l(\cos\theta). \qquad (4)$$

θ is the angle between $\mathbf{k}_1$ and $\mathbf{k}_3$, both momenta being restricted to the Fermi surface, the corresponding frequency arguments being set to zero. The P_l are the Legendre polynomials. If the normal Fermi liquid is to be the stable ground state all Γ_{cl} must be positive, an instability towards superconductivity will occur for any of the Γ_{cl} being negative. In the latter case (Γ_{cl} negative) the divergence in D_2 in eq.(2) will cause a pole in Γ_{fl}, signalling a bound two particle state in the l-channel. In the following we shall call the Γ_{cl} the "effective interaction" in the l-channel.

We will take up the scenario treated by KL (and several other groups [6,7], who developed the KL-theory further, reaching conclusions qualitavely similar to KL but of a more quantitative nature): Fermions with a bare interaction which is short ranged and purely repulsive (pure repulsion means that $V(r)$ is nonnegative in real space, decomposition of $V(r)$ into different angular momentum channels in an analogous way to Γ_c will lead to nonnegative V_l only). First we shall review the essential parts of the previous theories [4,6,7], and we shall give a detailed discussion of those features which we shall challenge. These parts will then be replaced by an alternative concept and the consequences will be derived. The main conclusion will be that the normal Fermi liquid remains stable against superconductivity.

2. Short range repulsion: The KL-theory. Kohn and Luttinger (KL) [4] argued that negative Γ_{cl} (i.e. effective attraction) will indeed develop even from a purely repulsive interaction: For short ranged $V(r)$ the direct repulsion V_l should – according to KL – be negligible for large enough l and polarization contributions to the "effective interaction" Γ_{cl} (i.e. higher order contributions to the irreducible vertex in the Cooper channel) should be negative: screening effects should lead to effective attraction.

The details of this approach [as worked out quantitatively in refs.6] are as follows:

Since close to the Fermi surface the contributions to Γ_c do not contain "dangerous" diagrams ("dangerous " meaning divergent like D_2), it is hoped that Γ_c may be calculated directly in perturbation theory. For a strong short ranged interaction – the case discussed here – the expansion parameter is $\lambda = ak_F$, where a is the scattering length and k_F is the Fermi momentum. The perturbation expansion for Γ_c is

$$\Gamma_c(p_1,p_3) = V(p_1-p_3) + 2V^2(p_1-p_3)\int dp_2 G(p_2)G(p_1+p_2-p_3) -$$

$$-V(p_3-p_1)\int dp_2 V(p_2-p_1)G(p_2)G(p_2-p_1+p_3)-$$

$$-V(p_3-p_1)\int dp_2 V(-p_1-p_2)G(p_2)G(p_2-p_3+p_1)+$$

$$+\int dp_2 V(p_2-p_1)V(p_3-p_2)G(p_2)G(p_2-p_3-p_1)+ \text{ higher order terms.} \tag{5}$$

For the calculation of the second and higher order diagrams contributing to Γ_c a momentum cutoff has to be introduced: For a very short ranged $V(r)$ these diagrams would otherwise be dominated by high momentum contributions even leading to divergencies, if the short ranged interaction is replaced by a δ-function. The standard recipe ("SR") for the cutoff is:

$V(p)$, the Fourier-transformed interaction, is replaced by the scattering length $4\pi a$ and the momentum cutoff is chosen to be of order k_F [6,7]. Within this scenario the first order contribution to the finite angular momentum components of Γ_c vanishes for a δ-function interaction (or becomes negligible for the range being sufficiently small), the second order contributions will be of order $a(ak_F)$, and further contributions containing higher powers of $\lambda = ak_F$. Thus the "polarization diagrams" (the second order ones in eq.2) are argued to give the dominant parts of Γ_c for finite angular momentum channels.

The quantitative analysis [6] of this recipe "SR" yielded that the $l=1$ channel was the dominant one, it had the largest amplitude of all Γ_{cl} for finite l, its sign being negative, supposedly indicating superconductivity. Extending this approach to include retardation effects [7] (but again using the expansion in $\lambda = ak_F$ in conjunction with "SR") resulted in quantitative corrections only, but qualitatively the same superconducting instability was claimed to exist.

3. The effective interaction, a renormalization approach. In the following an alternative approach for the evaluation of the effective interaction Γ_c is presented. Again the crucial question to be addressed is how the polarization contributions in eq.(5) influence the Γ_{cl} for finite l. But whereas the previous approaches [4,6,7] assumed that Γ_c could be calculated directly using $\lambda = ak_f$ as a small parameter (in conjunction with "SR"), the result of the renormalization approach presented below suggests that perturbation theory in $\lambda = ak_F$ fails for for finite l! The doubtful point here is the use of the recipe "SR" in dealing with high momenta: Simply replacing $V(p)$ by the scattering length and imposing a cutoff of order k_F is not accurate enough for the evaluation of the effective quasipartcle interaction for finite angular momentum channels. A distinction has to be made between the effective interaction between external test charges on one side and the quasiparticle–quasiparticle interaction on the other side. Possibly for the former a perturbation calculation in $\lambda = ak_f$ may be adequate, but not for the interaction between quasiparticles.

Here a renormalization approach is taken: Far away form the Fermi surface (that is for very high momenta k), where the distinction between particles and quasiparticles is negligible, the bare interaction is purely repulsive. Eliminating high momenta successively in a renormalization procedure will replace the bare interaction by an effective one, and the crucial question is whether in the procedure of this successive elimination of high momenta the effective interaction for finite l-channels changes sign from positive to negative when approaching the Fermi surface. Remark that this change of sign is not a "small perturbation" but rather a drastic and qualitative one. We will not attempt to carry out the full renormalization procedure quantitatively, but we will restrict ourselves to only this aspect: *Can polarization contributions produce a sign change from positive to negative (turning bare repulsion into effective attraction)?* We will concentrate the discussion on the $l=0$ and $l=1$ channel, higher l-channels will be addressed only briefly.

For the renormalization procedure three different k-regions can be distinguished:

α) externely large k, such that $r_0 k >> 1$, where r_0 is the range of the bare interaction. For this k-range Γ_{c1} and Γ_{c0} will be comparable, polarization contributions to Γ_c being negligible, but this k-region will not be of interest.

β) in this second region, k is still large enough compared to k_F, such that polarization contributions to Γ_c are still negligible, but $r_0 k$ (as well as ak, in case that scattering length a and bare range r_0 differ) are small: $ak << 1$.

γ) the third region is where k is small enough such that polarization contributions become important, it extends down to k_F, where the Γ_{cl} are the objects of interest.

We start the renormalization procedure in region β: All momenta larger than the k considered (which itself is still large compared to k_F) are already eliminated, and at this starting point $\Gamma_{c0}^{(k)}$ is large compared to $\Gamma_{c1}^{(k)}$. The superscript k indicates the chosen cutoff. We now want to reduce the cutoff from k to $k-\delta k$ (where δk is small), leading to a renormalization of the interaction parameters. The contribution of the polarization diagrams will be

$$\begin{aligned}\delta\Gamma^{(k)}_{c,pol}(p_1,p_3) &= 2V^2(p_1-p_3)\int'_{[|\epsilon-\mu|-\delta\epsilon,|\epsilon-\mu|]} dp_2 G(p_2)G(p_1+p_2-p_3) - \\ &-V(p_3-p_1)\int'_{[|\epsilon-\mu|-\delta\epsilon,|\epsilon-\mu|]} dp_2 V(p_2-p_1)G(p_2)G(p_2-p_1+p_3) - \\ &-V(p_3-p_1)\int'_{[|\epsilon-\mu|-\delta\epsilon,|\epsilon-\mu|]} dp_2 V(-p_1-p_2)G(p_2)G(p_2-p_3+p_1) + \\ &+\int'_{[|\epsilon-\mu|-\delta\epsilon,|\epsilon-\mu|]} dp_2 V(p_2-p_1)V(p_3-p_2)G(p_2)G(p_2-p_3-p_1). \qquad (6)\end{aligned}$$

The subscript "*pol*" at $\Gamma_{c,pol}$ indicates that only the polarization contributions have been retained, ϵ is the single particle energy $\epsilon(k)$, μ is the chemical potential ϵ_F.

The region of p_2 to be integrated over needs specification, first we discuss the case that k is still larger than $2k_F$: The integrand contains two single particle Green functions, one of the arguments has to be in the unoccupied (=particle) region, the other must be in the occupied (=hole) region of k-space. We take p_2 to be the "particle" variable, to be restricted to the shell of k-space to be eliminated in the renormalization step, i.e. in the interval $[k-\delta k, k]$. The variable of the second Green function – for the last contribution this is $(p_2-p_3-p_1)$ – then has to be restricted to the "hole" region (as long as k is still larger than $2k_F$ this will be the Fermi sphere).

When k becomes smaller than $2k_F$ the region of integration has to be modified: Following the principle that "high energy degrees of freedom have to be integrated out", we also have to reduce the k-region far below the Fermi energy. The integrals will have two contributions: The first (called "F_1" below) is similar to the one described above, except that the "hole" variable only runs over that part of the Fermi sphere, which has not yet been integrated out (i.e. close enough to ϵ_F). The second contribution (called "F_2") is due to eliminating the shell of energies $[| \epsilon - \mu | - \delta\epsilon, | \epsilon - \mu |]$ inside the Fermi sphere. Here the "hole" variable runs over the interval $[| \epsilon - \mu | - \delta\epsilon, | \epsilon - \mu |]$ and the particle variable runs over the region above ϵ_F up to $\epsilon(k)$.

The corresponding restrictions are indicated by the "prime" attached to the integration symbol.

Carrying out the integrals becomes cumbersome, due to the restrictions described above. For the question we want to answer ("Can polarization effects turn bare repulsion into effective quasiparticle attraction?") the analytic expressions for the integrals are not needed. If we define

$$\delta\Gamma_{cl}^{(k)} = \eta_l^{(k)} \delta k, \tag{7}$$

the sign of the coefficients $\eta_l^{(k)}$ can be obtained quite easily. Within the applicability of perturbation theory for the renormalization step we can replace the $V(p)$ appearing in the integrals by a constant. The perturbation parameter here is $\nu(k) = \Gamma_{c1}^{(k)}/\Gamma_{c0}^{(k)}$, which is small in the k-region, where the renormalization procedure is started. The conclusions which we will reach are valid under the condition of "weak coupling", that is that $\nu(k)$ has to remain small throughout. Under this condition only the last contribution in equ. (6) remains:

$$\delta\Gamma_{c,pol}^{(k)}(p_1, p_3) \sim \int'_{[|\epsilon-\mu|-\delta\epsilon,|\epsilon-\mu|]} dp_2 G(p_2) G(p_2 - p_3 - p_1). \tag{8}$$

The variables p_i contain momenta $\mathbf{k}_i$ and frequencies ω_i, the momenta $\mathbf{k}_1$ and $\mathbf{k}_3$ have essentially the same magnitude as k. To obtain $\delta\Gamma_{c0}$ we have to average over the angle θ between $\mathbf{k}_1$ and $\mathbf{k}_3$. Carrying out the frequency integration implied in equ. (7) yields a negative integrand for the momentum integration.

$$\delta\Gamma_{c,pol}^{(k)}(p_1, p_3) \sim - \int'_{[|\epsilon-\mu|-\delta\epsilon,|\epsilon-\mu|]} d^3\mathbf{k}_2 [\epsilon(\mathbf{k}_2) - \epsilon(\mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_1)]^{-1}. \tag{9}$$

As a consequence the coefficient $\eta_0^{(k)}$ is negative, $\Gamma_{c0}^{(k)}$ decreases when approaching the Fermi surface. The physical interpretation is that in reducing the cutoff, polarization contributions to the effective interaction will reduce its effective strength, certainly a result to be expected. The negative sign of $\eta_0^{(k)}$ obtained is valid for all values $k > k_F$, the geometrical constraints implied in the "prime" at the integration symbol (and described in detail above) do not influence the sign of $\eta_0^{(k)}$.

For $\eta_1^{(k)}$, determining the sign of $\delta\Gamma_{c1}^{(k)}$, we have to multiply by $\cos\theta$ and integrate over θ. Now the geometrical constraint signalled by the "prime" is essential: Due to this constraint the angle between $\mathbf{k}_1$ and $\mathbf{k}_3$ is *larger* than $\pi/2$, the $\cos\theta$ yielding an extra minus-sign. For the case that k is still larger than $2k_F$ the geometrical constraint restricts the angle θ to the interval

$$1/2 < -\cos\theta < \frac{1}{2k}(k + k_F). \tag{10}$$

For the case that k is between k_F and $2k_F$ the geometrical constraint changes as described above, due to the elimination of the "hole" region deep inside the Fermi sphere. Let k_h be the radius of this eliminated hole region. For the contribution from the region "F_1" (see above for its definition) the angle θ is restricted to the two intervals

$$\frac{1}{2k}(k+k_h) < -\cos\theta < \frac{1}{2k}(k+k_F) \tag{11}$$

and

$$\frac{1}{2k}(k-k_F) < -\cos\theta < \frac{1}{2k}(k-k_h). \tag{12}$$

The contribution from the second region "F_2" restricts the angle to

$$\frac{1}{2k}(k-k_h) < -\cos\theta < \frac{1}{2k}(k+k_h). \tag{13}$$

As a result $\eta_1^{(k)}$ has the opposite sign to $\eta_0^{(k)}$, again valid for all values of $k > k_F$. The physical interpretation of this result is:

Reducing the cutoff

i) decreases the effective strength of the interaction (i.e. Γ_{c0}),

ii) increases the effective range, Γ_{c1} increases when approaching the Fermi surface.

Since this is valid for all $k > k_F$ polarization effects cannot cause a sign change from positive to negative in the effective interaction, which therefore remains repulsive. While this was shown explicitely only for the $l = 1$-channel (the one which previously [6] was argued to be "most attractive"), the tendency to increase the effective range by reducing the cutoff invalidates the arguments of the previous treatments [4,6,7] for higher angular momenta as well.

We close by commenting on the reason for the different conclusions reached here through a renormalization procedure and the result of the previous treatments [4,6,7], where the recipe "SR" was used in dealing with high momenta when calculating the integrals of eq.(5): The negative sign obtained for Γ_{c1} in the latter is caused by the logarithmic singularity in the usual dielectric function for $k = 2k_F$, which is the usual Friedel-singularity to which the Friedel oscillations are associated. Let us consider the last integral on the right hand side of eq.(5): The singularity is due to the contributions from "particle" momentum $\mathbf{k}_2$ and "hole" momentum $(\mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_1)$ both having magnitude of order k_F but having opposite direction. The intermediate states are particle – hole excitations of momentum transfer $2k_F$ but very low energy, the excitations occuring from just below the Fermi surface on one side to just above the Fermi surface on the opposite side. These low energy excitations will contribute to the screening of static external test charges, but they do not contribute to the renormalization of the effective interaction between quasiparticles with momenta of order $2k_f$. These quasiparticles have energies $\epsilon(2k_F)$ much larger than the particle – hole excitation energy of similar momentum. But these quasiparticles of energy $2k_F$ can only be affected by excitation energies higher than $\epsilon(2k_F)$ itself: To construct an effective quasiparticle interaction at a given energy scale ϵ we can only use excitation processes at a higher energy scale than ϵ itself. It is essential to recognize the different relations between momentum and energy for quasiparticles on one side and particle – hole excitations on the other side. The particle – hole excitations will contribute to the renormalization of the effective interaction between quasiparticles under the principle that their energy must be higher than that of the quasiparticles whose interaction they are supposed to renormalize. It is the implication of this requirement which leads to the geometrical constraints indicated by the "prime" at the integration symbols of eqs.(6), (8), (9). These constraints ensure via eqs.(10)–(13) that the effective quasiparticle interaction remains repulsive.

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