Pis'ma v ZhETF, vol.62, iss.11, pp.873 - 878

MAXIMUM IN THE TEMPERATURE DEPENDENCE OF THE CRITICAL CURRENT IN BULK RANDOM JOSEPHSON NETWORKS

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Submitted 2 November 1995

Superconducting properties of the metastable alloy Zn–Sb were studied in a set of intermediate states of a bulk sample during its transformation toward the insulating state. When the temperature reduces below the transition, the critical current rises till $0.7\,T_c$, then drops and finally reaches the limiting value. The latter scales with the normal resistance of the sample as R_n^{-2}, whereas in the maximum at $0.7\,T_c$ the scaling power is different, $R_n^{-3/2}$. Comparison with properties of small superconducting tunnel junctions is made.

Some substances exhibit weak superconductivity and demonstrate Josephson properties in a bulk. Superconducting ceramics with poor contacts between grains are an example of such type of materials [1]. In this study, we investigated behavior of such substances using quenched high-pressure phase of alloy Zn-Sb [2]. Being stored at liquid nitrogen temperature after quenching, this alloy remained as metastable crystalline superconductor. Slow watchful heating induced gradual transition into an insulating state. We could trace the evolution of the superconducting response chopping the transformation by abrupt cooling at different stages and studying the intermediate states at low temperatures. With practically fixed temperature of the transition onset T_c, the resistive transition curves get a low-temperature tail first, then the transition becomes uncomplete and, next, quasireentrant. The huge interval of resistivities of the material while spanning this set of states is due to the inhomogeneous character of the transition: the insulating phase appears presumably in a fractal-like fashion making the current paths along the metallic phase long and confined [3]. Similar behavior has been also found in alloys Ga-Sb [4] and Cd-Sb [5].

At the preceding stage, we studied J-V-characteristics in a quasireentrant state [2, 6], with the resistance in the minimum of $R_{min} \approx 0.5R_n$. Maximum of the critical current observed at $T/T_c \approx 0.7$ was the most noteworthy feature of those experiments. In this paper, we go ahead with investigating the nature of this maximum.

Consider a 3D-lattice of Josephson junctions. There exist two logical possibilities. First, each single junction may have the well known temperature dependence of the critical current [7] with the maximum for the bulk sample critical current being caused by the temperature dependent nonuniformity of the current distribution over the network [6]. Secondly, the observed maximum may reflect the properties of individual junctions. The aim of this paper is to study this second possibility. For this purpose, we selected moderate-resistance states of the alloy $Zn_{41}Sb_{59}$ with tails in the superconducting transition and matched their behavior

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against the behavior of single small superconducting tunnel junctions studied by Tinkham group [8, 9].

We present here the measurements of the set of ten successive states of a $Zn_{41}Sb_{59}$ sample with normal resistance values R_n from 3 Ohm to 1500 Ohm (approximate resistivity values, correspondingly, from 0.3 Ohm·cm to 150 Ohm·cm). These were the states with tailed transitions. In the states with smaller R_n the transition was sharp and we could not reach critical current because of overheating of the contacts. The transition in the last of the presented states, one with $R_n = 1500$ Ohm, was already uncomplete at the lowest temperature used (1.2 K).

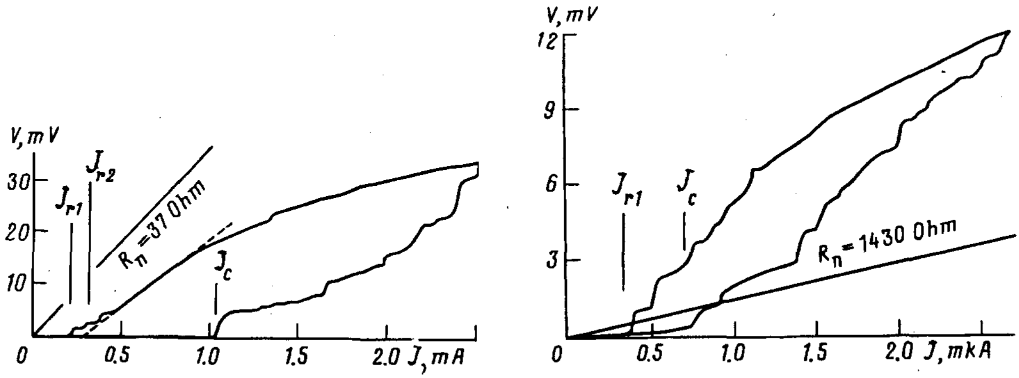

Fig.1. Hysteresis loops on the *J-V* curves in two states: that with R_n = 37 Ohm (above) and with R_n = 1430 Ohm (below). T = 1.2 K. Dashed line on the upper loop is the tangent to the inflection point. Also shown are straight lines which correspond to values of the normal resistance. The positions of the critical currents are marked

Two typical *J-V* curves with hysteresis loops are shown in Fig.1. Both branches of the loops have random steps, presumably, due to discrete structure of the network. Voltage jumps up (down) indicate switching off (on) the shortening Josephson currents in some bonds of the network. The *J-V* loops display two critical currents, J_c on the rising branch and J_r (called the recapture critical current [8]) on the descending branch of the loop. For J_c rather poor reproduction was typical: the value of J_c changed from run to run. The value of J_r reproduced much better but it had its own disadvantage. Fig.1 shows that there are different possibilities to determine it. Point J_{r1} is defined as the position of the last jump down of the voltage, point J_{r2} is the intersection with the *J*-axis of the tangent to the inflection point $\partial^2 V/\partial^2 J = 0$. However, this uncertainty did not affect our conclusions.

The initial part of the *J-V* curve in the high-resistance state has a finite slope because the resistance in this state did not reach zero even at 1.2 K. Second, in this state most of the (J, V) points on the loop correspond to resistances $R = V/J$ much larger then R_n. Hence, the resistance in the loop region is controlled by the single-particle tunneling between superconducting parts of the network.

The loops presented in Fig.1 were obtained at low temperature. On rising the temperature the loops become narrower. Then both the hysteresis and the steps disappear. At $t \equiv T/T_c \gtrsim 0.5$, the critical currents coincides $J_c = J_r$. At $t \gtrsim 0.8$, the precision of measurements of J_r falls down because the curve $V(J)$ becomes very smooth, the inflection point shifts to higher voltages and its position becomes uncertain.

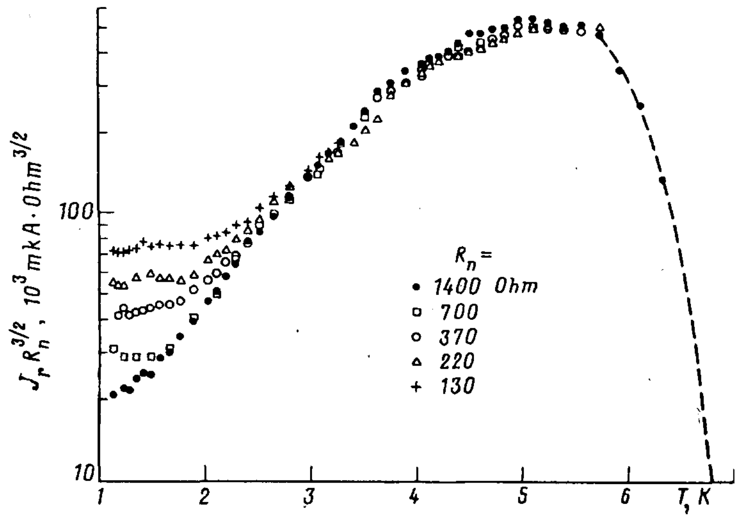

Fig.2. Temperature dependence or the critical current J_r normalized by the normal resistance R_n in the power 3/2 in five different states

In Fig.2, we display the temperature dependence of $J_r(T)$ for a number of different states. Being normalized by the factor $R_n^{3/2}$ they coincide precisely above $t \gtrsim 0.3$, in the vicinity of maximum $J_r(T) = J_{r,max}$. At low temperatures, the curves diverge and each one approaches its own limiting value $J_{r,min}$. According to Fig.3,

$$J_{r,max} \propto R_n^{-3/2}, \tag{1}$$

$$J_{r,min} \propto R_n^{-2}. \tag{2}$$

Relations (1), (2) are the main experimental results of this paper.

Note the remarkable similarity between properties of our samples and those of single small high-resistance tunnel junctions: for such junctions, the critical current also has maximum in the temperature dependence at $T \approx 0.8\,T_c$, more pronounced in the states with high normal resistance [8], it also ceases to decrease and reaches a limiting value at low temperature [9], the low temperature critical current is also inversely proportional to the square of the normal resistance [8]. That is why we start discussion listing the main expressions and relations for a small single Josephson junction which has shunting capacitance C and shunting resistance $r(T)$. We use below lower case for currents, resistances, and voltage for a single junction to distinguish them from those in the bulk samples. The dynamics of a junction depends on relations between the temperature T and two energies: the Josephson coupling energy

$$E_J = \frac{\pi}{4}\left(\frac{\hbar/e^2}{r_n}\right)\Delta \tag{3}$$

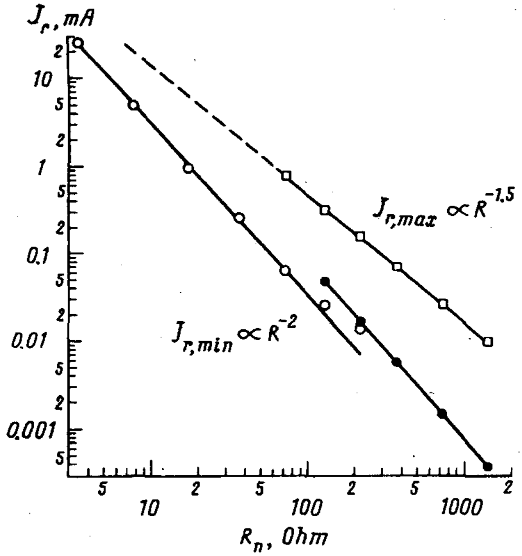

Fig.3. Dependence of the critical currents J_r on the normal resistance R_n of the state. Squares – in the maximum of the $J_r(T)$ curve, approximately at $T = 0.7\,T_c$. Curcles – the low temperature limiting value, open curcles – defined by steps, full curcles – defined by the derivative

(here $\Delta(T)$ is the superconducting gap) and the Coulomb energy $E_C = e^2/2C$. It is clear from (3) that when the normal resistance is small, i.e. when

$$r_n \ll \hbar/e^2 \approx 4\,\text{kOhm}, \qquad \text{then} \qquad E_J \gg \Delta > T_c \qquad \text{and hence} \qquad E_J \gg T \tag{4}$$

holds below the transition temperature. Then the thermal fluctuations may be neglected and the critical current j_{c0} is [7]

$$j_{c0}(T) = \frac{\pi}{4}\,\frac{2\Delta(T)}{e\,r_n}\,\tanh\frac{\Delta(T)}{2T} = \frac{2e}{\hbar}\,E_J\,\tanh\frac{\Delta(T)}{2T}. \tag{5}$$

The phase difference φ of the superconducting order parameter across the junction biased with current i satisfies the equation [10]

$$C\ddot{\varphi} + \dot{\varphi}/r + (2e/\hbar)^2 U'(\varphi) = 0, \qquad U(\varphi) = -E_J\cos\varphi - (\hbar/2e)i\varphi. \tag{6}$$

Eq.(6) has two solutions for average voltage across the junction in some interval of currents $j_{c0} > j > j_{r0}$, i.e. it describes hysteretic behavior of the junction. The recapture critical current j_{r0} can be expressed through the dimensionless damping parameter β_c [10]

$$\beta_c = (2e/\hbar) j_{c0}\, r^2\, C. \tag{7}$$

When the parameter is large, $\beta_c \gg 1$, then

$$j_{r0} = (4/\pi)\, j_{c0}\,\beta_c^{-1/2} = \frac{2}{\pi}\left(\frac{2\hbar}{eC}\right)^{1/2}\frac{j_{c0}^{1/2}}{r}. \tag{8}$$

If r in expression (4) were determined by the single-particle tunneling current, it would be equal to [5, 11]

$$r = r_{tun} = r_n(T/\Delta)\exp(\Delta/T). \tag{9}$$

Then j_{r0} wound tend to zero at low temperature.

For junctions with higher r_n, when inequalities (4) are not valid any more, fluctuations become important. Then a term with a Gaussian fluctuating current should be added into eq.(6). The fluctuations shift the currents j_{c0} and j_{r0} toward each other and bring other critical currents j_c and j_r into line. According to [8], the shift of j_{c0} is larger:

$$j_{r0} \lesssim j_r \leq j_c < j_{c0}, \tag{10}$$

so that in nonhysteretic regime, when $j_r = j_c$, the measured critical current should be interpreted as j_{r0} [8].

We can successfully apply this interpretation to our results. Since the dissipative channel of the junction is determined by a single-particle tunneling current, assuming upper case for j's and r's in expressions (2)–(5) we get from eqs.(2), (4) and (9) the scaling relation (1). Note that this does not necessarily mean that our sample is in essence a chain of small junctions. If it were so, it would follow from (4) that the number of junctions in the chain and the capacitance C of each of them remain the same while transformation of the sample. This seems improbable. Here and below, we want only to emphasize that the behavior of a bulk sample fits the theory intended for a single junction.

The comments upon the second relation, (2), are not so straightforward.

To starts with, J_{r0} does not tend to zero with $T \to 0$ despite eqs. (4) and (9). The similar behavior of single junctions [9] is explained by the effect of the loading impedance Z_{ac} of the leads. According to [9], there exists a minimum value j_{r0}^{min} which is determined by the balance between the dc power fed to the junction and the power dissipated at high frequency in the leads. The Josephson channel sources the ac current $j_{c0}\sin(\omega t)$ and the average power dissipated at ω is of the order of $j_{c0}^2 Z_{ac}$. The power in the dc single-particle tunneling channel being $j_{r0}^{min}(2\Delta/e)$, we get relation [9]

$$j_{r0}^{min} \propto e\, j_{c0}^2\, Z_{ac}/\Delta. \tag{11}$$

The relation (2) follows from eq. (6) if we assume Z_{ac} independent of r_n and $j_{c0} \propto (1/r_n)$ in accordance with eq. (2), and after we return to upper case.

The scaling law $\propto r_n^{-2}$ had been found in [8] also, though for the value of j_{c0} instead of j_{r0}. The explanation which was proposed in [8] used different terms.

The eq. (6) describes also a particle moving with friction along φ axis in the titled cosinusoidal potential U (the "titled washboard model") – see, for example, review [12]. According to the Josephson relation $v = (\hbar/2e)\dot{\varphi}$, the phase-ball being trapped in a potential well means zero voltage $v = 0$ across the junction and the current without dissipation. In classical description, the particle can be shifted toward the neighbor well due to either regular force $\partial U/\partial\varphi$ or thermoactivated fluctuations. However, if the capacitance C is small and the Coulomb energy E_C is comparable to E_J then quantum description comes into play. Then the phase-ball may tunnel to the neighbor well or even become completely delocalized as an electron in a periodic potential. The energy of the ground state (of the "bottom of the Bloch band") was calculated in [8] in the limit $E_C \gg E_J$ and turned to be $-E_J^2/8E_C$ instead of $-E_J$ in the opposite limit. The similar result was obtained in [13]. It was argued in [8] that, because of this crossover, E_J

should be replaced by $E_J^2/8E_C$ in the expression (2) for j_{c0} of a single junction with small C. From here follows the scaling law similar to relation (2).

These arguments relate to j_{c0}, as well as experimental observations obtained at very low temperatures [8]. Our experimental relation (2) contains J_r. Hence, the resemblance at this point is not so obvious.

In summary, similarity is established between the properties of bulk metastable Zn-Sb alloy in different intermediate states and of small high-resistance low-capacitance Josephson junctions. Hence, the equations describing these two subjects should be similar. We doubt that our substance should be treated as a multitude of separated and well defined small Josephson junctions. In particular, this would mean that for all the states under consideration the network of junctions and their density are the same and that only the parameters of each of them alter from state to state. The existing structure model of the transformation in the Zn–Sb alloy [3] more favors thin superconducting wires and constrictions. Note that a network of the phenomena of phase tunneling is possible for such objects as well [14].

It is also unclear to what extend the discreteness of the substance is important and whether the finite probability of the phase slipping cannot be attributed to each point of the 3D-space as it happens when thin long 1D wires are considered. This is the central point of the problem and we plan to study other similar materials from the same point of view.

We wish to thank M.V.Feigelman for discussions and comments. The work is supported by the Russian Fund for Fundamental Research under Grant 93-02-3271 and by the International Science Foundation under Grant RE7300.

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1. T.H.Lin, X,Y.Shao, M.K.Wu et.al., Phys.Rev.B **29**, 1493 (1984).
2. V.F.Gantmakher, V.N.Zverev, V.M.Teplinskii, and O.I.Barkalov, Pis'ma ZhETF **59**, 418 (1994) [JETP Lett. **59**, 446 (1994)].
3. V.F.Gantmakher, S.E.Esipov, and V.M.Teplinskii, ZhETP **97**, 373 (1990) [JETP **70**, 211 (1990)].
4. V.F.Gantmakher, V.N.Zverev, V.M.Teplinskii et al., ZhETF **104**, 3217 (1993) [JETP **77**, 513 (1993)].
5. V.F.Gantmakher, V.N.Zverev, V.M.Teplinskii, and O.I.Barkalov, ZhETF **105**, 423 (1994) [JETP **78**, 226 (1994)].
6. V.F.Gantmakher, V.N.Zverev, and V.M.Teplinskii. Pis'ma ZhETF **59**, 837 (1994) [JETP Lett. **59**, 874 (1994)].
7. V.Ambegoakar and A.Baratoff, Phys. Rev. Lett.**11**, 104 (1963).
8. M.Iansiti, M.Tinkham, A.T.Johnson et.al. Phys. Rev. B **39**, 6465 (1989).
9. A.T.Johnson, C.J.Lobb, and M.Tinkham, Phys. Rev. Lett. **65**, 1263 (1990).
10. D.E.McCumber, J. Appl. Phys. **39**, 3113 (1968).
11. Y.C.Chen, M.P.A.Fisher, and A.J.Leggett, J. Appl. Phys. **64**, 3119 (1988).
12. V.I.Mel'nikov, Physics Reports **209**, 1 (1991).
13. B.Mirhashem and R.A.Ferrel, Phys. Rev. Lett. **61**, 483 (1989).
14. N.Giordano, Phys. Rev. Lett. **61**, 2137 (1988).