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MAGNETOEXCITONS IN QUANTUM RINGS AND IN ANTIDOTS

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An opening (antidot) in two-dimensional (2D) electron gas placed in a strong perpendicular magnetic field makes it possible the formation of ring-shaped excitons stemming from the skipping -states of electrons and holes.The binding energy of the ground states of such excitons is logarithmically larger than that of the "bulk" 2D excitons.The possibility of tunneling electron to hole around the antidot results in a shift for each of the excitons levels that oscillates with magnetic field.

The problem of the Mott-excitons in a strong magnetic field has a rather long history (see [1–5]). It has been shown that the exciton binding energy as well as all the other characteristics (effective mass, dipole moment, dispersion law) are strongly magnetic field dependent. In particular a very strong magnetic field ($a_H \ll a_B$, where a_H, a_B are the magnetic length and Bohr radius, correspondingly) makes the 3D exciton effectively 1D with logarithmically deep ground state.

Recent successes in microstructuring technology aroused interest to the excitons in nanostructures of various dimensionalities (e.g.excitons in quantum dots with parabolic confinement have been considered by this author and Govorov in ref.[6]).

The aim of this paper is to call attention to the fact that in a strong magnetic field localized excitons can be formed without any confining potential both for electrons and for holes. Suppose we have an opening of the radius R in a 2D electron gas. Such an antidot is considered in the present work just as a region forbidden for both particles to penetrate inside.Thus the contour of the antidot $\rho = R$ (the polar coordinates are used) is a hard wall. The magnetic field perpendicular to the plane of 2D gas presses both types of particles to the wall forming the skipping orbits. Electrons and holes will circulate around the antidot in the opposite directions (as long as the Coulomb interaction is not accounted for). Hence one can expect the formation of quasi-1D exciton localized in a narrow ring embracing the antidot.

Let me start with an idealized model: electron and hole in a 1D ring placed in a magnetic field. The Hamiltonian of the problem can be written in a form:

$$\hat{\mathcal{H}} = \frac{\hbar^2}{2m_n R^2}\left(-i\frac{\partial}{\partial\varphi_n} + \lambda\right)^2 + \frac{\hbar^2}{2m_p R^2}\left(-i\frac{\partial}{\partial\varphi_p} - \lambda\right)^2 - \frac{e^2}{2\varepsilon R}\left|\sin\frac{\varphi_n - \varphi_p}{2}\right|^{-1}, \quad (1)$$

where $\lambda = \Phi/\Phi_0$, Φ is the magnetic flux through the ring ($\Phi = \pi R^2 B$), Φ_0 is the magnetic flux quantum, m_n, m_p are the effective masses of electron and hole, respectively; φ_n and φ_p are the azimuthal coordinates of the particles, and ε is the dielectric constant. To separate the internal motion in the exciton from that of center-of-mass one needs to go over to the variables

$$\varphi_c = \frac{m_n\varphi_n + m_p\varphi_p}{M}, \qquad \theta = \varphi_n - \varphi_p; \qquad M \equiv m_n + m_p. \quad (2)$$

Then the Hamiltonian reads

$$\hat{\mathcal{H}} = -B\frac{\partial^2}{\partial \varphi_c^2} + \beta\left(i\frac{\partial}{\partial \varphi} - \lambda\right)^2 + U_c(\theta), \tag{3}$$

where $B = \hbar^2/2MR^2$, $\beta = \hbar^2/2\mu R^2$, $\mu = m_n m_{p/M}$and $U_c(\theta)$ is the Coulomb energy (the last term in Eq.(1)). The total wave function may be written as $\Psi = \exp(iJ\varphi_c + i\lambda\theta)\chi(\theta)$, where J is a real number and $\chi(\theta)$ obeys the 1D Schrödinger equation

$$-\beta\frac{\partial^2\chi}{\partial\theta^2} + U_c(\theta)\chi = (E - BJ^2)\chi \equiv w\chi. \tag{4}$$

Here E is the total energy of the exciton, w is its internal energy.

To determine J and allowed solutions of the Eq.(4) we have to make the total wave function independently periodic in φ_n and φ_p with the period 2π. On the other hand the Eq.(4) has, formally, the Bloch type solutions

$$\chi = e^{ip\theta}v(\theta); \quad -\frac{1}{2} < p \leq \frac{1}{2}, \tag{5}$$

where v is a periodic function of θ with the same period 2π because this is a period of the potential: $U_c \sim |\sin\theta/2|^{-1}$. By adding 2π to φ_n and φ_p independently we come to the relations

$$J\frac{m_n}{M} + \lambda + p = N_n, \quad J\frac{m_p}{M} - \lambda - p = N_p, \tag{6}$$

where N_n, N_p are arbitrary integers. It follows from Eq.(6) that $J = N_n + N_p$, thus J is integer - the rotational quantum number of the exciton as a whole.

Now consider Eq.(4). In a small vicinity of the point $\theta = 0$, as well as at $\theta = \pm 2\pi, \pm 4\pi, \ldots$, one can expand the Coulomb energy: $U_c \approx e^2/\varepsilon R\theta$, so we have a 1D hydrogen "atom" with the charge e^2/ε and effective mass μ. As it is known (see e.g. [4] and references therein) the ground state of such a system is logarithmically deep

$$E_0 = -\frac{2\mu e^4}{\varepsilon^2\hbar^2}\ln^2\left(\frac{a_B}{a_0}\right), \tag{7}$$

where a_0 is a "cut off" radius. For 3D exciton in a strong magnetic field a_0 equals the magnetic length a_H. In our case $a_0$is connected ,obviously, with the size of skipping orbits and will be determined below.

Besides the ground state 1D exciton has the hydrogen series of the energy levels $E_n = -\mu e^4/2\varepsilon^2\hbar^2 n^2, n = 1, 2, 3, \ldots.$ All these levels form (approximately!) the set of allowed values for the eigenwert w in Eq.(4) provided the effective Bohr radius $a_B = \varepsilon\hbar^2/\mu e^2$ is much smaller than the distance $2\pi R$ to the next well in the Coulomb energy U_c. For the antidots reported till now the condition $a_B \ll R$ is very good satisfied and we may solve Eq.(4) in the tight binding approximation.

Any solution of the type (5) corresponds to the energy

$$w = E_n - \Delta_n \cos 2\pi p, \quad n = 0, 1, 2, \ldots, \quad \Delta_n > 0 \ , \tag{8}$$

where "quasimomentum" should be determined by Eq.(6).Then for the total energy of the exciton one obtains:

$$\begin{aligned} E(J,n) &= BJ^2 + E_n - \Delta_n \cos 2\pi(\Phi/\Phi_0 + Jm_n/M) = \\ &= BJ^2 + E_n - \Delta_n \cos 2\pi(\Phi/\Phi_0 - Jm_p/M); \quad n = 0,\ 1,\ 2\ \ldots \end{aligned} \tag{9}$$

The quantity Δ_n in Eq.(9) is an amplitude corresponding to the tunneling electron to the hole (or vice versa) around the antidot. Its order of magnitude is $\Delta_n \sim |E_n| \exp(-2\pi R/na_B)$ for $n \geq 1$ and $\Delta_0 \sim |E_0| \exp((-2\pi R/a_B) \ln(a_B/a_0))$ for the ground state.The possibility of such tunneling shifts the energy levels from their unperturbed positions E_n and results in a periodic dependence of the binding energy of the ring exciton on the magnetic flux with the period Φ_0. The latter statement has a general character and is not connected with the tight binding approximation used.The eigenvalues w of the Eq.(4) are always periodic functions of p from Eq.(5) and via Eq.(6) they are periodic functions of the magnetic flux.

Consider now a more realistic model and take into account the radial motion of electrons and holes in the skipping states.To find the energy spectrum in this regime I will use the small parameter a_H/R. The above introduced quantity λ is of the order $(R/a_H)^2 \gg 1$. The radial coordinates of the particles can be written as $\rho_n = R + x_n$, $\rho_p = R + x_p$. Then x_n, x_p are substituted by X (center-of-mass) and $x = x_n - x_p$ (relative radial distance), and the Hamiltonian is expanded in X and x. By substituting $\Psi = \chi \exp(i\kappa\theta + iJ\phi_c)$ with

$$\kappa = \frac{2J\mu}{M}\frac{x}{R} - \lambda\left(1 + \frac{2X}{R} + \frac{2\gamma x}{R}\right), \tag{10}$$

we get the Schrödinger equation with the linear in X and x accuracy:

$$\left[-\frac{\hbar^2}{2}\left(\frac{1}{M}\frac{\partial^2}{\partial X^2} + \frac{1}{\mu}\frac{\partial^2}{\partial x^2}\right) + 2(\beta\lambda^2 - BJ^2)\frac{X}{R} + \right.$$
$$\left. +(2\gamma\beta\lambda^2 + 4JB)\frac{x}{R}\right]\chi + \hat{H}_{int}\chi = (E - BJ^2)\chi\ , \tag{11}$$

where

$$\begin{aligned} \gamma &= \frac{m_p - m_n}{M}, \\ \hat{H}_{int} &= -\beta\left(1 - \frac{2X}{R} - \frac{2\gamma x}{R}\right)\frac{\partial^2}{\partial\theta^2} - \frac{e^2}{\varepsilon}\left(1 - \frac{X}{R} - \frac{\gamma x}{2R}\right)\left(x^2 + 4\sin^2\frac{\theta}{2}\right)^{-1/2} \end{aligned} \tag{12}$$

The dependence of the wave function on φ_c is as before but still a question remains how to separate the radial center-of-mass motion from the internal one. In a homogeneous electric field the internal and the center-of mass degrees of freedom can be completely separated (see [3,4]). Our case is spatially nonuniform and the exact separation is not possible. Nevertheless approximately such a procedure can be carried out for sufficiently strong magnetic fields. As one can see from Eq.(11) the frequencies of motion of the particles in the skipping states are governed by the effective potentials

$$\begin{aligned} W_1(X) &= 2(\beta\lambda^2 - BJ^2)X/R, \\ W_2(x) &= (2\gamma\beta\lambda^2 + 4JB)x/R, \\ &\qquad x, X > 0\ . \end{aligned} \tag{13}$$

The skipping states exist of course if the rotational energy BJ^2 is not larger than $\beta\lambda^2$ but this does not cause any problem due to the condition $\lambda \gg 1$ (besides, the probability of the exciton formation by the optical absorption is proportional to the factor $|\int \Psi(X, \varphi_c) d\varphi_c dX|^2$ which is nonzero only for $J=0$. I keep J in the calculations just for generality). For not too close effective masses of electron and hole $\gamma \sim 1$ and the frequency corresponding to x- and X-degrees of freedom are estimated as

$$\omega_x \sim \omega_X \sim \hbar\lambda^{4/3}/\mu R^2. \tag{14}$$

The internal azimuthal motion of the exciton is characterized by the Bohr frequency $\omega_B \sim \hbar/\mu a_B^2$, and the rotation around the antidot is the slowest motion: $\omega_R \sim \hbar/MR^2$. Hence for the magnetic field at which

$$a_H^2 \ll \sqrt{R a_B^3}\ , \tag{15}$$

we have ω_x, $\omega_X \gg \omega_B$ and the situation is quite similar to the one in molecules when considering the vibrational-rotational spectra. For GaAs with a_B =100Å, R =1000Å the magnetic field determined by Eq.(15) must exceed 4T.

In the spirit of the theory of molecules we have to solve the "internal" equation $\hat{H}_{int}\chi = w(x, X)\chi$ for given x and X (like rotation at fixed nuclei) and then to find corrections stemming from the fast degrees of freedom x and X (like interaction of rotation and vibrations). These corrections renormalize the effective mass and the charge of the hydrogen-like system described by the Hamiltonian $\hat{H}_{int}$ from Eq.(12):

$$\mu \rightarrow \tilde{\mu} = \mu\left(1 - \frac{2X}{R} - \frac{2\gamma x}{R}\right)^{-1}, \qquad e^2 \rightarrow \tilde{e}^2 = e^2\left(1 - \frac{X}{R} - \frac{\gamma x}{2R}\right). \tag{16}$$

It is essential that in the effective Rydberg energy $\tilde{\mu}\tilde{e}^4/2\hbar^2\varepsilon^2$ the X-dependent terms are canceled with the accuracy accepted (no terms linear in X). That means that all the higher energy levels E_n (n=1, 2 ...) do not depend on X and only the ground state energy E_0 contains X under logarithm via effective Bohr radius $\varepsilon\hbar^2/\tilde{\mu}\tilde{e}^2$. Thus with the "logarithmic accuracy" (meaning by that a very weak dependence) the center-of-mass radial motion is separated from the internal degrees of freedom.

The term x^2 under square root in Eq.(12) gives the above mentioned cut off radius of the quasi-1D exciton. Its estimate follows from the potential $W_2(x)$ in Eq.(13): $a_0 \sim R\lambda^{-2/3}$ - the size of ground state of a particle in the triangular well. Hence the lowest energy level of the exciton equals $E_0 = -(2\mu e^4/\varepsilon^2\hbar^2)\ln^2 A$ where $A = a_B R^{1/3}/a_H^{4/3} \gg 1$ in accord with Eq.(15).

The shift of the exciton levels due to tunneling depends both on X and x and should be averaged with appropriate wave functions of the radial motion in the skipping states (λ, determining the magnetic flux is now also renormalized, see Eq.(10)):

$$w = E_n + \left\langle \Delta_n(x, X)\cos 2\pi\left[\lambda\left(1 + \frac{2X}{R} + \frac{2\gamma x}{R}\right) - \frac{2J_\mu}{M}\frac{x}{R} - J\frac{m_n}{M}\right]\right\rangle. \tag{17}$$

By modelling the wave function of the triangular well in the form $\psi \sim X\exp(-X/a_0)$ we obtain for J =0:

$$w - E_n \sim \frac{\langle\Delta_n\rangle\cos 2\pi\lambda}{\lambda^{2/3}}. \tag{18}$$

This is a quite clear result: in comparison with 1D case given by Eq.(9) a decreasing factor $\lambda^{-2/3}$ has appeared because the magnetic flux embraced by electron-hole trajectories is now not strictly fixed to the value $\pi R^2 B$ but fluctuates in the skipping orbit states.

In conclusion, it is shown that in a strong magnetic field the ring shaped excitons can be formed around antidots due to skipping states of electrons and holes. The ground state energy level of such excitons is logarithmically deep in comparison with that of the "bulk" 2D exciton. Besides there is an exponentially small contribution to all the exciton levels oscillating as a function of magnetic flux with fundamental period hc/e.

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