Pis'ma v ZhETF, vol.62, iss.12, pp.923 - 928

SPECTRAL FLOW IN JOSEPHSON JUNCTIONS AND EFFECTIVE MAGNUS FORCE

Yu.G.Makhlin, G.E.Volovik

Low Temperature Laboratory
Helsinki University of Technology
02150 Espoo, Finland

L.D. Landau Institute for Theoretical Physics
117940 Moscow, Russia

Submitted 14 November 1995

Momentum production during the phase slip process in SNS Josephson junction is discussed. It is caused by the spectral flow of bound states of fermions localized within the junction. This effectively reduces the Magnus force acting on vortices which provides an explanation for the experimental observation of the negligible Magnus force in 2D Josephson junction arrays.

Vortex dynamics in 2D Josephson junction arrays has been studied intensively in recent years. The vortices in the arrays could be considered as massive particles with long-range Coulomb interaction [1, 2]. A particular attention was devoted to the forces acting on the vortices, one of them being Magnus force.

In the experiment [3] the straightforward ballistic motion of vortices was observed which implies the absence of reactive forces acting on a vortex perpendicular to its velocity. This was also confirmed by more recent experiments: vortices move perpendicular to the driving current [4], and no Hall effect was detected in the system [5].

An explanation of the absence of the Magnus force was proposed in Ref.[6]. The authors claim that the Magnus force is proportional to the offset charges on the superconducting islands, the effect of which being negligible. In a recent work [7] Gaitan and Shenoy argued that the Magnus force is proportional to the density of superconducting electrons on the islands averaged over distances large compared to the lattice constant of the array rather than to the charge of the island which is given by the difference in the numbers of electrons and protons. On the other hand Zhu, Tan and Ao [8] have shown the force to be proportional to the *local* superconducting density at the point where the vortex is situated. Since the vortex does not move through superconducting islands but through the junctions, the Magnus force on the vortex can be substantially reduced.

In the present paper we consider the forces on a vortex moving in a Josephson junction array, and propose a different explanation to the experiments mentioned above. It should be emphasized that one contribution to the force was missed in previous considerations [6–8] which is the force from the *spectral flow.* In uniform superfluids there exists the so called zero branch of energy levels of fermions localized within vortex core which crosses zero as a function of angular momentum [9]. Vortex motion leads to the flow of the levels along this branch, and energy of some levels crosses zero value. At low temperature these levels become occupied (or unoccupied, depending on whether they cross zero upwards or downwards). During this process the whole number of quasiparticles localized within the core is conserved, while the linear momentum of the quasiparticles is not conserved. This implies a transfer of the linear momentum from the vortex to the heat bath and thus an additional force acting on the moving vortex [10]. This force from the spectral flow can almost completely neutralize Magnus force [11] though in some regimes the spectral flow is suppressed and the net force appears. This scenario reproduces the microscopic calculations by Kopnin and co-authors [12].

We argue that an analogous situation takes place in Josephson junction arrays. Here the role of fermions localized within a vortex core is played by fermions localized within junctions. The phase slip events in junctions during vortex motion are accompanied by the spectral flow of the bound state fermions. The force from this spectral flow neutralizes Magnus force, and the reactive force on a vortex becomes negligible.

Since the cancellation of two contributions to reactive force is of topological origin and persists for several different geometries we consider the simplest case of the SNS junction. Other geometries and possible sources of difference between Magnus and spectral flow forces are under investigation and briefly discussed in conclusion section. Here we consider a square lattice of superconducting islands as plaquettes, the edges of the lattice representing normal layers in SNS-junctions. Vortices could be considered as sitting at vertices of the lattice (Fig.1). The 2D particle density of electrons n is assumed to be the same in the superconducting and normal regions.

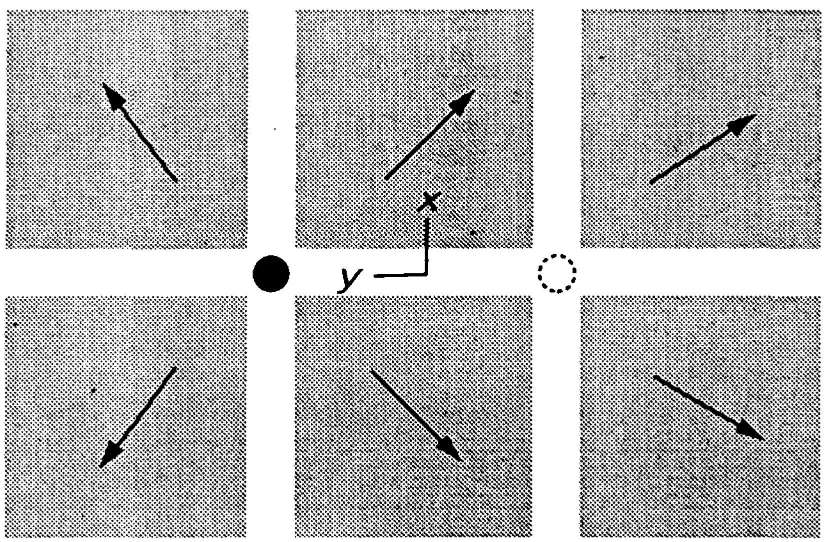

Fig.1. Josephson junction array. The squares represent superconducting islands and white spacings between them are normal intermediate layers in junctions. The distribution of phases of superconducting islands for a vortex sitting on a site of the lattice marked by a black circle is shown by arrows. When the vortex moves to the site marked by a dashed circle some levels of fermions localized within the junction between two central squares cross zero, and some amount of linear momentum k_x is produced. This corresponds to an additional force on the vortex which compensates conventional Magnus force.

As in the review [13] and Ref.[11] (see also [14]) we take it as granted that the conventional Magnus force on a vortex at low temperature is determined by the local n because n is the variable which is canonically conjugated to the phase

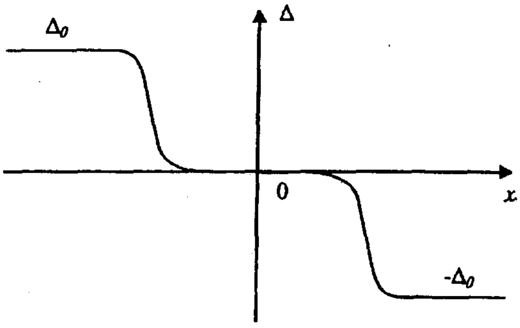

Fig.2. The dependence of superconducting order parameter on the coordinate x normal to a junction for phase difference of π between superconducting islands. Corresponding Bogolyubov-Nambu hamiltonian has zero energy eigenvalues, i.e. at the moment when ϕ crosses π the levels cross zero.

of the Bose-condensate:

$$\mathbf{F}_M = \pi n \hat{\mathbf{z}} \times \mathbf{v}_L \quad . \tag{1}$$

Here $\mathbf{v}_L$ is the velocity of the vortex with respect to the superfluid velocity, which is chosen to coincide with the velocity of the heat bath ($\mathbf{v}_s = \mathbf{v}_n = 0$). Note that in the low-temperature limit the superfluid density n_s tends to n irrespective of the magnitude of the order parameter gap Δ (actually due to the Iordanskii force from the heat bath, the Eq.(1) is valid even at nonzero T [13, 14]). This can be applied also to the electrons in "normal" regions with small magnitude of Δ. This expression does not contradict to Ref.[7] at $T = 0$, but we want to demonstrate that there is another force which nearly completely compensates the Magnus force.

During one step a vortex moves from a vertex to an adjacent one. We are going to calculate production of momentum due to spectral flow of fermions for such a process, which gives rise to the compensating force. The problem is in many details similar to the evolution of fermionic bound states, which exist within topological solitons in polymers, superfluid ^{3}He and other ordered systems in condensed matter with fermions [15]. The same type of evolution of the fermionic spectrum in topological solitons and sphalerons in particle physics leads to baryogenesis [16, 17], while in our case the spectral flow leads to "momentogenesis".

Let us consider one particular Josephson junction with x being the coordinate axis normal to the junction (Fig.1). The dependence of the eigenfunctions of Bogolyubov$-$Nambu hamiltonian

$$\mathcal{H} = \hat{\tau}_3 \cdot \hat{\epsilon} + \hat{\tau}_1 \mathrm{Re}\Delta - \hat{\tau}_2 \mathrm{Im}\Delta \tag{2}$$

on $\mathbf{r}_\perp = (y, z)$ is given by a factor $\exp(i\mathbf{k}_\perp \mathbf{r}_\perp)$. Here $\hat{\vec{\tau}}$ are Pauli matrices in Bogolyubov$-$Nambu space and $\hat{\epsilon} = (-\nabla^2 - k_F^2)/2m^*$ is the operator of energy of quasiparticles in normal liquid; k_F is the Fermi momentum. We suppose that the order parameter varies slow on the length scale of the coherence length, while the Fermi momentum remains constant, and that the mean free path is large enough for the electronic levels to be well defined. We are interested in the energies close to Fermi surface, so, we suppose that $k_\perp < k_F$ and the eigenfunction is represented in eikonal approximation:

$$\exp(iqx) \cdot \psi(x) \tag{3}$$

where $q^2 = k_F^2 - k_\perp^2$ (q plays a part of Fermi momentum of 1D Fermi liquid for given $\mathbf{k}_\perp$), the exponent represents fast oscillations in space, and $\psi(x)$ varies slowly. In this case we may substitute $\hat{\epsilon}$ by $q(-i\nabla)/m^*$.

We shall investigate the dependence of the energy spectrum on the phase difference $\Delta\phi$ between two superconducting islands. For simplicity we suppose that the order parameter is real $\Delta(x) = |\Delta(x)|$ on the one side of the junction ($x < 0$), and on the other side ($x > 0$) the order parameter is given by $\Delta(x) = |\Delta(x)|\cdot\exp(i\phi)$. For $\phi = \pi$ the order parameter is an odd real function of x (Fig.2) and the hamiltonian is supersymmetric: $\{\mathcal{H}, \tau_2\} = 0$. This $0 - \pi$ soliton corresponds to the sphaleron in particle physics [16, 17], i.e. the intermediate state between vacua with different topological charges. Due to supersymmetry the Hamiltonian has an eigenfunction

$$\tilde{\psi}_0 = \mathrm{const}\begin{pmatrix} 1 \\ -\mathrm{sign(q)}i \end{pmatrix}\psi_0,$$

$$\psi_0 = \exp\left(-\frac{m^*}{|q|}\int_0^x dx'\Delta(x')\right), \tag{4}$$

with zero eigenvalue. So, at $\phi = \pi$ for each q one energy level crosses zero. For small $\phi - \pi$ the perturbation of the hamiltonian is

$$\mathcal{H}_{int} = -(\phi - \pi)\Delta(x)\hat{\tau}_2 \tag{5}$$

for $x > 0$ and $\mathcal{H}_{int} = 0$ for $x < 0$. The energy level is shifted to

$$E = \mathrm{sign}(q)(\phi - \pi)\omega(|q|), \tag{6}$$

where

$$\omega(|q|) = \frac{\int_0^\infty dx|\Delta(x)|\psi_0^2(x)}{\int_0^\infty dx\psi_0^2(x)}. \tag{7}$$

From (6) it follows that at $\phi = \pi$, i.e. at the sphaleron, the energy level crosses zero upwards for $q > 0$ and downwards for $q < 0$. The similar phenomenon (cf. Refs.[18, 19] as well) was found for sphalerons in particle physics [16, 17].

This leads to the production of the x-component of the linear momentum

$$\Delta P = 2\cdot\frac{1}{2}A\int\frac{d^2k_\perp}{(2\pi)^2}2|q| = \frac{k_F^3}{3\pi}A. \tag{8}$$

The prefactor 2 stands for double spin degeneracy, and 1/2 removes the double counting of particle and hole momenta. Here A is the area of the cross-section of the junction, which is given by the product of the lattice constant a and the thickness l of the film along z-axis (which is perpendicular to the plane): $A = al$.

Using this equation one can find the spectral-flow force experienced by the vortex moving with respect to the heat bath. Since the velocity of the vortex is $v_L = a/\Delta t$, where Δt is the period of time during which the vortex crosses one junction, one has for the spectral-flow force in x direction

$$F_{sp.flow} = \frac{\Delta P}{\Delta t} = \frac{\Delta P v_L}{a} = \pi C_0 v_L, \tag{9}$$

where $C_0 = lk_F^3/3\pi^2$. In vector notations

$$\mathbf{F}_{sp.flow} = \pi C_0 \mathbf{v}_L \times \hat{\mathbf{z}}. \tag{10}$$

C_0 is very close to the 2D particle density $n = \rho l$ since the 3D density $\rho \approx k_F^3/3\pi^2$ with the accuracy of the order of $(\Delta/E_F)^2$. Therefore, the force induced by the spectral flow nearly neutralizes the Magnus force as anticipated.

Discussion. We have shown that in an ideal situation the energy levels for all values of q cross zero simultaneously at the moment when the phase difference between the islands equals π. In general case the energy levels may cross zero at different moments for different q. However, the production of linear momentum is still given by (8): The number of levels which cross zero is related to the asymmetry index which is a half of the difference between the numbers of negative and positive energy levels at given q. This index $N(q)$ can be expressed in terms of Green's function $\hat{G} = (i\omega - \hat{\mathcal{H}})^{-1}$:

$$N(q) = \int \frac{d\omega}{2\pi} \mathrm{Tr}\hat{G} = -\frac{1}{2}\sum_n \mathrm{sign}\, E_n(q), \tag{11}$$

where the summation is performed over eigenvalues for given q [20]. The change in $N(q)$ cannot depend upon the details of phase slip event but only on the initial and final states. Therefore, our derivation for a particular case is generalized to arbitrary phase slip events. One gets the same result calculating the Green's function in gradient expansion.

Our discussion shows that the production of momentum due to spectral flow is of topological origin, and we suppose that the result would not change with impurities taken into account when the levels cannot be classified in terms of q. This question is under investigation.

Let us now make some remarks on a more general case of the geometry of Josephson junction array when superconducting islands do not cover almost all area of a sample. If the core size of a vortex r_c is small compared to the lattice spacing a then one has the situation discussed in [8]: the vortex can be considered as a point-like object moving in the locally homogeneous environment with the slowly changing density of the electrons. In this case one can apply the bulk results [11]: the *en route* Magnus force will be canceled by the *en route* spectral-flow force for any route.

Let us consider another extreme limit $r_c \gg a$, which takes place if the Josephson coupling between the islands is small or superconducting islands cover a small part of the area. In this case, as it was discussed in Ref.[7], one can average over the distances of the order of the core size r_c and obtains the case of conventional homogeneous superfluid with r_c as a coherence length and a as "interatomic distance". The Magnus force in this case will be determined by the average number density $\bar{n}$ as it was stated in Ref.[7], however the spectral flow force on a vortex will be also proportional to average value of C_0. Thus they again will nearly compensate each other, but possibly with the relative accuracy of the order of $(a/r_c)^2$.

Thus there can be at least 3 different reasons which violate the fine tuning between the Magnus force and the spectral flow force: (1) The particle hole asymmetry gives the difference $\bar{n} - \bar{C}_0 \sim \bar{n}(\Delta/E_F)^2$ [10]. (2) Finite spacing ω_0 between the quasiparticle energy levels, which supresses the spectral flow of fermions, gives $\bar{n} - \bar{C}_0 \sim \bar{n}(\omega_0\tau)^2$ [11, 21]. In conventional Abrikosov vortices the spacing is $\omega_0 \sim \Delta^2/E_F$, while in the Josephson junction the discreteness arises due to the dimensional quantization along the axis y, which is parallel to the junction, or due to impurities. (3) The inhomogeneity discussed above may also give $\bar{n} - \bar{C}_0 \sim \bar{n}(a/r_c)^2$: for example, the vortex can prefer the pathes with appreciable $\omega_0\tau$, which leads to different *en route* values of n and C_0.

We are grateful to G. Schön and N.B. Kopnin for valuable discussions, and to F. Gaitan for sending us the preprint [7] and to S. Kashiwaya for the preprint [18]. This work was supported through the ROTA co-operation plan of the Finnish Academy and the Russian Academy of Sciences. G.E.V. was also supported by the Russian Foundation for Fundamental Sciences, Grant Nos. 93-02-02687 and 94-02-03121. Yu.G.M. was also supported by the International Science Foundation and the Russian Government, Grant No.MGI300, and by the "Soros Post-Graduate Student" program of the Open Society Institute.

---

1. U.Eckern and A.Schmid, Phys. Rev. **B 39**, 6441 (1989).
2. R.Fazio and G.Schön, Phys. Rev. **B 43**, 5307 (1991).
3. H.S.J. van der Zant, F.C.Frischy, T.P.Orlando, and J.E.Mooij, Europhys. Lett. **18**, 343 (1992).
4. S.G.Lachenmann, T.Doderer, D.Hoffmann et al., Phys. Rev. **B 50**, 3158 (1994).
5. H.S.J. van der Zant, M.N.Webster, J.Romijn, and J.E.Mooij, Phys. Rev. **B 74**, 4718 (1995).
6. R.Fazio, A. van Otterlo, G.Schön et al., Helv. Phys. Acta **65**, 228 (1992).
7. F.Gaitan and S.R.Shenoy, Preprint, cond-mat/9505088.
8. X.-M.Zhu, Yong Tan, and P.Ao, Preprint, cond-mat/9507126.
9. C.Caroli, P.G. de Gennes, and J.Matricon, Phys. Lett. **9**, 307 (1964).
10. G.E.Volovik, Pis'ma v ZhETF **57**, 233 (1993) [JETP Lett. **57**, 244 (1993)]; Zh. Eksp. Teor. Fiz. **104**, 3070 (1993) [JETP **77**, 435 (1993)].
11. N.B.Kopnin, G.E.Volovik, and Ü.Parts, Preprint, cond-mat/9509157, to be published in Europhys. Lett.
12. N.B.Kopnin and V.E.Kravtsov, Pis'ma ZhETF **23**, 631 (1976) [JETP Lett. **23**, 578 (1976)]; ZhETF **71**, 1644 (1976) [JETP **44**, 861 (1976)]; N.B.Kopnin and A.V.Lopatin, Phys. Rev. **B 51**, 15291 (1995).
13. E.B.Sonin, Rev. Mod. Phys. **59**, 87 (1987).
14. G.E.Volovik, Pis'ma ZhETF **62**, 58 (1995).
15. R.Jackiw and J.R. Schrieffer, Nucl. Phys. **B 190**, 253 (1981); A.J.Heeger, S.Kivelson, J.R.Schrieffer, and W.-P.Su, Rev. Mod. Phys. **60**, 781 (1988); T.L.Ho, J.R.Fulco, J.R.Schrieffer, and F.Wilczek, Phys. Rev. Lett. **52**, 1524 (1984).
16. N.Turok, Electroweak Baryogenesis, Preprint Imperial/TP/91-92/33
17. D.Diakonov, M.Polyakov, P.Sieber et al., Phys. Rev. **D 49**, 6864 (1995).
18. S.Kashiwaya, Y.Tanaka, M.Koyanagi, and K.Kajimura, "Bound states in superconductors", to be published in JJAP.
19. Y.Tanaka and S.Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
20. G.E.Volovik, Pis'ma ZhETF **49**, 343 (1989) [JETP Letters, **49**, 391 (1989)].
21. A. van Otterlo, M.V.Feigel'man, V.B.Geshkenbein, and G.Blatter, Phys. Rev. Lett. **75**, 3736 (1995).