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SUPERFLUID PROPERTIES OF MAGNETIZED ^{3}He-B IN DISORDERED MEDIUM

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Influence of nonmagnetic impurity scattering on the anisotropy of magnetized B phase of superfluid ^{3}He is investigated theoretically. Contribution of the impurity-induced spin-singlet superfluid correlations in presence of the p-wave Cooper pairing is revealed.

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A fundamental property of the Cooper-pair condensate is the symmetry of its order-parameter in spin and orbital spaces. Among various probes of this property the response of a superfluid state to the scattering of quasiparticles on the static (and dynamic) disorder is of great importance. One of the classical examples in this respect is strong influence of a small amount of paramagnetic impurities on the properties of low temperature superconductors with spin-singlet s-wave Cooper pairs, when spin-reversing scattering process destroys coherency of the state, leading to the suppression of T_c and to the formation of an ordered but gapless regime [1].

In contrast to magnetic impurities the isotropic scattering on nonmagnetic static disorder does not affect equilibrium properties of a superfluid state with isotropic orbital order-parameter (like in case of an s-wave pairing). On the other hand it was recognized long ago that in case of an ordered state with spontaneously broken orbital symmetry (spin-triplet Cooper pairing in the p-wave being and example) the nonmagnetic impurities should have a strong influence on superfluid properties of an ordered Fermi-system [2,3].

The idea of using nonmagnetic scattering as an efficient probe of the order-parameter symmetry became especially popular in the context of the Cuprate Superconductors and Heavy–Fermionic systems which are characterized (presumably) by unconventional Cooper pairing with yet unknown orbital symmetry of the order- parameter [4–6]. There are several experimental attempts to apply existing theoretical predictions in the aim to discriminate between possible s-wave-like and d-wave-like order-parameters in the Cuprate Superconductors (see, for instance, [7,8]).

Nontrivial influence of nonmagnetic impurities on the properties of superfluid phases of ^{3}He with spin-triplet p-wave order-parameters seemed to have an academic value due to the fact that this substance is one of the purest Fermi-systems with no sizable trace of impurities. Recent experiments [9], carried out on superfluid ^{3}He in the aerogel environment, transformed this academic problem to a practical one. As a first (but not the best) approximation one can model an intricate net of polymer strands in an aerogel by a disordered system of nonmagnetic scattering

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centers (with some effective concentration). Such an attempt have been made in recent investigations (see, for instance, [10]). In what follows we shall use this rather crude model in order to investigate nonmagnetic scattering effects on superfluid properties of ^{3}He-B placed in a magnetic field.

Our main concern is the evaluation of components of the superfluid density tensor of magnetically distorted B phase of liquid ^{3}He in a nonmagnetic disordered environment (aerogel). In the presence of a magnetic field $\mathbf{H} = H_0\mathbf{h}$ initially isotropic B phase acquires orbital anisotropy characterized by an axis $\hat{l}_B = \mathbf{h}\overleftrightarrow{R}$, where $\overleftrightarrow{R}$ is the matrix of 3D rotations of the spin space with respect to the orbital one. Uniaxial anisotropy in the flow effects is manifested in the tensorial character of superfluid density (below we drop the subscript at $\hat{l}_B$):

$$\rho^{(s)}_{ij} = \rho^{(s)}_{\parallel}\hat{l}_i\hat{l}_j + \rho^{(s)}_{\perp}(\delta_{ij} - \hat{l}_i\hat{l}_j) \ . \tag{1}$$

The degree of an anisotropy is characterized by the ratio $\rho^{(s)}_{\perp}/\rho^{(s)}_{\parallel}$. The influence of nonmagnetic scattering on the value of the anisotropy ratio for intrinsically anisotropic A phase of ^{3}He was considered some time ago in [11]. Here we address the same question to the case of magnetically induced anisotropy of ^{3}He-B.

In order to calculate $\rho^{(s)}_{\parallel}$ and $\rho^{(s)}_{\perp}$ we start with a general expression for the current density

$$j_i = 2\pi i k_F N_F T \sum_{\omega} < \hat{k}_i g_{\omega+iq}(\hat{k}) >, \tag{2}$$

where $g_\omega(\hat{k})$ is a scalar component of the ξ-integrated quasi-classical Green's function of the superfluid Fermi-system (in the Matsubara representation) and $q(\hat{k}) = k_F(\hat{k}\mathbf{v}_s)$ with $\mathbf{v}_s$ being the superfluid velocity. The unite vector $\hat{k}$ fixes the position on the Fermi- sphere and angular brackets in (2) denote averaging over these positions (k_F and N_F are Fermi-momentum and density of states at the Fermi-level, respectively).

Solving a set of equations (cf. Ref.[12]) for quasi-classical Green's functions

$$\hat{g}_\omega = g_\omega\hat{1} + \mathbf{g}_\omega\hat{\vec{\sigma}}^{tr}, \qquad \hat{f}_\omega = (f_\omega\hat{1} + \mathbf{f}_\omega\hat{\vec{\sigma}})i\hat{\sigma}_y, \tag{3}$$

it can be shown that in the presence of a magnetic field (which defines the value of Larmor frequency $\omega_0 = \gamma H_0$)

$$f_\omega = a_\omega g_\omega, \qquad \mathbf{f}_\omega = \mathbf{A}_\omega g_\omega, \qquad \mathbf{g}_\omega = -a_\omega\mathbf{A}_\omega g_\omega, \tag{4}$$

with a scalar component

$$g_\omega = -\frac{\omega}{\sqrt{\omega^2 + \mathbf{A}^2_\omega\omega^2}}\ \frac{1}{\sqrt{1 + a^2_\omega}}. \tag{5}$$

In Eq.(4)

$$\mathbf{A}_\omega = \frac{1}{\omega}(\vec{\Delta} + \frac{i}{2}\omega_0 a_\omega\mathbf{h}) \tag{6}$$

and a_ω is the solution of the quadratic equation

$$a_\omega = \frac{i}{2}\frac{\omega_0(\mathbf{h}\vec{\Delta})}{\omega^2 + \vec{\Delta}^2 + \frac{1}{4}\omega_0^2 + \frac{i}{2}\omega_0(\mathbf{h}\vec{\Delta})a_\omega} . \tag{7}$$

The order-parameter in case of spin-triplet Cooper pairing is given by $\vec{\Delta}(\hat{k})$ – a vector in the spin-space. $\vec{\Delta} = \vec{\Delta}_\parallel + \vec{\Delta}_\perp$, where $\vec{\Delta}_\parallel(\hat{k})$ and $\vec{\Delta}_\perp(\hat{k})$ are longitudinal and transversal components (with respect to the direction $\mathbf{h}$ of the magnetic field).

Below we shall keep lowest order terms in $\omega_0 \ll T_c$. In this approximation

$$g_\omega(\hat{k}) = -\frac{\omega}{\sqrt{\omega^2 + \vec{\Delta}^2(\hat{k})}}\left[1 + \frac{3}{8}\;\frac{\omega_0^2}{\omega^2 + \vec{\Delta}^2}\;\frac{\vec{\Delta}_\parallel^2}{\omega^2 + \vec{\Delta}^2}\right] \tag{8}$$

so that up to terms linear in $\mathbf{v}_s$ (in q)

$$g_{\omega+iq} \simeq g_\omega^{(0)} + g_\omega^{(1)} + \ldots, \tag{9}$$

where

$$g_\omega^{(1)} = -\frac{i}{3}\phi_\omega(\hat{k})q(\hat{k}) \tag{10}$$

with

$$\phi_\omega(\hat{k}) = \frac{3}{(\omega^2 + \vec{\Delta}^2(\hat{k}))^{3/2}}\left[\vec{\Delta}^2 - \frac{3}{2}\;\frac{\omega_0^2}{\omega^2 + \vec{\Delta}^2}\left(1 - \frac{5}{4}\;\frac{\vec{\Delta}^2}{\omega^2 + \vec{\Delta}^2}\right)\vec{\Delta}_\parallel^2\right]. \tag{11}$$

By definition

$$\rho_{ij}^{(s)} = \rho\pi T\sum_\omega < \phi_\omega(\hat{k})\hat{k}_i\hat{k}_j > \tag{12}$$

where ρ is the density of liquid ^{3}He. Expending Eq.(11) to the lowest order in $\epsilon = \vec{\Delta}^2 - \Delta_0^2(T) \sim \omega_0^2(\Delta_0(T)$ being an amplitude of the order-parameter of isotropic ^{3}He-B in zero magnetic field) and noticing that in the term proportional to ω_0^2

$$\vec{\Delta}^2(\hat{k}) \simeq \Delta_0^2(T), \qquad \vec{\Delta}_\parallel^2(\hat{k}) \simeq \Delta_0^2(T)(1 - (\hat{k}\hat{l})^2), \tag{13}$$

we easily obtain that in case of a pure magnetized ^{3}He-B

$$\rho_\parallel^{(s)}/\rho = Z_3(T) + \frac{1}{\Delta_0^2}\left(\frac{2}{5}\Delta_\perp^2(T) + \frac{3}{5}\Delta_\parallel^2(T) - \Delta_0^2\right)\left(Z_3 - \frac{3}{2}Z_5\right) - \\ - \frac{9}{10}\frac{\omega_0^2}{\Delta_0^2}\left(Z_5 - \frac{5}{4}Z_7\right), \tag{14}$$

$$\rho_\perp^{(s)}/\rho = Z_3(T) + \frac{1}{\Delta_0^2}\left(\frac{4}{5}\Delta_\perp^2 + \frac{1}{5}\Delta_\parallel^2 - \Delta_0^2\right)\left(Z_3 - \frac{3}{2}Z_5\right) - \\ - \frac{3}{10}\frac{\omega_0^2}{\Delta_0^2}\left(Z_5 - \frac{5}{4}Z_7\right), \tag{15}$$

where

$$Z_n(T) = \pi T \sum_{\omega} \frac{\Delta_0^{n-1}}{(\omega^2 + \Delta_0^2)^{n/2}}. \tag{16}$$

The missing Fermi-liquid effects can be easily incorporated in a standard way.

Turning now to nonmagnetic impurity effects we have to perform following renormalization procedure:

$$\begin{aligned} \omega &\rightarrow \tilde{\omega} = \omega + iM_{\omega}, \\ \omega_0 &\rightarrow \tilde{\omega}_0 = \omega_0 - 2\mathbf{h}\mathrm{M}_{\omega}, \\ \vec{\Delta} &\rightarrow \tilde{\vec{\Delta}}_{\omega} = \vec{\Delta} + \mathbf{m}_{\omega}, \end{aligned} \tag{17}$$

where in the Born approximation with respect to isotropic nonmagnetic impurity scattering potential u_0

$$M_{\omega} = \frac{i}{2\tau} < \tilde{g}_{\omega}(\hat{k}) >, \quad \mathrm{M}_{\omega} = \frac{i}{2\tau} < \tilde{\mathrm{g}}_{\omega}(\hat{k}) >, \quad \mathbf{m}_{\omega} = -\frac{i}{2\tau} < \tilde{\mathbf{f}}_{\omega}(\hat{k}) >, \tag{18}$$

with $1/\tau = 2\pi c_i u_0^2 N_F$, c_i being the concentration of impurities.

In equilibrium (in the absence of a superflow) order-parameter renormalization $\mathbf{m}_{\omega} = 0$ from the symmetry reasons and this property of spin-triplet p-wave Cooper pairing gives life to strong nonmagnetic impurity effects in superfluid ^{3}He. At $\mathbf{v}_s \neq 0$, $\mathbf{m}_{\omega+iq} \neq 0$ and, as we shall see, in the presence of magnetic field and of impurity potential a spin-singlet component

$$\tilde{\Delta}_{\omega} = 0 + m_{\omega+iq} = -\frac{1}{2\tau} < \tilde{f}_{\omega+iq}(\hat{k}) > \neq 0 \tag{19}$$

appears in the expression for the superfluid density. It seems to be a unique situation when combined action of impurity scattering and of magnetic field generates a spin-antisymmetric superfluid correlations in the Fermi-system with Cooper pairing in the *p*-wave.

Performing impurity-induced renormalization it can be shown that in the linear approximation with respect to $\mathbf{v}_s$

$$\begin{aligned} \tilde{g}_{\omega}^{(1)} \simeq \frac{1}{(\omega^2 + \Delta_0^2)^{3/2}} \Bigg\{ & -i\vec{\Delta}^2 q + \tilde{\omega}(\vec{\Delta}\mathbf{m}_{\omega}^{(1)}) + \\ + \frac{3}{2} \; \frac{\tilde{\omega}_0^2}{\tilde{\omega}^2 + \vec{\Delta}^2} & \left[i(1 - \frac{5}{4} \; \frac{\vec{\Delta}^2}{\tilde{\omega}^2 + \vec{\Delta}^2})\vec{\Delta}_{\parallel}^2 q - \frac{1}{2}\tilde{\omega} \left(\vec{\Delta}_{\parallel}\mathbf{m}_{\parallel}^{(1)} - \right.\right. \\ \left.\left. - \frac{5}{2}\vec{\Delta}\mathbf{m}_{\omega}^{(1)} \; \frac{\vec{\Delta}_{\parallel}^2}{\tilde{\omega}^2 + \vec{\Delta}^2} \right) \right] & + i\tilde{\omega}_0(\mathbf{h}\vec{\Delta}) \left(1 - \frac{3}{2} \; \frac{\vec{\Delta}^2}{\tilde{\omega}^2 + \vec{\Delta}^2} \right) m_{\omega}^{(1)} \Bigg\}, \end{aligned} \tag{20}$$

where

$$\mathbf{m}_{\omega}^{(1)} = -\frac{1}{2\tau} < \tilde{\mathbf{f}}_{\omega}^{(1)}(\hat{k}) >, \quad m_{\omega}^{(1)} = -\frac{1}{2\tau} < \tilde{f}_{\omega}^{(1)}(\hat{k}) >, \tag{21}$$

Using explicit expressions for $\tilde{\mathbf{f}}_{\omega}^{(1)}$ and $\tilde{f}_{\omega}^{(1)}$ (which are not reproduced here) and addressing to Eq.(2) it can be shown that in the Ginzburg-Landau region (where $(\Delta_{\parallel}, \Delta_{\perp}) \ll T_c$) the contribution of the spin-singlet component to the superfluid density tensor (steaming from the last term in (20)) is given by

$$\delta\rho_{ij}^{(s)} \simeq -\frac{1}{3}\pi T\sum_{\omega}\frac{1+1/4\tau|\omega|}{2\tau|\omega|}\left(\frac{\omega_0}{\tilde{\omega}}\right)^2\frac{\Delta_\|^2}{|\tilde{\omega}|^3}\hat{l}_i\hat{l}_j, \tag{22}$$

where $\tilde{\omega}=\omega+1/2\tau$. Near T_c the singlet contribution to $\rho_{ij}^{(s)}$ (which is proportional to the square of longitudinal component of the order-parameter) is small in comparison with the main part (see Eq.(23) below) but at low temperatures it becomes significant. Disregarding (22) we obtain that in the Ginzburg–Landau regime

$$\rho_\perp^{(s)}/\rho=\pi T\sum_{\omega}\frac{1}{|\tilde{\omega}|^3}\left(\frac{4}{5}\Delta_\perp^2+\frac{1}{5}\Delta_\|^2+\frac{1}{6\tau|\omega|}\Delta_\perp^2\right),$$

$$\rho_\|^{(s)}/\rho=\pi T\sum_{\omega}\frac{1}{|\tilde{\omega}|^3}\left(\frac{2}{5}\Delta_\perp^2+\frac{3}{5}\Delta_\|^2+\frac{1}{6\tau|\omega|}\Delta_\|^2\right), \tag{23}$$

where $\Delta_\|$ and $\Delta_\perp$ are amplitudes of longitudinal and transversal components of the order-parameter of magnetically distorted ^{3}He-B in presence of nonmagnetic scattering centers.

For the superfluid density anisotropy ratio we have:

$$\rho_\perp^{(s)}/\rho_\|^{(s)}=2\frac{1+\frac{1}{4}\frac{\Delta_\|^2}{\Delta_\perp^2}+\frac{1}{2}I(w)}{1+\frac{3}{2}\frac{\Delta_\|^2}{\Delta_\perp^2}+\frac{\Delta_\|^2}{\Delta_\perp^2}I(w)}, \tag{24}$$

where impurity scattering effects are accumulated in

$$I(w)=\frac{5}{6}w\frac{S_5(w)}{S_3(w)},\quad w=\frac{1}{4\pi\tau T_c} \tag{25}$$

with

$$S_3(w)=\sum_{n\geq 0}\frac{1}{(n+1/2+w)^3},\quad S_5(w)=\sum_{n\geq 0}\frac{1}{(n+1/2)(n+1/2+w)^3}. \tag{26}$$

In the vicinity of T_c the two regions are to be distinguished:

a) Immediate proximity to the planar state, where $\Delta_\| \ll \Delta_\perp$ ($\Delta_0\simeq\omega_0$). For the planar state ($\Delta_\|=0$)

$$\rho_\perp^{(s)}/\rho_\|^{(s)}=2+I(w)\ . \tag{27}$$

This result (signaling the impurity enhancement of the anisotropy) exactly coincides (not unexpectadly) with an answer found for the A phase [11].

b) Temperatures not too close to the planar state, where $\Delta_\|\simeq\Delta_\perp$ ($\Delta_0\gg\omega_0$). In this case

$$\Delta_\|^2/\Delta_\perp^2\simeq 1-\frac{5}{4}\frac{\omega_0^2}{\Delta_0^2}, \tag{28}$$

and

$$\rho_{\perp}^{(s)}/\rho_{\parallel}^{(s)} \simeq 1+\frac{1}{2}\ \frac{\omega_0^2}{\Delta_0^2}\ \frac{1+I(w)}{1+\frac{2}{5}I(w)}. \tag{29}$$

It should be stressed that our consideration is based on the weak coupling approximation justified at low pressures.

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