

## THEORY OF THE DE HAAS VAN ALPHEN EFFECT IN DOPED SEMICONDUCTORS

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Field dependence of magnetisation in doped semiconductors, like InSb, InAs, GaAs etc., under strong magnetic fields is studied. The standard theory of de Haas van Alphen effect, mostly applicable to metals, is modified to include the long range fluctuations of charged carriers. The experimental investigation of this effect can brighten up some open questions of semiconductor physics, e.g. problem of tails in the electronic density of states. It is shown that in such systems the mean magnetisation is sensitive to magnetic interactions.

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1. Quantum oscillations of magnetisation, the de Haas van Alphen effect, (dHvA), are known to be a very effective tool for studying the Fermi surfaces of metals [1]. In semiconductors, however, the observation of dHvA is much more difficult, because of low electron concentration, and it is the quantum oscillations of magnetoresistance, the Shubnikov de Haas effect (ShdH) which found a very wide application. The rich information about the concentration of carriers and scattering mechanisms in semiconductors, contained in ShdH is not easily interpreted, since both, the density of states and the relaxation time, entering the magnetoresistance, oscillate with the field.

dHvA is a thermodynamic probe and is, therefore, much easier to interpret. Recent progress in dHvA measurements in low-dimensional semiconductors [2], makes the study of dHvA in doped, degenerate, semiconductors to be very actual.

The principal difference of the dHvA theory in semiconductors from the standard Landau-Lifshitz-Kosevich (LLK) theory [3,1] which was so successful in metals follows from much lower electron concentration and, as consequence, weaker screening of charged impurities. Low concentration in high magnetic fields results in small number of occupied Landau levels,  $n_F \equiv \frac{\mu_0}{\hbar\omega_c}$ , here  $\mu_0$  is the Fermi energy and  $\hbar\omega_c$  is the distance between the Landau levels,  $\omega_c = \frac{eB}{m^*c}$  is the cyclotron frequency of an electron in a magnetic field  $B$ . This contradicts to the basic approximation of LLK theory, that  $n_F \gg 1$  and to the related assumption that the Fermi energy is practically field independent. In semiconductors with small effective masses,  $m^*$ , of carriers, the distance between the Landau levels is huge, compared to the temperature, which results in strong quantum oscillations of chemical potential.

Small effective masses and large dielectric constant,  $\epsilon_0$ , result in the fact that the extreme quantum limit can be reached already in magnetic fields of  $B \simeq 10$  T,

i.e. the chemical potential,  $\mu$ , is of order of  $\hbar\omega_c$ , and only the lowest Landau level is occupied.

The weakly nonideal electron gas is described by set of parameters [4]:

$$g = \frac{e^2}{\epsilon_0 \pi \hbar v_F}, \quad n = \frac{p_F^3}{3\pi^2 \hbar^3}, \quad \mu_0 = \frac{p_F^2}{2m^*}, \quad v_f = \frac{p_F}{m^*}. \quad (1)$$

The condition of weakly compressed gas,  $g \ll 1$ , is fulfilled, even if the electron density in a doped semiconductor, say  $n_0 \simeq 10^{16} \text{ cm}^{-3}$  is by six orders of magnitude lower than that in normal metals. The Landau magnetic susceptibility,  $\chi_0$ , is, therefore, of the same order of magnitude as that a metal:

$$\chi_0 = -\frac{e^2}{12\pi^2 \hbar v_F} \left(\frac{v_F}{c}\right)^2; \quad (2)$$

and can be measured by standard methods [1].

2. Here we develop a theory of magnetic orbital properties of the electron gases in the limit  $g \ll 1$ . In these limit three characteristic parameters can be introduced: the magnetic length,  $\ell_B = \sqrt{\frac{\hbar c}{eB}}$ ; the mean distance between the electrons,  $a \propto \frac{1}{n^{1/3}}$  and the screening length,  $r_0 \propto \frac{a}{\sqrt{g}}$ .

We introduce, in the limit  $\ell_B < r_0$ , the mean potential  $V$ , which is the effective shift of the Landau levels in smooth potential fluctuations,  $V$ :

$$E_n^\sigma(p, V) = V + \frac{p_x^2}{2m^*} + \hbar\omega_c \left(n + \frac{1}{2}\right) \pm \frac{\hbar\omega_\sigma}{2}; \quad (3)$$

here  $p_x$  is the electron momentum along the magnetic field  $B$  and  $\hbar\omega_\sigma = \frac{\hbar e B}{m_0 c}$  is the Zeeman splitting of the electron energy levels.

To calculate the thermodynamic potential,  $\Omega$ , and the magnetic moment  $M = -\frac{\partial \Omega}{\partial B}$  we proceed as follows. First we calculate the local value,  $\Omega(V)$ , at fixed value of  $V$ , and then we average over  $V$ :

$$\bar{\Omega} = \frac{1}{V_0 \sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{V^2}{V_0^2}} \Omega(V) dV. \quad (4)$$

The justification of such an average could be given using the standard technics [4].

Similar procedure can be applied to the electron Green function:

$$G = \frac{1}{\sqrt{\pi} V_0} \int_{-\infty}^{\infty} \frac{e^{-\frac{V^2}{V_0^2}} dV}{\epsilon - \frac{p^2}{2m^*} + \mu + V} \quad (5)$$

which yields the following characteristic fluctuation scale:

$$V_0^2 = 2n \left(\frac{4\pi e^2}{\epsilon_0}\right)^2 \int \frac{d^3 k}{(2\pi)^3 (k^2 + \kappa^2)^2} \simeq \mu_0^2 g^{\frac{2}{3}} \frac{8\pi}{3},$$

$$\kappa^2 = \left(\frac{4\pi e^2}{\epsilon_0}\right) \frac{\partial n}{\partial \mu} \simeq \frac{4g p_F^2}{\hbar^2}. \quad (6)$$

In estimations, Eq.(6), we assume the value of  $V_0$  and of the inverse screening length,  $\kappa$ , at  $B = 0$ . In strong magnetic field, however, the dependence of these parameters on  $B$  is strong due to the magnetic field dependence of the parameters  $n$  and  $\frac{dn}{d\mu}$ , defined as:

$$n = -\frac{\partial \bar{\Omega}}{\partial \mu}; \quad \frac{dn}{d\mu} = -\frac{\partial^2 \bar{\Omega}}{\partial^2 \mu} \quad (7)$$

It follows, from Eq. (6), that in the limit  $g \rightarrow 0$  potential  $V$  is smooth, since the integral, defining the value of  $V$ , diverges as  $\kappa \rightarrow 0$ . The smallness of  $g$  results, therefore, in the smallness:  $V_0 \ll \mu_0$  and in the smallness of the inverse of the screening length,  $\kappa \ll \frac{2F}{\hbar}$ . This justifies the assumption of the linear screening of the density fluctuations of the charged impurities.

3. Let us calculate now the thermodynamic potential,  $\Omega(V)$ . In contrast to the standard theory of dHvA in metals, [3], we do not assume that the Fermi energy,  $\mu_0$ , is much higher than the distance between the Landau levels,  $\hbar\omega_c$ .

The standard expression for the thermodynamic potential is:

$$\Omega = -\lambda \hbar\omega_c \sum_{n,\sigma} \int_0^\infty \frac{\sqrt{\epsilon_1} d\epsilon_1}{\exp\left[\frac{E_n - \mu_\sigma^\pm}{T}\right] + 1} \quad (8)$$

Here:

$$\lambda = \frac{(2m^*)^{\frac{3}{2}}}{4\pi^2 \hbar^3}, \quad E_n = \epsilon_1 + \hbar\omega_c \left(n + \frac{1}{2}\right) \text{ and } \mu_\sigma^\pm = \mu + V \pm \frac{\omega_\sigma}{2}$$

To perform the summation over the Landau levels,  $n$ , we apply the Poisson summation formula in the following form:

$$\hbar\omega_c \sum_{n=0}^\infty \phi(n) = \int_{-\frac{\hbar\omega_c}{2}}^\infty \phi(\epsilon_2) d\epsilon_2 + 2 \sum_{k=1}^\infty \int_{-\frac{\hbar\omega_c}{2}}^\infty \phi(\epsilon_2) \cos\left(2\pi k \frac{\epsilon_2}{\hbar\omega_c}\right) d\epsilon_2 \quad (9)$$

where, in our case:

$$\phi(\epsilon_2) = \left[1 + \exp\left(\frac{\epsilon_1 + \epsilon_2 - \mu_\sigma^\pm}{T}\right)\right]^{-1} \quad (10)$$

After several integrations by parts and going to the polar coordinates in the  $\epsilon_1, \epsilon_2$  plane, we arrive at the following expression for the thermodynamic potential:

$$\Omega(V) = -\lambda \sum_\sigma \int_0^\infty \frac{\rho(E) dE}{4T \operatorname{ch}^2\left(\frac{E - \mu_\sigma^\pm}{2T}\right)}, \quad (11)$$

$$\rho(E) = \frac{4}{15} E^{\frac{5}{2}} - \frac{\hbar^2 \omega_c^2}{24} \sqrt{E} + \frac{1}{4\pi^2} \sum_{k=1}^\infty (-1)^{k+1} \left(\frac{\hbar\omega_c}{k}\right)^{\frac{5}{2}} [\cos(u^2) C(u) + \sin(u^2) S(u)] \quad (12)$$

where:  $u^2 = 2\pi k \frac{E}{\hbar\omega_c}$ ;  $C(u)$  and  $S(u)$  are the Fresnel' functions:

$$C(u) = \sqrt{\frac{2}{\pi}} u \int_0^1 \cos(u^2 t^2) dt; \quad S(u) = \sqrt{\frac{2}{\pi}} u \int_0^1 \sin(u^2 t^2) dt.$$

In the interval  $0 < E < \frac{\hbar\omega_c}{2}$  the density of states,  $\rho(E)$ , in Eq. (12), vanishes. The oscillatory part of  $\rho(E)$  is obtained using the following relation:

$$\cos(u^2)C(u) + \sin(u^2)S(u) = \frac{1}{\sqrt{2}} \cos\left(u^2 + \frac{\pi}{4}\right) + \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-2ut} \sin t^2 dt. \quad (13)$$

4. In the low-temperature limit,  $T \rightarrow 0$ , the function  $\rho(E)$  can be taken out of the integral, Eq. (11), at  $E = \mu_\sigma^+$  and the oscillatory part of the local thermodynamic potential,  $\Omega_0(V)$  takes the following form:

$$\Omega_0(V) = \sum_{k=1}^{\infty} \Omega_0^k(V), \quad (14)$$

where:

$$\Omega_0^k(V) = \frac{\lambda}{2\sqrt{2}\pi^2} (-1)^k \left(\frac{\hbar\omega_c}{k}\right)^{\frac{5}{2}} \cos\left(\frac{m^*}{m}\pi k\right) \cos\left[2\pi k \frac{(\mu + V)}{\hbar\omega_c} + \frac{\pi}{4}\right].$$

At finite, but low ( $T \ll \mu$ ), temperatures [3] an additional prefactor

$$\Psi_k(Z) = \frac{Z}{\text{sh}Z}; \quad Z = \frac{2\pi^2 kT}{\hbar\omega_c} \quad (15)$$

will appear in Eq. (14)

The averaged value of  $\bar{\Omega}_0^k(V)$  is that of  $\Omega_0^k(V=0)$  with an exponential prefactor:

$$\bar{\Omega}_0 \equiv \sum_{k=1}^{\infty} \bar{\Omega}_0^k; \quad \bar{\Omega}_0^k = \Omega_0^k(V=0) \exp\left\{-\frac{\hbar^2 \pi^2 V_0^2}{k^2 \omega_c^2}\right\} \quad (16)$$

Let us analyze, now, the obtained results. It follows, from Eq.(14), that as the local potential,  $\Omega(V)$ , so and the local magnetic moment,  $M(V)$ , will oscillate with  $B$ . The phase of these oscillations, however, has the Gaussian distribution, Eq. (4).

In a weak field,  $\hbar\omega_c < V_0$ , the magnetic quantum oscillations of the whole sample are exponentially small, Eq. (16). In a strong field limit,  $\hbar\omega_c > V_0$ , however, the magnetic moment, averaged over the whole sample, will oscillate with field.

Since  $V_0 \propto n^{\frac{5}{2}}$  and  $\mu_0 \propto n^{\frac{3}{2}}$ , Eq.(6), there is always a region, in the electron density, such that  $\hbar\omega_c > V_0$  and  $\hbar\omega_c < \mu_0$  simultaneously. At such densities the quantum oscillations in doped semiconductors are similar to ones in a metal, Eq. (16).

5. In sufficiently pure metals at low temperatures the electron spectrum,  $\omega_c = \frac{eB}{mc}$ ; is defined by the magnetic induction,  $B = H + 4\pi M(B)(1 - n)$ , where  $M(B)$  is the magnetic moment of a sample, and  $n$  is the demagnetizing factor, depending on the sample shape. This results in a richer harmonic content of dHvA oscillations, (the Shoenberg effect, or magnetic interactions, [1]), than it follows from the Lifshitz-Kosevich theory, [3], where the electron spectrum is defined as  $\omega_c^0 = \frac{eH}{mc}$ . Adding impurities usually reduces the amplitude of magnetic quantum oscillations,  $M(B)$ , thus making the difference between  $H$  and  $B$  negligible and additional harmonics disappear.

Amazingly enough, while the random potential smears out the magnetic oscillations, the average magnetisation contains a measurable term, following from the difference between the external magnetic field  $H$  and the magnetic induction,  $B$ :

$$\bar{M} = \chi_0 H \left( 1 - C_0 \chi_0 \frac{\mu}{\hbar \omega_c^0} \right) \quad (17)$$

where:

$$\omega_c^0 = \frac{eH}{m^*c} \gg T$$

and  $C_0$  is a number.

While the first term in Eq. (17) is the usual Landau susceptibility, the second term is a nonanalytical in field,  $H$ , contribution to  $\frac{\bar{M}}{H}$ , which arises from the difference between  $\omega_c$  and  $\omega_c^0$ .

To see why the average magnetic susceptibility is so sensitive to magnetic interactions, consider a system of disoriented small metallic clusters with anisotropic Fermi surface. Assuming that each metallic particle is clean enough, we expect it to display magnetisation oscillations, influenced by magnetic interaction.

The magnetic induction:

$$B(V) = H + 4\pi(1 - n)M(V) \quad (18)$$

contains, now, the oscillatory magnetic moment,  $M(B, V)$ . It follows that, while averaging over impurity realisations, the thermodynamic values, like  $\Omega(V)$  and  $M(V)$ , will contain quadratic in oscillatory amplitude, terms, surviving the averaging over the random phase of the magnetic oscillations in each metallic cluster. Note, that the measured value is  $\bar{M}(H)$  rather than  $\bar{M}(B)$ .

Consider now the experimental feasibility for observation of magnetic interaction term in the average magnetisation. At  $T < \hbar \omega_c^0 < \mu$ , this contribution can be experimentally observed, since it contains a large factor,  $\frac{\mu}{\hbar \omega_c^0} \simeq 10^3$  in metals, compensating a  $\chi_0 \simeq 10^{-4}$ . Similar consideration can be given for a doped semiconductor.

6. Let us estimate, now, the upper limit of the doping, when our theory is still valid. In low fields,  $\hbar \omega_c < \mu_0$ , the thermodynamic parameters,  $\chi, \kappa$  and  $V_0$ , Eq. (6), have small oscillatory, with magnetic field, corrections to their  $B = 0$  value. In a strong field, however, this will result in weakening of the long wave potential fluctuations of charged impurities. In the limit,  $\hbar \omega_c > \mu_0$ , we obtain, from Eq. (11):

$$\mu = \frac{1}{2} \hbar \omega_c + \frac{4}{9} \left( \frac{\mu_0}{\hbar \omega_c} \right)^2,$$

$$\kappa^2(B) = \frac{3}{4} \left( \frac{\hbar \omega_c}{\mu_0} \right)^2 \kappa^2(0),$$

$$V_0^2(B) = V_0^2(0) \frac{\kappa_0}{\kappa} \quad (19)$$

with  $V_0^2(0)$  and  $\kappa^2(B)$  defined by Eq.(6). It follows, from Eq. (19) that in strong enough magnetic field the screening length,  $r_0 = \kappa^{-1}$ , becomes smaller than

magnetic length ,  $\ell_B$ , and the magnetic length becomes smaller than the mean distance between the electrons:

$$r_0 \equiv \frac{1}{\sqrt{g}} \frac{\ell^2}{a} \ll \ell_B; \quad \ell_B = \sqrt{\frac{\hbar c}{eB}} \ll a. \quad (20)$$

In this limit the metall-insulator transition will take the place [5], and the electrons will be localized on the charged impurities, thus smoothing out the potential fluctuations:

$$V_0(B) < V_0(0) < \hbar\omega_c \quad (21)$$

To conclude, we have developed the theory of dHvA in doped semiconductors. We outline that dHvA can be used to shed new light on some open questions in semiconductor physics. Among them is the problem of tails in the electronic density of states, which, while treated by standard methods, yields the results, which contradict with the Urbach rule, known in the optical studies of semiconductors [2]. That is, the Gaussian distribution of charged impurities fluctuation density (4.5) contradicts with the existing experimental data. The reasons of this contradiction between theory and experiment is not clear up to now. It is shown also that the mean magnetization is sensitive to nonlinear effects, like magnetic interactions.

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