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GLASS STATE OF SUPERFLUID ^{3}He-A IN AEROGEL

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The glass states formed in superfluid ^{3}He confined in aerogel are discussed. If the short range order corresponds to the A-phase state, the glass state is nonsuperfluid in the long wave length limit. The superfluidity can be restored by application of a small mass current. The transitions between the superfluid and nonsuperfluid glass states can be triggered by small magnetic field and by the change of the tipping angle of magnetization in NMR experiments.

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In aerogel there is 98% of the empty volume, which is occupied by ^{3}He, while the other 2% has mainly the shape of tangled silica strings or strands. The string radius is about $R = 20 - 30$ A°, and the average distance between them is about $l \sim 1000 A^\circ$ [1, 2]. Since R is much less than the superfluid coherence length ξ_0, which changes in the range 200-800 A^o depending on the pressure, these strings have an effect of anisotropic and randomly oriented impurities. In the triplet p-wave condensate such impurities have several different effects: (i) They suppress both the condensate and the superfluid transition temperature T_c, which is experimentally measured [1, 2]. (ii) They produce the renormalization of the coefficients in fourth order terms in the Ginzburg–Landau functional, which according to [3] triggers the transition between different superfluid phases in the anisotropic heavy fermionic superconductors. The same arguments applied to the ^{3}He in aerogel [4] explain why instead of the B-phase state in the bulk superfluid ^{3}He the magnetic experiments [2] provide an evidence of the A-phase (or of another equal pairing state). (iii) Also the random local anisotropy produced by the string network leads according to Imry and Ma [5] to the violation of the orientational long range order at large distances. This was discussed for anisotropic superconductors in [6], where the state without the long range orientational order was called the superconducting glass.

Here we discuss some properties of the "superfluid glass" states of the ^{3}He in the aerogel. In the case of the exotic superconductivity described by the vector order parameter the effect of the random anisotropy is suppressed by the regular anisotropy of the crystal lattice, as a result the superconducting glass can arise only in a small strip of the phase diagram very close to T_c [6]. In superfluid ^{3}He the external regular anisotropy is absent that is why the superfluid ^{3}He in aerogel is always in the glass state in the absence of the external magnetic field. Application of the rather weak magnetic field can restore the conventional long range order.

The main property of the glass state is that the expectation value of the order parameter (averaged over the large enough volume) vanishes: $< A_{\alpha i}(\mathbf{r}) >= 0$. In other words the correlation of the order parameter decays at large distances:

$< A_{\alpha i}(\mathbf{r}_1)A_{\beta j}(\mathbf{r}_2) >= 0$ if $(\mathbf{r}_1 - \mathbf{r}_2)/L_0 \to \infty$. Here L_0 is the characteristic length, within which the order parameter is homogeneous. The value of L_0 will be discussed below. Some part of the long range order can still be remained and it is described in terms of bilinear combinations, such as

$$Q_{\alpha\beta ij} =< A_{\alpha i}(\mathbf{r})A^*_{\beta j}(\mathbf{r}) >, \quad (1)$$

or

$$P_{\alpha\beta ij} =< A_{\alpha i}(\mathbf{r})A_{\beta j}(\mathbf{r}) > . \quad (2)$$

Let us assume that the short range order corresponds to the A-phase, which is consistent with the magnetic measurements [2]. The local order parameter of the A-phase is $A_{\alpha i}(\mathbf{r}) = \Delta d_\alpha(e_{1i}(\mathbf{r}) + ie_{2i}(\mathbf{r}))$ [7], where $\hat{d}$ is the unit vector of spin anisotropy, and the orthogonal unit vectors $\hat{e}_1$ and $\hat{e}_2$ describe the orbital part of the order parameter, which is randomly oriented in space. Assuming that the strands violate only the orbital orientational order, one has

$$P = 0, \quad Q_{\alpha\beta ij} = Qd_\alpha d_\beta \delta_{ij} . \quad (3)$$

This long range order corresponds to the nonsuperfluid spin nematic. The quantity Q is gauge invariant and thus the state with $Q \neq 0$, $P = 0$ is nonsuperfluid.

The difference between the superfluid and nonsuperfluid states is given by the loop function [8]:

$$< e^{i(2\pi/\kappa)\oint_L \mathbf{v}_s \cdot d\mathbf{r}} >,$$

where $\kappa = \pi\hbar/m_3$ is the circulation quantum in ^{3}He and integral is along the loop of length L. In superfluids the loop function decays as e^{-L} or slower, while in the nonsuperfluid state it decays e^{-S}, where $S \sim L^2$ is the area of the loop. Using the Mermin-Ho relation [9], which couples the superfluid velocity with the spatial variation of the randomly oriented orbital anisotropy vector $\hat{l} = \hat{e}_1 \times \hat{e}_2$:

$$\vec{\nabla} \times \mathbf{v}_s = \frac{\kappa}{4} e_{ijk} \hat{l}_i \vec{\nabla} \hat{l}_j \times \vec{\nabla} \hat{l}_k , \quad (4)$$

one finds the area law for the state in Eq.(3):

$$< e^{i(2\pi/\kappa)\oint_L \mathbf{v}_s \cdot d\mathbf{r}} > \propto e^{-L^2/L_0^2}, \quad L \gg L_0 . \quad (5)$$

The Eq.(5) means that the linear response of the current to the superfluid velocity $\vec{v}_s$ vanishes in the long wave length limit: $\rho_s(q \ll 1/L_0) = 0$. The superfluid density is however restored to its local value if the velocity is large enough: $v_s \gg \kappa/L_0$. This can be directly checked in the experiments of the type described in [1], if one measures ρ_s as a function of v_s. This type of behavior can be named "weak nonsuperfluidity".

Till now we ignored the tiny spin-orbit interaction $-g_D(\hat{l} \cdot \hat{d})^2$ between the spin and orbital degrees of freedom, which is characterized by the so called dipole length $\xi_D \sim 10^{-3}$cm [7]. This is justified if $L_0 < \xi_D$. If $L_0 > \xi_D$, the vector $\hat{d}$ is "dipole-locked" with $\hat{l}$ and also becomes randomly oriented. In this case $Q_{\alpha\beta ij}$ becomes isotropic and at large distances the symmetry of the state corresponds to that of the normal ^{3}He without any long range order.

Situation changes if the magnetic field $H \gg 30$Gauss is applied. In this case the spin part of the order parameter is oriented perpendicular to magnetic field:

$\hat{d} \perp \vec{H}$. Since $L_0 > \xi_D$ the $\hat{l}$ vector is to be aligned with $\hat{d}$. The order parameter $P_{\alpha\beta ij}$ in Eq.(2) now contains a nonzero term:

$$P_{\alpha\beta ij} = P(\delta_{\alpha\beta} - \hat{z}_\alpha \hat{z}_\beta)(\delta_{ij} - 3\hat{z}_i\hat{z}_j)\ e^{2i\Phi}. \tag{6}$$

This state with $P \neq 0$ is equivalent to the superfluid condensate the elementary boson of which is not a Cooper pair of atoms, but rather consists of 4 atoms. Thus the transition from the weak nonsuperfluid to the superfluid states can be triggered by rather small magnetic field of order 100 G.

Another possible way to trigger the transition is to change the dipole energy. This can be made in the regime of nonlinear NMR with the large tipping angle β of the precessing magnetization. According to [10] the dipole energy which tends to keep the vector $\hat{l}$ in the plane perpendicular to $\mathbf{H} = H\hat{h}$ is

$$F_D = g_D\left(\frac{7}{8}\cos^2\beta + \frac{1}{4}\cos\beta - \frac{1}{8}\right)(\hat{l}\cdot\hat{h})^2. \tag{7}$$

The dipole energy decreases with increasing β and at some moment the dipole length $\xi_D(\beta) = \xi_D(0)\ (\frac{7}{8}\cos^2\beta + \frac{1}{4}\cos\beta - \frac{1}{8})^{-1/2}$ becomes larger than L_0. The critical value of β at which the transition should occur is smaller than the value of β at which the dipole energy changes sign ($F_D = 0$ at $\cos\beta = 0.26$). This correlates with the abrupt change of the NMR signal observed in [2] at $\beta \sim 40^\circ - 50^\circ$. The details of this transition are however unclear.

Now let us discuss the magnitude of the correlation length L_0 at which the random anisotropy produced by anisotropic silica strands kills the orientational and superfluid orders in ^{3}He-A. Aerogel can be represented as the randomly distributed cylinders of the radius R with the distance l between the cylinders. According to [11] the characteristic energy of the order parameter distortion produced by a small object of size $R \ll \xi_0$ is

$$k_F^2 R\Delta^2/T_c \tag{8}$$

per unit length of the strand. This is the measure of the anisotropy energy which tends to orient the $\hat{l}$-vector. Now let us consider the box of size $L \gg l$ and apply Imry-Ma arguments [5]. The orientational energy, which comes from the typical fluctuation within this box of volume $V = L^3$ and which tends to fix the orientation of $\hat{l}$ in the box, is

$$E_{random} = (L/l)^{3/2} k_F^2 R l \Delta^2/T_c. \tag{9}$$

The gradient energy which arises due to different preferred orientation of $\hat{l}$ in neighbouring boxes is

$$E_{gradient} \sim K\int_V (\nabla\hat{l})^2 \sim (\Delta/T_c)^2 (k_F^3/m) L. \tag{10}$$

These energies are equal at

$$L = L_0 \sim l\frac{\xi_0^2}{R^2} \gg l, \tag{11}$$

and this gives the size of the domain L_0 within which the $\hat{l}$-vector is homogeneous. In this model L_0 is temperature independent, while its value can be comparable with ξ_D, since $R \ll \xi_0 < l$.

The value of L_0 can be found by measurement of the linear term in the heat capacity of ^{3}He in aerogel at low temperature: $C(T) \propto N(0)T$. The nonzero density of states $N(0)$ arises in the $\hat{l}$ texture due to the gap nodes in ^{3}He-A[12]:

$$N(0) \sim N_F \xi_0 |\hat{l} \times (\vec{\nabla} \times \hat{l})| \sim N_F \frac{\xi_0}{L_0} \sim N_F \frac{R^2}{\xi_0 l}, \qquad (12)$$

where N_F is the density of states in the normal Fermi liquid. The observation of this effect will be interesting, since it will be the indication of the anomalies in ^{3}He-A, related to the point gap nodes, which are very similar to anomalies in particle physics [12].

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