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INELASTIC ELECTRON-ACOUSTIC-PHONON INTERACTION IN QUANTUM-WELL WIRES

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Due to the lack of transverse momentum conservation for the electron-acoustic-phonon interaction in quantum wires this interaction becomes strongly inelastic within a wide range of electron energies. As a result the electron distribution function has to be found from the integro-differential equation. We derive the new nonequilibrium distribution functions for these conditions and present the electric field dependences for the kinetic coefficients. Our approach can be applied as well for two-dimensional electron systems or for electrons subjected to an external quantizing magnetic field.

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In a quantum wire (QWR) the component of the electron wave function which describes the motion of the electrons along quantization directions represents a packet of plane waves whose characteristic size is order of the quantum well width $L_\perp$. This means that all electron states have an uncertainty for the wave-vector along quantization directions $\Delta k_\perp \simeq k_\perp \simeq 2\pi/L_\perp$, and there is no precise momentum conservation for the electron-acoustic-phonon interaction with bulk-like phonon modes in these directions. In accordance with [1] the duration of a collision τ_c between an electron and an acoustic-phonon under the quantization conditions cannot be given by the formula $\tau_c \sim h/\varepsilon_\parallel$, where $\varepsilon_\parallel$ is the electron kinetic energy in the free direction, when $\varepsilon_\parallel$ becomes the same order or smaller than some characteristic energy $\hbar\omega^*_{\mathbf{q}}$ of the acoustic-phonon with wave-vector q. When $\varepsilon_\parallel$ is small, the electron velocity is small as well, but in such a case τ_c is defined by the motion of the scatterers, i.e. acoustic-phonons. An acoustic-phonon is moving with the velocity of sound s, so that τ_c cannot be longer than $L_\perp/s$. This is the time during which the acoustic-phonon crosses the quantum well where the electrons are confined. The formula $\tau_c \sim h/\varepsilon_\parallel$ cannot be applied when $h/\varepsilon_\parallel \geq L_\perp/s$, viz.

$$\varepsilon_\parallel \leq hs/L_\perp. \tag{1}$$

From the quantum picture of the electron-acoustic-phonon interaction it follows that the cross-section of the interaction differs from zero only if the phonon wave function "overlaps" the electron wave function, i.e. when the component of wave-vector of the phonon along quantization directions $q_\perp$ is smaller or the same order as $q^*_\perp \equiv 2\pi/L_\perp$,

$$q_\perp \leq q^*_\perp. \tag{2}$$

The corresponding phonon energy is equal to $\hbar\omega^*_{\mathbf{q}} \simeq \hbar s q^*_\perp = 2\pi\hbar s/L_\perp$. Hence, we may expect novel peculiarities of the electron kinetics in a QWR if the kinetic energy of electrons is within the range $\varepsilon_\parallel \leq \hbar\omega^*_{\mathbf{q}} = 2\pi\hbar s/L_\perp$. The electron-acoustic-phonon interaction in this case is strongly inelastic, because the electron energy is of the same order as the acoustic-phonon energy.

In this article we study the nonequilibrium kinetic problem for a non-degenerate 1D electron gas in a QWR which is confined between infinitely deep potential interfaces and interact predominantly with bulk deformation acoustic-phonons. We show that the kinematic peculiarities of the electron-acoustic-phonon interaction under the quantization conditions manifest themselves in non-trivial electron kinetics since the small parameter of the quasi-elasticity [2] does not exist in this case and the corresponding Boltzmann kinetic equation for the distribution function is an integro-differential equation.

In the case when several subbands are involved in the relaxation process the electrons take part simultaneously in inelastic intrasubband and quasielastic intersubband [3] scattering. As a result the peculiarities of the inelasticity of the electron-phonon interaction are smoothed out. It is obvious from Eq.(1) that the electrons will occupy only the first subband and the scattering will be inelastic for the majority of electrons even in the thermal equilibrium state if the electron kinetic energy $\varepsilon_{\|}(k_z)$ and the lattice temperature T_0 (which defines also the mean electron energy in the equilibrium) are within the range

$$\varepsilon_{\|}(k_z),\ T_0 < \sqrt{8m^* s^2 W_0}. \tag{3}$$

Here m^* is the electron effective mass, k_z is the longitudinal electron wave-vector, and $W_0 = \pi^2\hbar^2/m^* L_\perp^2$ is the quantum energy of the ground state in a QWR. For a GaAs rectangular QWR with $L_\perp = 50\,\mathring{A}$ we obtain for the right-hand side in Eq.(3) value about 70 K. As one can see this corresponds to the actual energy range for usual experimental situations.

From the energy conservation for the electron-acoustic-phonon interaction it follows [3] that $|q_z| \ll q_\perp$ for the discussed conditions, and we can put everywhere $q \simeq q_\perp$. In this approximation the antisymmetric part of the collision operator in the Boltzmann kinetic equation can be described [4] by the momentum relaxation time $\tau(\varepsilon_{\|})$. This gives the solution for the antisymmetric distribution function $F^-(k_z)$ in a usual form

$$F^-(k_z) = \frac{1}{\hbar} e E_z \tau(\mathcal{E}_{\|}) \frac{dF_0(\mathcal{E}_{\|})}{dk_z}, \tag{4}$$

where

$$\frac{1}{\tau(\mathcal{E}_{\|})} = \frac{1}{\tau_0}\frac{1}{\Psi(\mathcal{E}_{\|})}, \qquad \frac{1}{\Psi(\mathcal{E}_{\|})} = \int_0^\infty \frac{N_\omega}{(\mathcal{E}_{\|}+\omega)^{1/2}}\omega^2 d\omega + \int_0^{\mathcal{E}_{\|}} \frac{N_\omega + 1}{(\mathcal{E}_{\|}-\omega)^{1/2}}\omega^2 d\omega, \tag{5}$$

Here E_z is an applied electric field, $N_\omega = (e^\omega - 1)^{-1}$, $\mathcal{E}_{\|} = \varepsilon_{\|}/T_0$, $\omega = \hbar s q_\perp/T_0$, $\tau_0 = 2\sqrt{2}\pi\rho\hbar^4 s^4/(\Xi_a^2 (m^*)^{1/2} T_0^{5/2})$, Ξ_a is the deformation acoustic potential, ρ is the matter density.

By using $F^-(k_z)$ from Eq.(4) we obtain the kinetic equation for the symmetric distribution function $F_0(\mathcal{E}_{\|})$

$$-\varepsilon_E^2 \mathcal{E}_{\|}^{1/2}\frac{d}{d\mathcal{E}_{\|}}\left[\mathcal{E}_{\|}^{1/2}\Psi(\mathcal{E}_{\|})\frac{dF_0(\mathcal{E}_{\|})}{d\mathcal{E}_{\|}}\right] = \int_0^\infty \frac{\omega^2}{(\mathcal{E}_{\|}+\omega)^{1/2}}\left[F_0(\mathcal{E}_{\|}+\omega)(N_\omega+1) - \right.$$

$$\left. - F_0(\mathcal{E}_{\|})N_\omega\right]d\omega + \int_0^{\mathcal{E}_{\|}} \frac{\omega^2}{(\mathcal{E}_{\|}-\omega)^{1/2}}\left[F_0(\mathcal{E}_{\|}-\omega)N_\omega - F_0(\mathcal{E}_{\|})(N_\omega+1)\right]d\omega, \tag{6}$$

where $\varepsilon_E^2 = E_z^2/E_c^2$, $E_c^2 = m^*T_0/2e^2\tau_0^2$. In Eqs.(5) and (6) in accordance with Eqs.(2),(3) we put the form-factor in the matrix element of the electron-phonon interaction equal unity. Changing the variable of integration we transform integro-differential equation (6) to the form

$$\hat{D}F_0(\mathcal{E}_\parallel) = -\int_0^\infty K(\omega, \mathcal{E}_\parallel)F_0(\omega)d\omega, \tag{7}$$

where the inhomogeneous part is a functional of $F_0(\mathcal{E}_\parallel)$, and $\hat{D}$ is the differential operator,

$$\hat{D} = \varepsilon_E^2 \frac{d}{d\mathcal{E}_\parallel}\Psi_1(\mathcal{E}_\parallel)\frac{d}{d\mathcal{E}_\parallel} - \frac{1}{\Psi_1\mathcal{E}_\parallel}. \tag{8}$$

Here we have introduced the new notations

$$\Psi_1(\mathcal{E}_\parallel) = \mathcal{E}_\parallel^{1/2}\Psi(\mathcal{E}_\parallel), \quad K(\omega, \mathcal{E}_\parallel) = \frac{(\mathcal{E}_\parallel - \omega)^2}{(\mathcal{E}_\parallel\,\omega)^{1/2}}\left|\frac{e^\omega}{e^{\mathcal{E}_\parallel} - e^\omega}\right|. \tag{9}$$

To find the solution of Eq.(7) we will consider it formally as a inhomogeneous differential equation [5]. The operator $\hat{D}$ has two linearly independent eigenfunctions with zero eigenvalues, which are given by

$$\tilde{F}_0^\pm(\mathcal{E}_\parallel) = \exp\left[\pm\frac{J(\mathcal{E}_\parallel)}{\varepsilon_E}\right], \qquad J(\mathcal{E}_\parallel) = \int_0^{\mathcal{E}_\parallel}\frac{d\mathcal{E}'_\parallel}{\Psi_1(\mathcal{E}'_\parallel)}. \tag{10}$$

Hence, the solution of Eq.(7) can be obtained by the ordinary method [6], and it reads

$$F_0(\mathcal{E}_\parallel) = A_0\tilde{F}_0^-(\mathcal{E}_\parallel) + B_0\tilde{F}_0^+(\mathcal{E}_\parallel) + \int_0^\infty R(\omega, \mathcal{E}_\parallel)F_0(\omega)d\omega, \tag{11}$$

where

$$R(\omega, \mathcal{E}_\parallel) = \frac{1}{2\varepsilon_E}\int_0^{\mathcal{E}_\parallel}\begin{vmatrix} \tilde{F}_0^-(\mathcal{E}_\parallel) & \tilde{F}_0^+(\mathcal{E}_\parallel) \\ \tilde{F}_0^-(\mathcal{E}'_\parallel) & \tilde{F}_0^+(\mathcal{E}'_\parallel) \end{vmatrix} K(\omega, \mathcal{E}'_\parallel)d\mathcal{E}'_\parallel. \tag{12}$$

The constants of integration A_0 and B_0 have to be determined from the boundary conditions to Eq.(7) which are the normalization condition and the absence of electrons with infinitely high energies, i.e. $F_0(\mathcal{E}_\parallel) \to 0$ if $\mathcal{E}_\parallel \to \infty$. The second condition gives for B_0

$$B_0 = \frac{1}{2\varepsilon_E}\int_0^\infty\int_0^\infty K(\omega, \mathcal{E}'_\parallel)\tilde{F}_0^-(\mathcal{E}'_\parallel)F_0(\omega)d\mathcal{E}'_\parallel\,d\omega. \tag{13}$$

After this Eq.(11) can be presented in the form

$$F_0(\mathcal{E}_\parallel) = A_0\exp\left[-\frac{J(\mathcal{E}_\parallel)}{\varepsilon_E}\right] + \int_0^\infty H(\omega, \mathcal{E}_\parallel)F_0(\omega)d\omega. \tag{14}$$

Here

$$H(\omega, \mathcal{E}_\parallel) = \frac{1}{2\varepsilon_E}\left[\int_0^{\mathcal{E}_\parallel} K(\omega, \mathcal{E}'_\parallel)\exp\left(-\frac{J(\mathcal{E}_\parallel) - J(\mathcal{E}'_\parallel)}{\varepsilon_E}\right)d\mathcal{E}'_\parallel + \right.$$

$$\left. + \int_{\mathcal{E}_\parallel}^\infty K(\omega, \mathcal{E}'_\parallel)\exp\left(\frac{J(\mathcal{E}_\parallel) - J(\mathcal{E}'_\parallel)}{\varepsilon_E}\right)d\mathcal{E}'_\parallel\right]. \tag{15}$$

We have transformed an initial integro-differential equation (6) to a pure integral equation (14) with a kernel given by Eq.(15) which is very convenient for the analytical analysis in practically important cases.

The case of warm electrons. The electron system is very close to the thermodynamic equilibrium state, i.e. the external electric field is sufficiently small $\varepsilon_E^2 \ll 1$.

It can be shown [4] that Eqs.(6) or (14) represent the regular rather than singularly perturbed equations with respect to a small parameter ε_E^2, and for the solution we can apply ordinary perturbation methods. The first term in Eq.(14) will be negligibly small for all electron energies which satisfies the condition $J(\mathcal{E}_\parallel) \gg \varepsilon_E^2$. In the range of small electric fields we find that this inequality holds for all actual electron energies $\mathcal{E}_\parallel \geq 1$, and it is possible to neglect the first term in Eq.(14). The equation resulting from is a homogeneous integral equation

$$F_0(\mathcal{E}_\parallel) = \int_0^\infty H(\omega, \mathcal{E}_\parallel) F_0(\omega) d\omega. \tag{16}$$

Note that Eq.(16) is not a Fredholm's type equation. This is because the kernel $H(\omega, \mathcal{E}_\parallel)$ does not satisfy the following condition [7] $\int_0^\infty \int_0^\infty H^2(\omega, \mathcal{E}_\parallel) d\omega d\mathcal{E}_\parallel < +\infty$. In fact, the integral over ω diverges logarithmically at the lower limit point $\omega = 0$, since $H^2(\omega, \mathcal{E}_\parallel) \propto 1/\omega$ if $\omega \to 0$.

Within the discussed conditions let us transform the kernel $H(\omega, \mathcal{E}_\parallel)$. We have under the integrals in Eq.(15) the products of a slowly varying function of $\mathcal{E}'_\parallel$, $K(\omega, \mathcal{E}'_\parallel)$, and very rapidly varying function of $\mathcal{E}'_\parallel$, $\exp\left[\pm\left(J(\mathcal{E}_\parallel) - J(\mathcal{E}'_\parallel)\right)/\varepsilon_E\right]$. The last function has its maximum value 1 at $\mathcal{E}'_\parallel = \mathcal{E}_\parallel$ and rapidly decreases as $\mathcal{E}'_\parallel$ varies over the narrow interval $\Delta\mathcal{E}'_\parallel = |\mathcal{E}_\parallel - \mathcal{E}'_\parallel| \simeq \varepsilon_E [dJ(\mathcal{E}_\parallel)/d\mathcal{E}_\parallel]^{-1} \simeq \mathcal{E}_\parallel [\varepsilon_E / J(\mathcal{E}_\parallel)] \ll \mathcal{E}_\parallel$. Consequently, the main contribution to the integrals comes from the range of $\mathcal{E}'_\parallel$ which is very close to $\mathcal{E}_\parallel$, i.e. to the upper limit for the first integral and to the lower one for the second integral. This allows us to apply the Laplace method [8] for the calculations of these integrals. We have restricted our calculations of the integrals to the accuracy which is proportional to the third order of the small parameter ε_E. With this approximation we obtain the following equation for $F_0(\mathcal{E}_\parallel)$

$$F_0(\mathcal{E}_\parallel) = \Psi_1(\mathcal{E}_\parallel) \int_0^\infty K(\omega, \mathcal{E}_\parallel) F_0(\omega) d\omega +$$

$$+ \varepsilon_E^2 \Psi_1(\mathcal{E}_\parallel) \frac{d}{d\mathcal{E}_\parallel} \left[\Psi_1(\mathcal{E}_\parallel) \frac{d}{d\mathcal{E}_\parallel} \left(\Psi_1(\mathcal{E}_\parallel) \int_0^\infty K(\omega, \mathcal{E}_\parallel) F_0(\omega) d\omega \right) \right]. \tag{17}$$

Employing an iterative procedure to this equation we present $F_0(\mathcal{E}_\parallel)$ in the form

$$F_0(\mathcal{E}_\parallel) = f^{(0)}(\mathcal{E}_\parallel) + \varepsilon_E^2 f^{(1)}(\mathcal{E}_\parallel). \tag{18}$$

This gives the equations

$$f^{(0)}(\mathcal{E}_\parallel) = A_0 \exp(-\mathcal{E}_\parallel), \tag{19}$$

$$f^{(1)}(\mathcal{E}_\parallel) = \Psi_1(\mathcal{E}_\parallel) \left[\int_0^\infty K(\omega, \mathcal{E}_\parallel) f^{(1)}(\omega) d\omega + \frac{d}{d\mathcal{E}_\parallel} \left(\Psi_1(\mathcal{E}_\parallel) \frac{df^{(0)}(\mathcal{E}_\parallel)}{d\mathcal{E}_\parallel} \right) \right]. \tag{20}$$

The inhomogeneous integral equation (20) does not contain any small parameter and this is,once more, not a Fredholm's type equation as well. Nevertheless, the

properties of the inhomogeneous term in this equation allow us to apply the Picar method [7] which is usually used for the solution of the Fredholm's type equations. This gives us the followin final expression for $F_0(\mathcal{E}_\parallel)$

$$F_0(\mathcal{E}_\parallel) = A_0 \left[1 + \frac{e^2 E_z^2 \lambda_\varepsilon^2(\varepsilon_\parallel)}{T_0^2} \left(1 - \frac{T_0}{\lambda_\varepsilon(\varepsilon_\parallel)} \frac{d\lambda_\varepsilon(\varepsilon_\parallel)}{d\varepsilon}\right)\right] \exp\left(-\frac{\varepsilon_\parallel}{T_0}\right), \tag{21}$$

where $\lambda_\varepsilon(\varepsilon_\parallel) = v(\varepsilon_\parallel)\tau(\varepsilon_\parallel)$ is the free path of the electron with energy $\varepsilon_\parallel$. The criterion of the validity of Eq.(21) can be written as $eE_z\lambda_\varepsilon(T_0) \ll T_0$.

The distribution functions in Eqs.(4) and (21) describe all the kinetic properties of warm electrons in D QWR associated with inelastic scattering by the acoustic-phonons. For example, we obtain that the current-voltage characteristics obey a sublinear dependence on the electric field and the mobility of the warm electrons is given by $\mu_e = (\alpha_0 e\tau_0/m^*)(1 - \beta_0\varepsilon_E^2)$, where α_0 and β_0 are the general temperature independent coefficients numerically equal to 0.27 and 0.31, respectively. The mean electron energy of the warm electrons is equal $\bar{\varepsilon_\parallel} = T_0/2(1 + \gamma_0\varepsilon_E^2)$ with $\gamma_0 = 0.10$.

The case of hot electrons. This situation is realized in a strong electric field, i.e. $\varepsilon_E^2 \gg 1$, when the majority of electrons are distributed within the energy range $\sqrt{8m^* s^2 W_0}/T_0 > \mathcal{E}_\parallel \gg 1$. We find that here $J(\varepsilon_\parallel) \leq \varepsilon_E^2$. The structure of Eq.(14) suggests a main role for the first term. This means that the symmetric distribution function is equal to

$$F_0(\mathcal{E}_\parallel) = A_0 \exp\left[-\frac{1}{\varepsilon_E}\int_0^{\mathcal{E}_\parallel} \frac{d\mathcal{E}'_\parallel}{\Psi_1(\mathcal{E}'_\parallel)}\right]. \tag{22}$$

To prove that this is a good approximation for the solution of Eq.(14) we carry out an analytical calculation of the second term in this equation with $F_0(\mathcal{E}_\parallel)$ from Eq.(22). It can be shown that for the actual values of the electron energy and electric field the ratio of the second term to the first one is much smaller than $[J(\mathcal{E}_\parallel)/\varepsilon_E]^{1/6} \exp[J(\mathcal{E}_\parallel)/\varepsilon_E] \simeq 1$.

Substituting $F_0(\mathcal{E}_\parallel)$ from Eq.(22) into Eq.(4) we find for the antisymmetric distribution function

$$F^-(k_z) = -\frac{k_z E_z}{|k_z||E_z|} F_0(\mathcal{E}_\parallel). \tag{23}$$

The total distribution function is equal

$$F(k_z) = A_0 \left[1 - \frac{k_z E_z}{|k_z||E_z|}\right] \exp\left[-\frac{1}{\varepsilon_E}\int_0^{\mathcal{E}_\parallel} \frac{d\mathcal{E}'_\parallel}{\Psi_1(\mathcal{E}'_\parallel)}\right]. \tag{24}$$

This is a strongly anisotropic function in the momentum space:

$$F(k_z) = \begin{cases} 2F_0(\mathcal{E}_\parallel), & \text{if } k_z E_z < 0, \\ 0, & \text{if } k_z E_z > 0. \end{cases} \tag{25}$$

Physically this means that electrons move ballistically in k_z-space along straightforward trajectories. In the steady state the electron motion is governed by the momentum balance equation

$$-eE_z\tau(\tilde{\varepsilon}_\parallel) = \hbar k_z(\tilde{\varepsilon}_\parallel). \tag{26}$$

The electric field pushes electrons into the high energy region, where the increase in energy results in the increase of the scattering probability in accordance with Eq.(5). There is some balance value of the electron energy $\tilde{\varepsilon}_{\|}$ which is given by the solution of Eq.(26) and which depends on the electric field E_z. When the electron reaches the energy $\tilde{\varepsilon}_{\|}$ and momentum $\hbar k_z(\tilde{\varepsilon}_{\|})$ it emits a 3D-acoustic-phonon with the wave vector $\mathbf{q} \equiv (q_z, q_\perp)$ and with the energy $\hbar\omega_{\mathbf{q}} = \hbar sq$. In accordance with longitudinal momentum conservation $q_z = k_z(\tilde{\varepsilon}_{\|})$. The portion of the phonon energy $\hbar s|q_z| = \sqrt{2m^* s^2 \tilde{\varepsilon}_{\|}}$ which corresponds to this "longitudinal" interaction is very small compared with $\tilde{\varepsilon}_{\|}$ and $\hbar sq_\perp$. In accordance with energy conservation and due to the inequality $q_\perp \gg |q_z|$ the electron loses its energy due mainly to the interaction with $\mathbf{q}_\perp$-component of $\mathbf{q}$, i.e. due to emission of the acoustic-phonon with the energy $\hbar\omega_{\mathbf{q}} \simeq \hbar sq_\perp \simeq \tilde{\varepsilon}_{\|}$.

The new strongly anisotropic electron distribution we have obtained corresponds to the electron streaming regime. The latter is more familiar in the context of the electron inelastic interaction with optical phonons [3]. Recently, a regime similar to that discussed here was investigated in [9] by using the Monte Carlo technique. Our analytical results coincides qualitatively with the Monte Carlo analysis. Some quantitative distinctions is connected with using in [9] expression for the scattering rate which is different from ours.

For the electron energy $\mathcal{E}_{\|} \gg 1$ we find from Eq.(5) for the scattering rate $1/\tau(\mathcal{E}_{\|}) = 16/15\tau_0^{-1}\mathcal{E}_{\|}^{5/2}$. This gives for the distribution function

$$F(k_z) = A_0 \left[1 - \frac{k_z E_z}{|k_z||E_z|}\right] \exp\left(-\frac{16}{45}\frac{\mathcal{E}_{\|}^3}{\epsilon_E}\right). \tag{27}$$

As a result we obtain for the mobility

$$\mu_e = \sqrt{\frac{2T_0}{m^*}}\frac{\Gamma(1/3)}{\Gamma(1/6)}\left(\frac{45}{16}\frac{|E_z|}{E_c}\right)^{1/6}\frac{1}{|E_z|}, \tag{28}$$

and for the mean energy of the hot electrons in a streaming regime

$$\bar{\varepsilon}_{\|} = T_0\frac{\Gamma(1/3)}{\Gamma(1/6)}\left(\frac{45}{16}\frac{|E_z|}{E_c}\right)^{1/3}. \tag{29}$$

The electron mobility is a decreasing function of the electric field. The physical explanation of this dependence consists in the increasing of the scattering rate $\tau^{-1}(\mathcal{E}_{\|})$ when electric field pushes electrons in the high energy region. This is also the reason for a slow increasing of the mean energy with the electric field.

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