AB INTERFACE IN SUPERFLUID ^{3}He AND CASIMIR EFFECT

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The friction force on the moving interface between two different vacuum states of superfluid ^{3}He is considered at low temperature. Since the dominating mechanism of the friction is the Andreev reflection of the massless "relativistic" fermions, which live on the A-phase side of the interface, the results are similar to that for the perfectly reflecting mirror moving in the quantum vacuum.

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1. **Introduction.** The AB interface is the boundary between two different superfluid vacua of ^{3}He. The dynamics of the interface is determined by the fermionic quasiparticles (Bogoliubov excitations). In the A-phase vacuum the fermions are chiral and massless, while in the B-phase vacuum they are massive. At the temperature T well below the temperature T_c of the superfluid transtition the thermal fermions are present only in the A-phase. Close to the gap nodes, ie at $p \approx \pm p_F \hat{l}$, the energy spectrum $E(p)$ of the gapless A-phase fermions becomes "relativistic" [1]:

$$E^2(\mathbf{p}) = g^{ik}(p_i - eA_i)(p_k - eA_k),\qquad(1.1)$$

where the vector potential is $\mathbf{A} = p_F \hat{l}$; $e = \pm$; and the metric tensor is

$$g^{ik} = c_\perp^2(\delta^{ik} - \hat{l}^i \hat{l}^k) + c_\parallel^2 \hat{l}^i \hat{l}^k .\qquad(1.2)$$

Here $\hat{l}$ is unit vector in the direction of the gap nodes in the momentum space; $c_\perp = \Delta/p_F$ and $c_\parallel = v_F$ (with $c_\perp \ll c_\parallel$) are "speeds of light" propagating transverse to $\hat{l}$ and along $\hat{l}$ correspondingly; p_F is the Fermi momentum; $v_F = p_F/m_3$ is the Fermi velocity; m_3 is the mass of ^{3}He atom; Δ is the gap amplitude in ^{3}He-A.

In the presence of superflow with the superfluid velocity $\mathbf{v}_s$ the following term is added to the energy $E(\mathbf{p})$:

$$\mathbf{p} \cdot \mathbf{v}_s \equiv (\mathbf{p} - e\mathbf{A}) \cdot \mathbf{v}_s + eA_0, \qquad A_0 = p_F \hat{l} \cdot \mathbf{v}_s .\qquad(1.3)$$

The second term corresponds to the scalar potential A_0 of the electromagnetic field, while the first one leads to the nonzero element $g^{0i} = v_s^i$ of the metric tensor and to the change of the elements $g^{ik} \to g^{ik}_{stationary} - v_s^i v_s^k$. As a result the Eq.(1.1) transforms to

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0, \qquad(1.4)$$

with $g^{00} = -1$, $p_\mu = (\mathbf{p}, E)$, $A_\mu = (\mathbf{A}, A_0)$.

Since the B-phase excitations are massive, the A-phase excitations cannot propagate through the AB interface. The scattering of the A-phase fermions

from the interface, which is known as Andreev reflection [2], is the dominating mechanism of the friction force experienced by the moving AB interface. Due to the relativistic character of the A-phase fermions the dynamics of the interface becomes very similar to the motion of the perfectly reflecting mirror in relativistic theories, which was heavily discussed in the relation to the Casimir effect (see eg [3, 4]). So the investigation of the interface dynamics at $T \ll T_c$ will give the possiblitity of the modelling of the effects of quantum vacuum. On the other hand, using the relativistic invariance one can easily calculate the forces on moving interface from the A-phase heat bath in the limit of low T or from the A-phase vacuum at $T = 0$. This can be done for any velocity of wall with respect to the superfluid vacuum and to the heat bath. We discuss here the velocities below the "speed of light" in ^{3}He-A. The case of the velocity exceeding $c_\perp$, which is typical for experiments made far from T_c [5], will be discussed later.

2. **Force on moving wall at finite temperature: Massless isotropic relativistic fermions.** The motion of the AB interface in the so called ballistic regime for the quasiparticles has been considered in [6–8] (see also [9]). In this regime the force on the interface comes from the mirror reflection at the interface (Andreev reflection) of the ballistically moving thermally distributed Fermi particles. Three velocities are of importance in this process: superfluid velocity of the condensate $\mathbf{v}_s$, normal velocity of the heat bath $\mathbf{v}_n$ and the velocity of the interface $\mathbf{v}_L$. The friction force is absent when the wall is stationary in the heat bath frame, ie $\mathbf{v}_L = \mathbf{v}_n$.

Let us first consider the nonrealistic model in which the speed of "light" is isotropic, ie $c_\perp = c_\parallel = c$, and the vector potential $\mathbf{A}$ is absent. In the next Section the results will be extrapolated to the real AB interface. In the reference frame of the interface the system is stationary thus the energy of the quasiparticles in this frame

$$E' = E + (\mathbf{v}_s - \mathbf{v}_L) \cdot \mathbf{p}, \quad E = cp, \tag{2.1}$$

is conserved during the scattering. In thermal equilibrium their distribution function is

$$f(\mathbf{p}) = \left[1 + \exp\left\{\frac{E' - (\mathbf{v}_n - \mathbf{v}_L) \cdot \mathbf{p}}{T}\right\}\right]^{-1} = \left[1 + \exp\left\{\frac{E - (\mathbf{v}_n - \mathbf{v}_s) \cdot \mathbf{p}}{T}\right\}\right]^{-1}. \tag{2.2}$$

In the ballistic regime one calculates the momentum transfer from the heat bath to the wall due to scattering at the wall

$$F = \sum_{\mathbf{p}} \Delta p_z \frac{dE'}{dp_z} f(\mathbf{p}). \tag{2.3}$$

Here

$$\frac{dE'}{dp_z} = c\cos\theta - (v_L - v_s) \tag{2.4}$$

is the group velocity of the particles in the wall frame;

$$\Delta p_z = 2p\frac{\cos\theta - (v_L - v_s)/c}{1 - (v_L - v_s)^2/c^2} \tag{2.5}$$

is the momentum transfer after reflection, where θ is the angle between the particle momentum $\mathbf{p}$ and the velocity of the wall $\mathbf{v}_L$. The momentum transfer Δp_z is

small compared to the cut-off parameter p_F, which corresponds to the Andreev reflection in condensed matter. The force per unit area is:

$$\frac{F(v_L - v_s, v_n - v_s)}{A} = -\hbar c \frac{7\pi^2}{60} \frac{T^4}{(\hbar c)^4} \alpha(u_L, u_n), \quad u_L = \frac{v_L - v_s}{c}, \quad u_n = \frac{v_n - v_s}{c},$$

$$\alpha(u_L, u_n) = \frac{1}{1 - u_L^2} \int_{-1}^{u_L} d\mu \; \frac{(\mu - u_L)^2}{(1 - \mu u_n)^4}. \tag{2.6}$$

Now we can consider several different cases.

2.1. $v_L \neq v_s = v_n$. In this most typical case the distribution function is the Fermi function $f(E) = 1/(1 + e^{E/T})$, with $E = cp$. From Eq.(2.6) one has

$$\alpha(u_L, 0) = \frac{1}{1 - u_L^2} \int_{-1}^{u_L} d\mu \; (\mu - u_L)^2 = \frac{1}{3} + u_L + \frac{4}{3} \frac{u_L^2}{1 - u_L}. \tag{2.7}$$

The force disappears at $v_L - v_n \to -c$, because the particles cannot reach the wall moving with the speed of light. At $v_L = v_s = v_n$ the first term in the rhs of Eq.(2.7) gives a conventional pressure P on the wall from the gas of particles, $F(v_L = v_s = v_n) = -AP$, where A is the area of the wall and

$$P = \hbar c \frac{7\pi^2}{180} \frac{T^4}{(\hbar c)^4}. \tag{2.8}$$

The second term, which is linear in $v_L - v_n$, is the friction force on the moving wall if the wall moves with respect to the heat bath:

$$F_{\text{friction}} = -(v_L - v_n)\Gamma A, \quad \Gamma = \hbar c \frac{7\pi^2}{60} \frac{T^4}{(\hbar c)^4}. \tag{2.9}$$

2.2. $v_L = v_s \neq v_n$. The spectrum of the particles in the reference frame of the wall is relativistic, $E' = cp$, while the distribution function is the Doppler shifted Fermi function $f(p) = 1/[1 + \exp\{\frac{E' + (\mathbf{v}_s - \mathbf{v}_n) \cdot \mathbf{p}}{T}\}]$. From Eq.(2.6) one has

$$\alpha(0, u_n) = \int_{-1}^{0} d\mu \; \frac{\mu^2}{(1 - \mu u_n)^4} = \frac{1}{3}(1 - \frac{v_L - v_n}{c})^{-3}. \tag{2.10}$$

For the small $v_L - v_n \ll c$ the results for the pressure and the friction force are the same as in previous subsection. Difference occurs at higher velocity: when $\mathbf{v}_s - \mathbf{v}_n$ approach c the vacuum becomes unstable.

2.3. $v_L = v_n \neq v_s$. When the interface moves with the heat bath the force is an even function of $v_L - v_s$:

$$\alpha(u_L = u_n) = \frac{1}{3}(1 - \frac{(v_L - v_s)^2}{c^2})^{-2}. \tag{2.11}$$

This means that the friction force is absent since the interface is in equilibrium with the heat bath. The effect of the superflow $\mathbf{v}_s - \mathbf{v}_L$ across the interface leads to the relativistic renormalization of the temperature in the expression for the pressure:

$$T_{\text{effective}} = \frac{T}{\sqrt{g_{00}}}, \quad g_{00} = 1 - \frac{v_s^2}{c^2}, \tag{2.12}$$

where the superfluid velocity is in the reference frame of the wall and heat bath. This is in agreement with the Unruh analogy in which the superfluid velocity plays the part of the gravitational potential [10].

3. **Force on moving AB interface at low T.** Now let us apply the obtained results to the A-phase, which has an anisotropic velocity of light and also contains the vector potential $\mathbf{A} = p_F \hat{l}$. The constant vector potential can be gauged away by shifting the momentum. If $\mathbf{p}$ is counted from $e\mathbf{A}$ the situation is the same as in previous Section with one exception: the Doppler shift leads also to the appearance of the scalar potential: $A_0 = \mathbf{A} \cdot \mathbf{v}$. In the reference frame of the interface the energy of the quasiparticles becomes

$$E' = E + (\mathbf{v}_s - \mathbf{v}_L) \cdot \mathbf{p} + eA_0, \quad E = \sqrt{g^{ik}p_i p_k}, \quad A_0 = \mathbf{A} \cdot (\mathbf{v}_s - \mathbf{v}_L). \tag{3.1}$$

Since the scalar potential $A_0 = \text{const}$, it does not influence the scattering of the quasiparticles at the wall. The scalar potential can influence only the thermal distribution function. But this does not happen in two cases: (i) when $v_L \neq v_s = v_n$: in this case the scalar potential arising from $\mathbf{v}_s$ is compensated by the contribution from $\mathbf{v}_L$, and (ii) if $\hat{l}$ is perpendicular to the flow the potential $A_0 = 0$. In both cases one has again the thermal distribution function $f(E) = 1/(1 + e^{E/T})$.

3.1. $v_L \neq v_s = v_n$. For the most symmetric solutions for the interface structure the anisotropy vector $\hat{l}$ is either parallel or perpendicular to the normal $\hat{n}$ to the wall (see Sections 3.14-15 in [1]). In both cases the result for the force on the interface can be obtained from the result in previous subsection by the rescaling of the momenta. Thus for $v_L \neq v_s = v_n$ one has

$$F(v_L)_{\hat{l} \parallel \hat{n}} = -A\hbar \frac{7\pi^2}{60} \frac{T^4}{\hbar^4 v_F c_\perp^2} \left[\frac{1}{3} + \frac{v_L}{v_F} + \frac{4}{3} \frac{v_L^2}{v_F^2} \frac{v_F}{v_F - v_L} \right], \tag{3.2a}$$

$$F(v_L)_{\hat{l} \perp \hat{n}} = -A\hbar \frac{7\pi^2}{60} \frac{T^4}{\hbar^4 v_F c_\perp^2} \left[\frac{1}{3} + \frac{v_L}{c_\perp} + \frac{4}{3} \frac{v_L^2}{c_\perp^2} \frac{c_\perp}{c_\perp - v_L} \right]. \tag{3.2b}$$

Here v_L is the velocity of the interface with respect to the heat bath.

In both cases the value of the pressure is the same, while the parameter Γ in the friction force is essentially different. The friction force for the case $\hat{l} \parallel \hat{n}$ coincides with that obtained by Kopnin in Ref.[7]. For $\hat{l} \perp \hat{n}$ the friction force is larger by the factor $c_\parallel/c_\perp \sim E_F/\Delta$. For these two directions the the pressure P and the friction parameter Γ can be written in the general form:

$$P = \hbar \frac{7\pi^2}{180} \frac{T^4}{\hbar^4} (-g)^{1/2}, \quad \Gamma = \hbar \frac{7\pi^2}{60} \frac{T^4}{\hbar^4} (-g)^{1/2} (g^{ik} n_i n_k)^{-1/2}. \tag{3.3}$$

Here $g_{\mu\nu} = (g_{ik}, g_{00} = -1)$ is the metric tensor of the stationary A-phase with g^{ik} from Eq.(1.2); $g = -1/(v_F^2 c_\perp^4)$ is the determinant of the metric tensor.

3.2 $v_L = v_n \neq v_s$, $\hat{l} \perp \hat{n}$. Since the interface is stationary in the heat bath frame the friction force is absent. Taking into account that for $\hat{l} \perp \hat{n}$ the scalar potential $A_0 = 0$ one obtains that the scattering of the fermions from the interface only to renormalizes the pressure:

$$P = \hbar \frac{7\pi^2}{180} \frac{T^4}{g_{00}^2 \hbar^4} (-g)^{1/2} = \frac{7\pi^2}{180\hbar^3} \frac{T^4}{v_F c_\perp^2} \left(1 - \frac{v_s^2}{c_\perp^2} \right)^{-2}, \tag{3.4}$$

where v_s is the superfluid velocity in the heat bath frame.

4. **Casimir force on vibrating interface, $T = 0$.** Let us now consider the friction force which arises when the AB interface is oscillating at $T = 0$. This corresponds to the dissipative Casimir force acting on the reflecting mirror in the form of the flat infinite plane, which oscillates in the electromagnetic vacuum along the normal to the plane. Such oscillations with frequency ω lead to the friction parameter [3]

$$\Gamma = \hbar \frac{1}{60\pi^2} \frac{\omega^4}{c^4} . \tag{4.1}$$

The dissipative Casimir force can arise also if the motion occurs along the plane: for the case of relative motion of two planes such force has been calculated in Ref.[11]. In superfluid ^{3}He this geometry corresponds to the A-phase vacuum between two flat boundaries moving with different velocities.

In the case of the reflecting AB interface oscillating in "relativistic" fermionic vacuum the result should be similar to Eq.(4.1). Vibrations of the interface lead to the production of pairs of fermions (see Refs.[6, 8]) and the friction force can be estimated by extrapolation of the results in Eqs.(3.2) for $T \neq 0$ if one substitutes $T \sim \hbar\omega/\pi$ [6, 8]. Let us find an exact expression for the force using again the covariance of the fermionic spectrum of the A-phase.

The motion of the interface with alternating velocity leads to the time dependence of the scalar potential A_0 in Eq.(1.3):

$$A_0(t) = p_F \hat{l} \cdot \mathbf{v}_s(t) , \tag{4.2}$$

where $\mathbf{v}_s(t)$ is the superfluid velocity in the reference frame of the vibrating interface. This however has no effect since such time dependence can be gauged away, ie compensated by the gauge transformation of the phase of the wave function: $\phi(t) \rightarrow \phi(t) + e \int^t dt' A_0(t')$.

The effect of the alternating velocity v_s comes from the time dependence of the metric tensor

$$g^{i0}(t) = v_s^i(t) , \qquad g^{ik}(t) = g^{ik}_{\text{stationary}} - v_s^i(t) v_s^k(t) , \tag{4.3}$$

with $g^{0i}(t) p_i = \mathbf{p} \cdot \mathbf{v}_s(t)$. If p_z is a good quantum number, then for each p_z the time dependence can be compensated by the gauge transformation, but due to the wall the momentum p_z is not conserved and this leads to mixing of states and finally to the production of the pair of fermions. If the motion of the wall is periodic, $\mathbf{v}_s(t) = \hat{z} v_\omega e^{-i\omega t}$, the term $\mathbf{p} \cdot \mathbf{v}_s(t)$ corresponds also to the action of the electromagnetic field with the finite frequency ω but with zero wave vector. This field provides the matrix element $M = p_z v_\omega$ for the "photon" absorption. This allows the annihilation of two particles, when they move to the wall. The energy of the fermions is $E(\mathbf{p}_1) + E(\mathbf{p}_2) = \omega$; their transverse momenta are opposite due to conservation of the momentum along the wall, $\mathbf{p}_{1\perp} = -\mathbf{p}_{2\perp}$; the deficite of the momentum along the normal $\hat{n}$ to the wall, $p_{1z} + p_{2z} \neq 0$, is absorbed by the wall. The inverse process corresponds to the production of fermion pair from the vacuum in the presence of the reflecting wall.

Let us consider first the case of isotropic fermions. The energy loss per unit time due to the pair creation is

$$\Gamma_\omega v_\omega^2 =$$

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$$= 4\omega \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{dp_{z1}}{2\pi} \int \frac{dp_{z2}}{2\pi} 2\pi \left(\frac{|M_1|^2}{\omega^2} + \frac{|M_2|^2}{\omega^2} \right) \frac{\partial E_1}{\partial p_{z1}} \frac{\partial E_2}{\partial p_{z2}} \delta(\omega - E(\mathbf{p}_1) - E(\mathbf{p}_2)) .$$

$$\tag{4.4}$$

Here $M_{1,2} = \mathbf{p}_{1,2} \cdot \mathbf{v}_\omega$; the factor 4 takes into account 2 spin species and two values of the "electric charge" $e = \pm 1$; $\partial E / \partial p_z$ is the group velocity of the particle moving towards the wall. Integration gives

$$\Gamma = \hbar \frac{1}{30\pi^2} \frac{\omega^4}{c^4} .$$

$$\tag{4.5}$$

Extrapolating to the case of the anispotropic fermions in the A-phase one obtains the friction parameter

$$\Gamma_{\hat{l}\|\hat{n}} = \hbar \frac{1}{30\pi^2} \frac{\omega^4}{v_F^2 c_\perp^2} , \quad \Gamma_{\hat{l}\perp\hat{n}} = \hbar \frac{1}{30\pi^2} \frac{\omega^4}{v_F c_\perp^3} .$$

$$\tag{4.6}$$

In the case of the moving AB-interface this effect can be observable since the velocities of "light" are small.

The relativistic description of the fermions in the A-phase of ^{3}He allows us to obtain easily many different results for the dynamics of the AB interface in the low temperature limit. On the other hand there is one to one correspondence between the motion of the interface and the Casimir effect for the objects moving in quantum relativistic vacuum, which will alow to model this effect in the experiments with the AB interface.

The next steps are (i) to extend calculations to the case of arbitrary angle between the normal of the interface and the orientation of the anisotropy vector $\hat{l}$; (ii) to find what happens when the velocity of the interface exceeds the smallest of the "speeds of light". This is interesting especially at $T = 0$, where some kind of Hawking radiation effect should arise due to analogy between the superfluid velocity and the gravity field.

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