Pis'ma v ZhETF, vol.63, iss.8, pp.652 - 657

HALL-EFFECT FOR NEUTRAL ATOMS

A.P.Kasantsev[*], *A.M.Dykhne*[□], *V.L.Pokrovsky*[*+].

[*] *Landau Institute for Theoretical Physics RAS*
142432 Chernogolovka Moscow Reg., Russia

[□] *Filial of the Kurchatov Institute of the Atomic Energy*
142092 Troitsk Moscow Reg., Russia

[+] *Dept. of Physics, Texas A&M University*
College Station, TX 77843-4242, USA

Submitted 27 March 1996

It is shown that polarizable neutral systems can drift in crossed magnetic and electric field. The drift velocity is perpendicular to both fields, but, contrary to the drift velocity of a charged particle, it exists only if fields vary in space or in time. We develop an adiabatic theory of this phenomenon and analyze conditions of its experimental observation.

PACS: 32.80.Rm, 64.30.+t

This work has been designed and partly performed several years ago. It has been interrupted by the untimely death of A.P.Kasantsev whose contribution to this article was decisive. By completing this work we pay our tribute to the memory of our dear friend, a profound scientist, a sincere and kind man.

Polarizable system in crossed fields. To our knowledge nobody considered the motion of a neutral particles in crossed electric and magnetic fields. The purpose of this work is to point out that neutral systems perform a drift in crossed fields, if these systems are polarizable. Consider a neutral system of point particles in external fields. Equations of motions for such a system are:

$$m_i\ddot{\mathbf{r}}_i = e_i\mathbf{E}(\mathbf{r}_i) + \frac{e_i}{c}\dot{\mathbf{r}}_i \times \mathbf{B}(\mathbf{r}_i) - \sum_{j\neq i}\nabla V_i(\mathbf{r}_{ij}) \tag{1}$$

Notations are obvious. Assuming the electric and magnetic fields to be classical, equation (1) is valid as an operator equation for the quantized motion of the neutral system.

Let the system size to be much smaller than the characteristic length L of the fields variation. Summing both sides of eqn (1) over i and expanding $\mathbf{E}(\mathbf{r}_i)$ and $\mathbf{B}(\mathbf{r}_i)$ over small deviations $\mathbf{r}_i - \mathbf{R}$ from the center-of-mass position $\mathbf{R}$, we obtain in the leading approximation:

$$M\ddot{\mathbf{R}} = (\mathbf{d}\nabla)\mathbf{E}(\mathbf{R}) + \frac{\dot{\mathbf{d}} \times \mathbf{B}(\mathbf{R})}{c} \tag{2}$$

Where $\mathbf{d} = \sum e_i\mathbf{r}_i$ is the operator of dipolar momentum and $M = \sum m_i$ is the total mass.

Further we accept the adiabatic approximation, i.e., we assume that the quantum state of our system adiabatically follows instant values of fields. It means that the center-of mass motion is slow enough to provide the characteristic time of motion to be much larger than the inverse frequency of the system. Quantitative implications of the adiabaticity requirement will be considered later. We additionally assume $B \gg E$. As a consequence of the adiabaticity the dipolar momentum is a linear function of the field $\mathbf{E}(\mathbf{R})$ at the center-of-mass location:

$$\mathbf{d} = \hat{\alpha}\mathbf{E}(\mathbf{R}) \tag{3}$$

The polarizability tensor $\hat{\alpha}$ may depend on magnetic field $\mathbf{B}(\mathbf{R})$. Thus, a closed differential equation for the center-of-mass radius-vector $\mathbf{R}$ coincides formally with equation (2), in which $\mathbf{d}$ is defined by eqn (3) and $\hat{\alpha}(\mathbf{B})$ is a given tensorial function of $\mathbf{B}$.

To get a tangible result we assume the velocity of the center-of-mass $\mathbf{v}_0 = \dot{\mathbf{R}}_0$ to be large, so that the change of velocity $\Delta\mathbf{v}$ caused by the fields $\mathbf{E}$ and $\mathbf{B}$ is small compared to $\mathbf{v}_0$. Then we find for $\Delta\mathbf{v}$:

$$\Delta\mathbf{v} = \frac{1}{M}\int\left[(\mathbf{d}\nabla)\mathbf{E}(\mathbf{R}(t)) + \frac{\dot{\mathbf{d}} \times \mathbf{B}(\mathbf{R})}{c}\right]dt \tag{4}$$

where $\mathbf{R} = \mathbf{R}(t) = \mathbf{R}_0 + \mathbf{v}_0 t$. Below several possible configurations are considered.

Constant Fields. First we consider static fields, varying in space. Let us focus on the second term in the r.-h.s. of eqn (4). A simplest possible geometry occurs when a condenser is located inside a solenoid in such a way that the electric field is perpendicular to the magnetic field. The beam of neutral atoms is supposed to propagate along the axis of the solenoid, parallel to the magnetic field and perpendicular to the electric field. The magnetic field $\mathbf{B}$ is assumed to be a constant. Then:

$$\Delta\mathbf{v} = \frac{\Delta\mathbf{d} \times \mathbf{B}}{Mc} \tag{5}$$

where $\Delta\mathbf{d}$ is the total change of the dipolar momentum. If the electric field $\mathbf{E}$ is the same at the beginning and at the end of trajectory, the total effect is zero. It is the case for an arbitrary electric field distribution, if it is fully located in the volume occupied by the magnetic field. Therefore, a more complex configuration is necessary. In order to obtain a finite drift velocity, one can shift the condenser in such a way that one its end, where the atomic beam enters the condenser, is located beyond the volume occupied by magnetic field, whereas the second end is located inside this volume. Then:

$$\Delta\mathbf{v} = -\frac{\alpha_{\perp}(B)(\mathbf{E} \times \mathbf{B})}{Mc}. \tag{6}$$

A modification of this idea is to excite the Rydberg state by a laser beam through a hole in the condenser. The effect in such a configuration differs by the sign from that given by Eq. (6). In the framework of this theory it does not depend on the position of a hole along the atomic beam path.

Another simple configuration is delivered by a variation of the magnetic field in space. Let us direct x-axis along the atomic beam and the magnetic field, y-axis along the electric field. We assume that $\mathbf{E}(x)$ is a constant in the condenser, whereas $B_x(x)$ changes from a value B_1 at a point where the beam enters the condenser till a value B_2 at a point where the beam leaves the condenser[1)]. Integrating by part we obtain a following result:

$$\Delta \mathrm{v} = -\frac{1}{Mc}\mathbf{E}\cdot\hat{\mathbf{B}}\int_{B_1}^{B_2}\alpha_\perp(B)dB \tag{7}$$

where $\hat{\mathbf{B}}$ is the unit vector along $\mathbf{B}$ direction. Notice that only the transverse to magnetic field polarizability $\alpha_\perp(B)$ enters expressions (6), (7) for the Hall drift velocity $\Delta\mathrm{v}$. A rough estimate of $\Delta\mathrm{v}$ magnitude can be found by substituting in (6) $\alpha_\perp \approx a^3$, where a is the radius of the external atomic orbital. In this way we arrive at an approximate formula:

$$\Delta \mathrm{v} \approx \frac{EBa^3}{Mc} \tag{8}$$

For $B = 10\,\mathrm{T}$, $E = 3\cdot 10^3\,\mathrm{V/cm}$, $M = M_p$ (M_p is the proton mass) and $a = 1\mathring{A}$ one finds from (8) $\Delta\mathrm{v} \approx 1.85\cdot 10^{-5}\mathrm{cm/s}$. This is far beyond the experimental resolution with no hope to increase the effect by changing fields. Even more polarizable molecules have no chance to enhance the effect to an appeciable value. However, the situation changes drastically for highly excited (Rydberg) atoms. This stems from the fact that $a \sim n^2$ where n is the principal quantum number. Therefore, the Hall velocity (8) is proportional to n^6. For a modest value of $n = 50$ the enhancement factor is $\sim 1.6\cdot 10^{10}$ producing $\Delta\mathrm{v} \sim 2.89\cdot 10^5\,\mathrm{cm/s}$. However, as we show below, a more accurate treatment reduces this value in approximately n/m times where m is the magnetic quantum number, which is not large at a conventional way of excitation.

An unexpected feature of the neutral atom Hall-effect is that is does not depend on atomic path length in crossed fields. Instead it depends on the variation of the electric or magnetic field or polarizability. Let us discuss eqn (7) in more details. We restrict our calculations with a range of fields $B \ll c/n^3$ in atomic units. This inequality guarantees that the diamagnetic energy $c^{-2}B^2n^4$ is much less than the Coulomb energy n^{-2}. For this range of fields the transverse polarizability $\alpha_\perp$ has been found in [1] to be:

$$\alpha_\perp(B) = \frac{9}{2}\frac{mn^2c}{B} \tag{9}$$

Here m is the magnetic quantum number, which is well-defined in external magnetic field. Plugging (9) into (7) we arrive at

$$\Delta\mathrm{v} = -\frac{9mn^2}{2M}\ln\frac{B_2}{B_1}\cdot\mathbf{E}\cdot\hat{\mathbf{B}} \tag{10}$$

Two or three lasers can excite an atom from the ground-state to a state with $m \leq 3$. Thus, in comparison to our apriori estimate (8) the result given by eqn. (10) differs by a factor $(9/2)m(c/Bn^4)$. For the most interesting range of parameters the factor c/Bn^4 is small (diamagnetic energy is much larger than the interlevel spacing). Nevertheless, an estimate for the same value of

[1)] Since $\nabla\mathbf{B} = 0$, a purely longitudinal variation of $\mathbf{B}$ is impossible and transverse components are inavoidable. However, they can be easily made equal to zero on the axis of solenoid, where the beam is located. It is possible, at least in principle, that only y-component of magnetic field appears, which does not produce any effect.

$E = 3 \cdot 10^3\,\mathrm{V/cm}$, $M = M_p$ (Hydrogen), $B_2/B_1 \approx 2.7$, $m = 3$ and $n = 50$ eqn(10) gives $|\Delta \mathrm{v}| \approx 2150\,\mathrm{cm/s}$. For the distance between the magnet and the screen $1m$ and the initial velocity of atoms $1\,\mathrm{km/s}$ the excited atoms form a spot at the distance $2.15\,\mathrm{cm}$ from the central spot, produced by non-excited atoms. These figures give a realization about expected magnititude of the Hall velocity. The effect can be used to separate excited atoms from nonexcited ones and for the measurement of $\alpha_\perp(B)$. For $m = 0$ the effect is much weaker. It is determined by the diamagnetic Hamiltonian. The polarizability for this case can be found in [1].

Let us analyze the contribution of the first term in the integral (4). All fields depend on the coordinate x only. On the other hand vector d is directed along y. Therefore, formally this term is equal to zero. However, small angular misalignment $\delta\theta$ can create a non-zero random contribution $\Delta \mathrm{v}_{rand}$, proportional to $\sim \frac{dE}{M\Delta \mathrm{v}_0}(\delta\theta)$. For $E = 3 \cdot 10^3\,\mathrm{V/cm}$, $B = 10\,\mathrm{T}$ the ratio $\Delta \mathrm{v}_{rand}/\Delta \mathrm{v}$ is $10^{-2}\delta\theta$ by the order of magnitude and can be neglected.

Alternating Fields. Consider an excited atom in an alternating electromagnetic field in a resonator. We assume that the alternating magnetic field $B_x = E \sin \omega t \sin kz$ is directed along x-axis, the alternating electric field $E_y = E \cos \omega t \cos kz$ is directed along y. The atomic beam propagates along x-axis. The frequency ω is assumed to be small enough to justify the adiabatic approximation. To find the effect, we employ eqn (4) neglecting the first term in the integral. In contrast to the case of permanent fields, the main contribution to the Hall velocity in alternating fields comes from the linear Stark effect, specific for the Hydrogen and Rydberg states. Indeed, in this case the dipolar moment d in the electric field E has a constant modulus, but changes its direction with E:

$$\mathrm{d} = d_{nk}\hat{\mathrm{E}} \tag{11}$$

where $d_{nk} = (3/2)nk$ in atomic units is the dipolar moment due to the linear Stark-effect, k is the parabolic quantum number associated with the quantized Runge–Lenz vector A and varying from $-(n-|m|-1)$ till $n-|m|-1$ [2]; $\hat{\mathrm{E}}$ is the unit vector parallel to E. In our geometry the vector $\hat{\mathrm{E}}$ is always collinear to the x-axis and changes its sign each half-period. Thus:

$$\dot{\mathrm{d}} = 3nk\hat{x} \sum_{n=-\infty}^{\infty} (-1)^n \delta\left(t - \frac{(n-1/2)\pi}{\omega}\right) \tag{12}$$

Integrating $\dot{\mathrm{d}} \cdot \mathrm{B}$, according to Eqs.(4) and (12), and neglecting oscillating terms, we find:

$$\Delta \mathrm{v}_z = \frac{3\omega nkEl}{\pi M c v_0} \sin kz \,\mathrm{sign}(\cos kz) \tag{13}$$

where l is the length of the path passed by atoms in the ac fields and v_0 is the velocity of atoms in the beam. We have assumed that the atomic beam width is much smaller than the wave-length of the EM-wave. The maximal effect is reached at $\sin kz = 1$. For $\omega = 2\pi 10^8\,\mathrm{s}^{-1}$, $E = 100\,\mathrm{V/cm}$, $l = 10\,\mathrm{cm}$, $v_0 = 1\,\mathrm{km/s}$, $n = k = 50$ we find $\Delta \mathrm{v}_z = 6,700\,\mathrm{cm/s}$. Note that the ac Hall-effect is proportional to the path length l, passed by the atoms in the ac field. This fact provides an additional opportunity to enhance the effect.

An important question is: what values of the quantum number k can be reached at the excitation of Rydberg states. We calculated matrix elements for dipolar transitions from the ground state to a highly excited state with the parabolic quantum numbers n_1 and n_2 and have found that they decrease slowly with n_i ($\sim n_i^{-3/2}$). Therefore the exitation of large $k = n_1 - n_2$ is reasonably probable.

In our calculations we did not take into account the finite size of the atomic remainder, which lifts the degeneracy over the total orbital moment l. Thus, if the frequency is large in comparison to the finite size splitting $\delta E \sim n^{-6}$ for s-wave the linear Stark effect is responsible for the effect. Otherwise the quadratic Stark-effect should be accounted for. For $n = 50$ the crossover frequency is about 1 MHz. The quadratic effect is much weaker. Details will be published elsewhere.

Limitations of the effect. There are two kinds of limitations for the Hall-effect magnitude: intrinsic limitations, wich stem from physical reasons, and methodical limitations associated with the range of validity for the theory i.e. with what we can calculate. An example of an intrinsic limitation is given by inequality $\Delta v < 4.5 n^3 E \ln(B_2/B_1)$, following from eqn. (10). Another limitation $\omega < 1/n^3$ means that at higher frequencies atomic electrons easily transit from one orbit to another and the adiabaticity is violated. It seems to be a kind of methodic limitations, since our description definitely fails at so high frequencies. On the other hand, for such a large frequency the atomic electrons on remote orbits do not have time enough for polarization. Thus, we expect the effect to decrease rapidly at $\omega > 1/n^3$.

An important methodic limitation is $B \ll c/n^3$. Expressions for polarizability etc., which we have employed earlier, are valid in this range of B. However, it does not mean that the effect vanishes at higher fields. It does exist, though we can not describe it properly. The range $B \sim c/n^3$ is of a great interest [3, 4] since it is a range of the quantum chaos, where the levels are located randomly according to the orthogonal ensemble [5]. The problem of polarizability of random quantum levels is still open. It is worthwile to note that the experimental data on atomic drift in this range of fields could give an important information on the properties of quantum states.

An important question is the validity of the adiabatic approximation. For static fields a necessary requirement is that the field variation in a frame of reference, associated with moving atoms, is slow in the scale of atomic frequency $1/n^3$ or even the smaller diamagnetic frequency $B^2 n^3/c^2$. The size of the edge inhomogeneity for electric field is determined by the distance h between condenser plates. Accepting $h = 0.5\,\mathrm{cm}$ and $\Delta v_0 = 10^5\,\mathrm{cm/s}$ we see that even the second requirement is satisfied reliably: $h/\Delta v_0 \ll B^2 n^3/c^2$.

The adiabatic approximation fails when an atom transits from the ground state to an excited state. However, the final state is established in the interval of time equal to the inverse transition frequency for this state by the order of magnitude. Therefore, one can neglect this interval, very short in the scale of the center-of-mass motion. Due to a long life-time of the Rydberg atoms (about 0.1 s for $n = 50$), one can neglect the possibility of reciprocal transitions to the ground state on the way of a Rydberg atom from the condenser to the observation point.

A serious obstacle for observation of *ac* fields effect may the fact that a strong *ac* field produces ionization of Rydberg atoms rather effectively [6]. Therefore, the length of the resonator l should be small enough to reduce the ionization probability. On the other hand, the drift velocity is proportional to l. Hopefully, ajusting the field parameters n, E_1, and l one can find a range where the *ac* Hall-effect is observable. However, an additional analysis is necessary.

In conclusion we propose to find experimentally a new phenomenon: Hall-effect (drift) of neutral atoms. The best objects for observation of this effect are highly excited (Rydberg) atoms, due to their giant polarizability. Another possibility may be eximeric molecules. For experiments with static fields simple estimates made in the text show that effect is not small. In a most natural geometry Hall velocity is perpendicular to the initial velocity of atoms. This deviation enables one to separate excited atoms. Measurements of the Hall velocity would be a source of additional information on the spectrum and wave functions of atomic electrons especially in the quantum chaos range $B \sim c/n^3$. In the case of static fields the theory predicts that the drift velocity does not depend on the time which atoms spend in fields, but only on spatial variations of fields.

The atomic drift can be observed also in alternating fields forming standing waves. In this situation it grows linearly with the path-length passed in the *ac* field and its amplitude.

One of the authors (V.L.P.) is grateful to G. Welch for an illuminating discussion on the experimental situation and indicating several references.

---

1. A.A.Belov, A.P.Kasantsev, D.V.Khveshchenko, and V.L. Pokrovsky, J.Phys. **B18**, 4683 (1985).
2. L.D.Landau and E.M.Lifshitz, *Quantum Mechanics*, Pergamon Press, 1986.
3. D.Delande and J.C.Gay, Phys. Rev. Lett. **57**, 2006 (1986); G.Wunner et al., ibid. **57**, 3261 (1986); D.Wintgen and H.Friedrich, ibid. **57**, 571 (1986).
4. G.R.Welch, M.M.Kash, C.Iu et al., Phys. Rev. Lett. **62**, 893 (1989).
5. F.J.Dyson, J.Math, Phys. **3**, 140 (1962); M.L.Mehta, *Random Matrices*, Ac. Press, NY, 1991; B.D.Simons and B.L.Altshuler, Phys. Rev. Lett. **71**, 2899 (1993); B.D.Simons, P.A.Lee and B.L.Altshuler, ibid. **72**, 64 (1993).
6. M.G.Littman, M.M.Kash, and D.Kleppner, Phys. Rev. Lett., **41**, 105 (1978).