Pis'ma v ZhETF, vol.68, iss.1, pp.15 - 20

ON THE SCREENING OF THE LEPTONIC CHARGE OF A BODY BY THE CONDENSATE OF CHARGED BOSONS

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Submitted 1 June 1998

The hypothesis of leptonic charge raises the problem of its neutralization which is crucial for the stability of material bodies. The screening of the leptonic charge by charged bosonic condensate is considered. The screening length, the structure of the skin layer are investigated in both the nonrelativistic and relativistic regimes.

PACS: 14.80.-j

1. Introduction. The possible existence of leptonic charge coupled to leptonic photons is discussed for a long time [1]. The stability of the bodies with respect to this additional interaction requires the neutralization of leptonic charge unless the leptonic interaction is extremely small [1]. The neutralization by antineutrinos does not insure the stability of the bodies [2]: the pressure of degenerate gas of antineutrinos occurs to be disruptive for the bodies. Another possibility is the neutralization by the condensate of leptonic bosons (scalar superpartners of antineutrinos, for example). In this case the neutralization is complete everywhere except of the skin layer [2] and the stability of the skin layer requires the constant of leptonic interaction α_l to be smaller than approximately 10^{-12} (α^6)[3]. In Ref.[3] the thickness of the skin layer was obtained from the dimensional analysis of the coupled Klein-Gordon and Poisson equations for scalar field ϕ and leptostatic potential A_0. The existence of the solutions for these equations was not considered. Here we present the numerical solutions of Klein – Gordon – Poisson equations in some specific cases. We will consider small pieces of matter, the balls with radius equal to several screening length, uniformly charged by electronic charge. Two limiting cases will be analyzed: nonrelativistic and relativistic regimes for the Klein-Gordon equation for the boson field ϕ. In the nonrelativistic case there exists the continuous set of solutions corresponding to different degrees of neutralization (from 0 to 100%). In the relativistic case (massless bosons) only the totally screened solution (100% neutralization) exists. The latter statement is in agreement with the analysis of Ref.[4] where the problem of screening (although in another context and for different distribution of screened charge) was also considered.

In section 2 we consider the system of equations to be solved. In section 3 we present the numerical solutions of this system in nonrelativistic case. In section 4 we consider relativistic case. In conclusion we outline the prospect of future investigations.

2. The set of equations. Let A_μ be leptonic gauge field coupled to massive boson field Φ of mass m that has leptonic charge $e = -g$ and electrons that have leptonic charge g. The Lagrangian of the system is

$$L = -\frac{1}{4}F_{\mu\nu}^2 + (\partial_\mu - ieA_\mu)\Phi^*(\partial_\mu + ieA_\mu)\Phi - m^2\Phi^*\Phi - j_\mu^{ext}A_\mu, \tag{1}$$

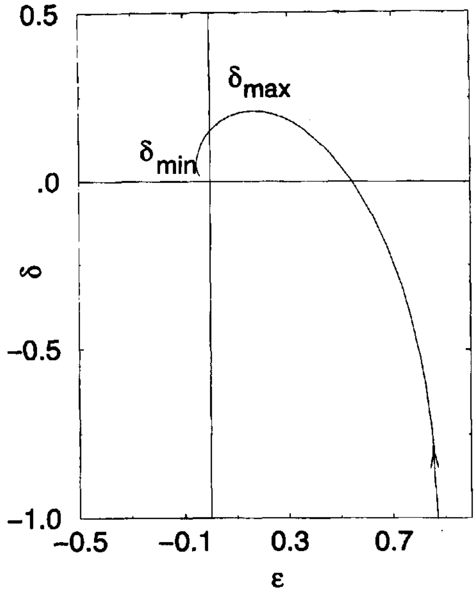

Fig.1. The curve in $\epsilon - \delta$ parameters plane for which there are the solutions of eqs.(6). The arrow on the curve corresponds to the strengthening of the screening

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $j_\mu^{ext} = g\delta_{\mu 0} n\theta(R-r)$ is the current of leptonic charge of electrons, n is the density of electrons inside a body of radius R. The equations of motion for fields A_μ and Φ are

$$\begin{aligned} -(\partial_\mu + ieA_\mu)^2\Phi - m^2\Phi = 0, \\ \partial_\nu^2 A_\mu = j_\mu^{ext} + e\Phi^*(i \overleftrightarrow{\partial}_\mu - 2eA_\mu)\Phi. \end{aligned} \tag{2}$$

In the static limit for leptonic gauge field,

$$A_i = 0 \quad , \dot{A}_0 = 0 \ ,$$

and for the bosonic ground state $\Phi = e^{-iEt}\phi$. We derive from eqs.(2) the coupled Klein-Gordon and Poisson equations for scalar field ϕ and leptostatic potential A_0

$$\begin{aligned} (E + gA_0)^2\phi + \Delta\phi = m^2\phi, \\ -\Delta A_0 = gn\theta(R-r) - g2(E + gA_0)\phi^2. \end{aligned} \tag{3}$$

These equations describe the screening of leptonic charge of a body by the condensate (E is a ground state energy of the bosons) of oppositely charged bosons. In what follows we will solve these equations numerically for nonrelativistic and ultrarelativistic bosons.

3. Nonrelativistic regime. Consider first the heavy boson, $gA_0 \ll m, E = m + \delta E, \delta E \ll m$ (nonrelativistic limit). Then from eqs.(3) one gets

$$\delta E\phi \approx (-\tfrac{1}{2m}\Delta - gA_0)\phi, \quad -\Delta A_0 \approx gn\theta(R-r) - g2m\phi^2. \tag{4}$$

To cast these equations into scale invariant form we make the following substitutions

$$\begin{aligned} r \to r(g^2nm)^{-1/4} \ , \quad \delta E = \epsilon \to \epsilon(g^2n/m)^{1/2}, \\ -gA_0 = V \to V(g^2n/m)^{1/2}, \quad \phi \to \phi(n/2m)^{1/2}. \end{aligned} \tag{5}$$

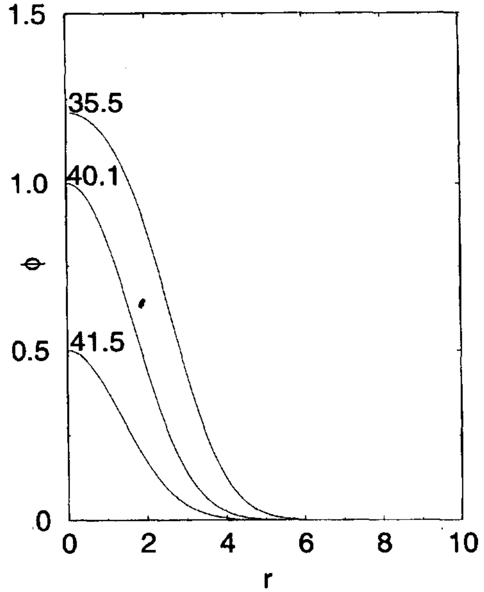

Fig.2. The growing of the boson field ϕ for first regime of the strengthening of the screening. The numbers near the curves are the values of $Q/4\pi$

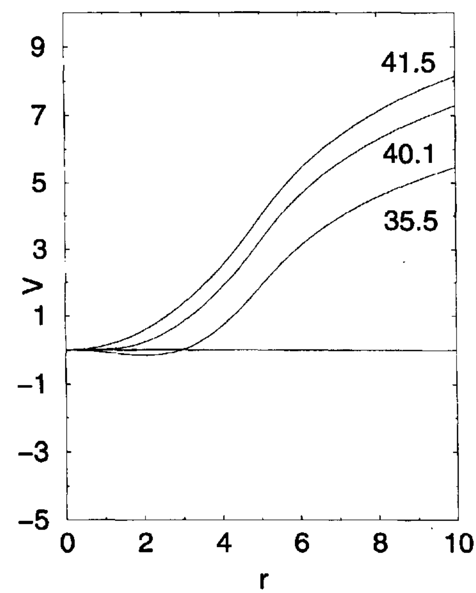

Fig.3. The lowering of the potential energy of bosons $V(r)$ for first regime of the strengthening of the screening. The numbers near the curves are the values of $Q/4\pi$

Then, the eqs.(4) read

$$\epsilon\phi = (-\tfrac{1}{2}\Delta + V)\phi, \quad \Delta V = \theta(R-r) - \phi^2. \tag{6}$$

We solve equations (6) with the following boundary conditions at the origin

$$V(0) = 0, \qquad V'(0) = 0,$$

$$\phi(0) = 1 + \delta, \qquad \phi'(0) = 0.$$

For a given δ we are looking for the lowest eigenvalue ϵ for which $\phi \to 0$ at the infinity. The resulting curve in $\epsilon - \delta$ plane is shown in Fig.1 ($R = 5$). This curve starts from the point $\epsilon = 0.85$, $\delta = -1$ for which

$$\phi = 0 \ \ , V(r) = \frac{1}{6}r^2 \ \ (r < R), \ \ V(r) = \frac{1}{2}R^2 - \frac{1}{3}R^3/r \ \ (r > R).$$

The eigenvalue $\epsilon = 0.85$ is close to the lowest energy eigenvalue of harmonic oscillator

$$V(r) = \frac{1}{6}r^2, \ \omega = 1/\sqrt{3}, \ \ \epsilon = \frac{3}{2}\omega = \sqrt{3}/2.$$

The solution corresponding to $\epsilon = 0.85$, $\delta = -1$ is the totally unscreened solution ($\phi = 0$) and its charge is equal to $Q_{max} = 4\pi/3R^3$ (in units $n(g^2nm)^{-3/4}$). Going from the starting point along the curve of Fig.1 we get more and more screened solutions. There are two qualitatively different regimes in this process of strengthening of the screening effect. The first one corresponds to δ growing from -1 to $\delta_{max} = 0.21$ for which ϕ

is growing everywhere and $V(r)$ is lowering everywhere (see Figs.2,3). The second one corresponds to δ lowering from δ_{max} to $\delta_{min} = 0.02$ for which ϕ is lowering and flattening at the origin and slightly bumping near the surface of the body, $V(r)$ is growing and flattening at the origin and slightly decreasing near the surface of the body (see Figs.4,5). The end point on the curve ($\delta = \delta_{min}$) corresponds to the total charge which is 60 times smaller than the initial unscreened charge of a body ($Q_{min} = \frac{1}{60} Q_{max}$). From the physical argumentation it is clear that the totally screened solution with zero charge ($Q_{min} \to 0$) is possible.

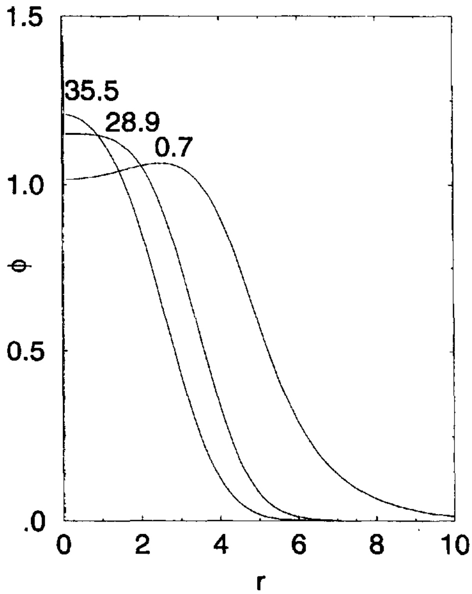

Fig.4. The flattening of the boson field ϕ near the origin and bumping of ϕ near the surface from inside for second regime of the strengthening of the screening. The numbers near the curves are the values of $Q/4\pi$

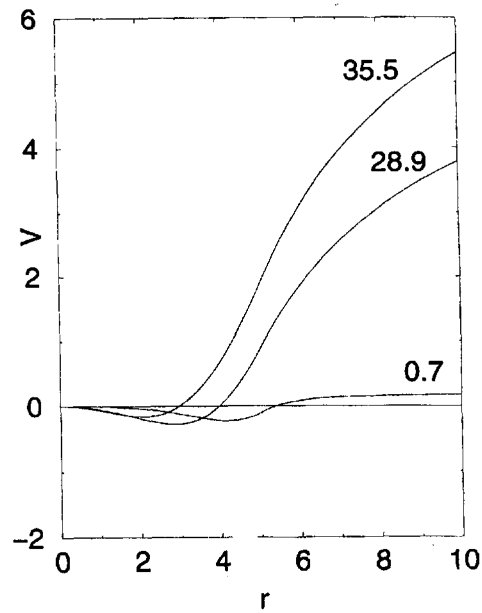

Fig.5. The flattening of the potential energy of bosons $V(r)$ near the origin and falling of $V(r)$ near the surface from inside for second regime of the strengthening of the screening. The numbers near the curves are the values of $Q/4\pi$

4. Relativistic regime. To consider the relativistic limit we put $m = 0$ in the equations (3). We set also $E = 0$ by shifting the potential A_0 and for the boson potential energy $V = -gA_0$ and boson field ϕ we arrive at the equations

$$V^2\phi + \Delta\phi = 0, \quad \Delta V = g^2 n\theta(R - r) + g^2 2V\phi^2. \tag{7}$$

As in the nonrelativistic case to cast these equations into scale invariant form we make the following substitutions

$$r \to r(g^2 n)^{-1/3}, \quad V \to V(g^2 n)^{1/3}, \quad \phi \to \phi(n/2|V(0)|)^{1/2}. \tag{8}$$

The substitutions (8) reduce the equations (7) to the scale invariant form

$$V^2\phi + \Delta\phi = 0, \quad \Delta V = \theta(R - r) - \frac{V}{V(0)}\phi^2. \tag{9}$$

We solve equations (9) with the following boundary conditions at the origin

$$V(0) = -\epsilon, \qquad V'(0) = 0,$$

$$\phi(0) = 1 + \delta, \qquad \phi'(0) = 0.$$

There is only one point for δ and ϵ where V and ϕ goes to zero at infinity. This is the point

$$\delta = 0.021, \qquad \epsilon = 0.265.$$

Figs.6,7 present the corresponding solutions for ϕ and V $(R = 5)$. The charge of the body is completely screened. It follows from both the asymptotic of V which has no Coulomb tail and from the uniqueness of the solution (there are no different degrees of neutralization) and is in agreement with the analysis of Ref.[4]. Physically it is obvious that for massless bosons any electric field associated with the growth of the potential near the surface produces the pairs boson-antiboson. The bosons neutralize the body and antibosons go away to the infinity.

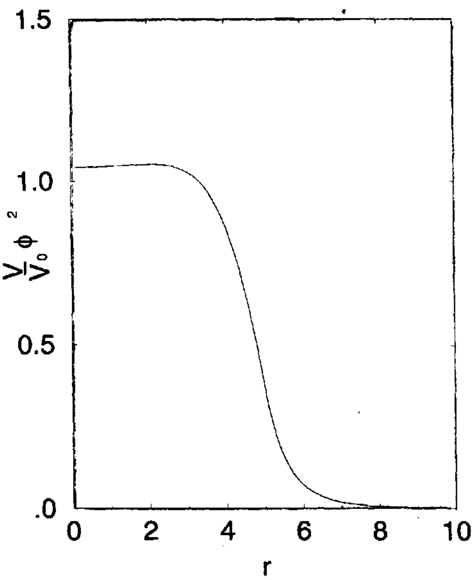

Fig.6. Minus the charge density of the boson field $(V/V_0)\phi^2$ for the totally screened leptonic charge of a body by massless boson field

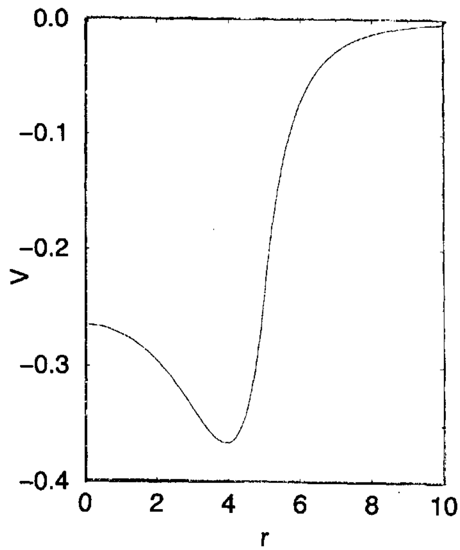

Fig.7. The potential energy of massless bosons for the totally screened leptonic charge of a body by massless boson field

5. Conclusion. We have presented the numerical solutions of Klein - Gordon - Poisson equations for the screening of hypothetical leptonic (electronic) charge of small balls of matter with radius equal to several (five in our case) screening length by the condensed boson field. We considered both the nonrelativistic and relativistic regimes for the Klein-Gordon equation for the boson field . In the nonrelativistic case we found the continuous set of solutions corresponding to different degrees of neutralization (from 0% to 100%). In the relativistic case (massless bosons) the equations (7) have only the totally screened solution (corresponding to the 100% neutralization).

There is an interesting problem of considering larger (in comparison to the screening length) balls of matter. From the numerical point of view computations for $R = 10$(in units of screening length) or larger are difficult. From physical point of view it is clear that the structure of the skin layer is the same for all bodies of sufficiently large size if all they are totally screened by bosons. This problem is now under investigation.

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