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LOW-FREQUENCY RELATIVISTIC ELECTROMAGNETIC SOLITONS IN COLLISIONLESS PLASMAS

T.Zh.Esirkepov, F.F.Kamenets, S.V.Bulanov, N.M.Naumova**

Moscow Institute of Physics and Technology
141700 Dolgoprudnii, Russia.

* *General Physics Institute RAS*
117333 Moscow, Russia

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A relativistic electromagnetic soliton solution in the model of a one-dimensional, unbounded, cold, collisionless plasma is obtained without using the envelope approximation. The breaking of solitons with overcritical amplitudes is observed. The stability of undercritical solitons and the breaking of overcritical solitons are demonstrated by a particle-in-cell computer simulation.

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1. The technique of ultrashort, ultraintense laser pulses has been developing rapidly in recent years [1]. It has called into being a vast area of nonlinear physics describing the dynamics of matter in ultraintense electromagnetic fields. Among the enormous variety of nonlinear effects induced by a relativistically strong laser pulse propagating through a plasma, we will select out as our topic the formation and evolution of relativistic solitary waves.

Electromagnetic solitons with relativistic amplitudes were first investigated by Kozlov, Litvak and Suvorov [2]. Their study was continued in Refs. [3 – 10]. For the most part these studies used the envelope approximation, i.e., they assumed that the solitary solution to be found is a wave packet whose length and frequency are much greater than those of an electron plasma wave.

Electromagnetic relativistic solitons in collisionless plasmas were observed in a PIC-code simulation of the interaction of ultraintense laser radiation with underdense [11] and overdense plasmas [12]. It was shown that the soliton formation is a significant channel of laser-pulse energy transformation [13].

The solitons obtained in computer simulations, in contrast to theoretical models using the envelope approximation, have lengths and frequencies of the order of the length and frequency of an electron plasma wave and move at nonrelativistic speed [13].

As is shown in [11, 14], the growth of instability induced by Raman scattering plays the key role in the mechanism of decay of a long, relativistically strong laser pulse. If the length of a high-frequency laser pulse is sufficiently greater than the electron plasma wavelength λ_{pe}, the propagation of the pulse through a plasma is inevitably accompanied by envelope modulation on a scale of the order of the electron plasma wavelength. This modulation is induced by Raman scattering and leads to decay of the pulse.

On the other hand, an intense laser pulse with length sufficiently smaller than the electron plasma wavelength excites wake-field plasma waves and quickly loses energy.

On the basis of this discussion one can suggest that the expected size of a relativistic electromagnetic soliton must be comparable with the electron plasma wavelength λ_{pe}.

The main task of this paper is to investigate such a "short" relativistic soliton.

2. Let us consider a model of unbounded cold collisionless plasma, described by Maxwell's equations and the hydrodynamic equations of an electron fluid on a fixed ion background:

$$\Delta \mathbf{A} - \mathbf{A}_{tt} - \nabla\varphi_t - \frac{n}{\gamma}(\mathbf{P}+\mathbf{A}) = 0\,,$$

$$n = 1 + \Delta\varphi\,, \quad \mathrm{div}\mathbf{A} = 0\,, \mathbf{P}_t = \nabla(\varphi - \gamma) + \frac{1}{\gamma}(\mathbf{P}+\mathbf{A}) \times \mathrm{rot}\mathbf{P}\,. \tag{1}$$

Here $n = n_e/n_i$ is the ratio of the electron and ion densities, n_i = const; $\mathbf{P}$ is the generalized momentum of an electron, $\mathbf{P} = \mathbf{p} - \mathbf{A}$; $\gamma^2 = 1 + (\mathbf{P}+\mathbf{A})^2$. The equations are written in dimensionless form with the help of the substitutions $(\omega_{pe}/c)\mathbf{x} \to \mathbf{x}$, $\omega_{pe}t \to t$, $(e/m_ec^2)\mathbf{A} \to \mathbf{A}$, $(e/m_ec^2)\varphi \to \varphi$, $(e/m_e\omega_{pe}c)\mathbf{E} \to \mathbf{E}$, $(e/m_e\omega_{pe}c)\mathbf{B} \to \mathbf{B}$, $(1/m_ec)\mathbf{p} \to \mathbf{p}$, where m_e and e are the electron mass and charge, and $\omega_{pe} = \sqrt{4\pi n_e e^2/m_e}$ is the electron plasma-wave frequency for the undisturbed density $n_e = n_i$ at infinity.

Let us consider the case when all the variables that characterize fields and plasma are independent of y and z, so that $\partial_y = \partial_z = 0$. Denoting $\mathbf{P} = (P^1, P^2, P^3)$ and $\mathbf{A} = (A^1, A^2, A^3)$, we can rewrite Eqs. (1) in terms of the vector components:

$$\begin{aligned}
&A^1_{xx} - A^1_{tt} - \varphi_{xt} - \frac{n}{\gamma}(P^1 + A^1) = 0\,,\\
&A^2_{xx} - A^2_{tt} - \frac{n}{\gamma}(P^2 + A^2) = 0\,, \quad A^3_{xx} - A^3_{tt} - \frac{n}{\gamma}(P^3 + A^3) = 0\,,\\
&n = 1 + \varphi_{xx}\,, \quad A^1_x = 0\,,\\
&P^1_t = \varphi_x - \gamma_x + \frac{1}{\gamma}\left(A^2P^2_x + A^3P^3_x + P^2P^2_x + P^3P^3_x\right)\,,\\
&P^2_t + \frac{1}{\gamma}P^1P^2_x = 0\,, \quad P^3_t + \frac{1}{\gamma}P^1P^3_x = 0\,.
\end{aligned} \tag{2}$$

We want to find a spatially localized solution corresponding to perturbations of the fields and of the electron density and momentum which move with a constant velocity V. Localization in the spatial coordinate entails the relation $A^1 = 0$.

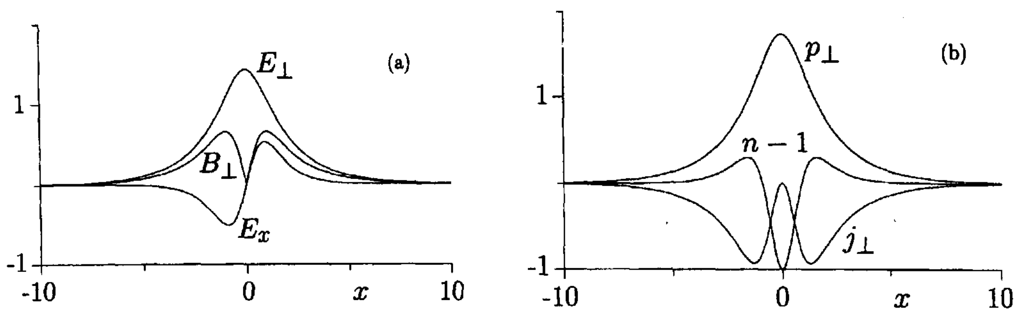

Fig.1. Electromagnetic fields and the momentum and density of electron fluid in the soliton. (a) Fields $E_\perp$, $B_\perp$, E_x. (b) Transverse momentum $p_\perp$ and density n of the electron fluid and the transverse current $j_\perp$

Let us assume that the solution to be found is induced by an ultraintense, circularly polarized, finite laser pulse propagating through an underdense plasma. (In the one-dimensional model the direction of laser pulse propagation coincides with the $0x$ axis).

Let us imagine that before the interaction the laser pulse was outside the plasma, in vacuum. Then the momentum of the laser pulse had only one nonzero component (in the direction of propagation). According to generalized momentum conservation, it is natural to expect that the transverse generalized momentum $\mathbf{P}_\perp$ (averaged over a sufficiently long period) in the region of the solution that we seek should be zero after the interaction. Therefore, the condition $P^2 = P^3 = 0$, which simplifies Eqs. (2) significantly, can be regarded as a strengthened form of the momentum conservation law.

We can also suppose the solution to be circularly polarized as a result of polarization of the initial laser pulse. This condition can be regarded as a strengthened form of the angular momentum conservation law and is a result of the symmetry of Eqs. (2) with respect to rotation in the plane (y, z).

Thus we are looking for a solution of the form (in terms of the new coordinates ξ, τ):

$$\begin{aligned} &\xi = x - Vt\,, \quad \tau = t\,, \\ &A^1 = 0\,, \quad A^2 + iA^3 = a(\xi)e^{i\omega((1-V^2)\tau - V\xi)}\,, \\ &P^1 = Vb(\xi)\,, \quad P^2 = P^3 = 0\,. \end{aligned} \tag{3}$$

Note that the components P^2, P^3 can be proved to be zero by making use of the requirement that the solution is localized in space.

After substituting expression (3) into (2) we obtain

$$\left(\gamma - V^2 b\right)'' = \frac{b}{\gamma - b}\,, \qquad \frac{a''}{a} = \frac{1}{1-V^2}\frac{1}{\gamma - b} - \omega^2\,, \tag{4}$$

where $\gamma = \left(1 + a^2 + V^2 b^2\right)^{1/2}$. This system of equations with the boundary conditions

$$a(\infty) = b(\infty) = 0\,, \quad a(\xi) < \infty\,, b(\xi) < \infty \tag{5}$$

describes a one-dimensional relativistic electromagnetic soliton propagating through a cold collisionless plasma. The velocity of the soliton is less than speed of light, $V < 1$. The frequency of the soliton is less than the electron plasma frequency, $\omega < 1$.

In the case $V = 0$ the component P^1 vanishes, and the system of equations (4) can be reduced to

$$b = \frac{\gamma\gamma''}{1+\gamma''}\,, \qquad \frac{a''}{a} = \frac{1+\gamma''}{\gamma} - \omega^2\,. \tag{6}$$

These equations are easily integrated with the help of the substitution $a = \operatorname{sh} u$, $\gamma = \operatorname{ch} u$, and we obtain:

$$\mathbf{A} = \frac{2\sqrt{1-\omega^2}\ \operatorname{ch}\left(\xi\sqrt{1-\omega^2}\right)}{\operatorname{ch}^2\left(\xi\sqrt{1-\omega^2}\right) + \omega^2 - 1}\ \exp(i\omega\tau)\,. \tag{7}$$

One can see that the relationship between the maximum soliton amplitude A_0 and the angular velocity of rotation ω of the plane of polarization of the soliton is given by

$$A_0 = \frac{2\sqrt{1-\omega^2}}{\omega^2}\,. \tag{8}$$

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The values of the fields, electron momentum, vector potential, electron density, and gamma factor in the soliton can be written as

$$\mathbf{E} = \left(\frac{2(1-\omega^2)}{\omega}\frac{\mathrm{sh}(\zeta)}{\mathrm{ch}^2(\zeta)-1+\omega^2}, \sin(t\omega), -\cos(t\omega)\right)\frac{2\omega\sqrt{1-\omega^2}\mathrm{ch}(\zeta)}{\mathrm{ch}^2(\zeta)-1+\omega^2},$$

$$\mathbf{B} = (0, \sin(t\omega), -\cos(t\omega))\frac{2(1-\omega^2)\,\mathrm{sh}(\zeta)}{\mathrm{ch}^2(\zeta)-1+\omega^2}\left(1+\frac{2(1-\omega^2)}{\mathrm{ch}^2(\zeta)-1+\omega^2}\right),$$

$$\mathbf{p} = \mathbf{A} = (0, \cos(t\omega), \sin(t\omega))\frac{2\sqrt{1-\omega^2}\,\mathrm{ch}(\zeta)}{\mathrm{ch}^2(\zeta)-1+\omega^2},$$

$$n = 1 + \frac{(1-\omega^2)^2\ \left(\mathrm{ch}(4\zeta)-2(2\omega^2-1)\,\mathrm{ch}(2\zeta)-3\right)}{\left(\mathrm{ch}^2(\zeta)-1+\omega^2\right)^3},$$

$$\gamma = 1 + \frac{2(1-\omega^2)}{\mathrm{ch}^2(\zeta)-1+\omega^2}. \tag{9}$$

Here $\zeta = \sqrt{1-\omega^2}\,(x-x_0)$. Fig.1 shows a corresponding graph for the case $A_0 = \sqrt{3}$, $\omega = \sqrt{2/3}$.

Let us consider the series expansion of the solution a, P^1 in powers of the soliton velocity V about zero:

$$P^1(\xi) = Vb_0(\xi) + V^3b_2(\xi) + O(V^5), \quad a(\xi) = a_0(\xi) + V^2a_2(\xi) + O(V^4). \tag{10}$$

The function $b_0(\xi)$ defined in the first equation in (6) can be singular when $1+\gamma'' = 0$. It is obvious that the electron density $n = 1+\gamma''$ reaches its minimum value when the amplitude $a_0(\xi)$ is maximum. Thus the minimum envelope amplitude A_0 for which the function $b_0(\xi)$ is singular is determined by the condition

$$n|_{\xi=0} \equiv 1 - \frac{4(1-\omega^2)^2}{\omega^4} = 0 \tag{11}$$

and is equal to $\sqrt{3}$.

We see that there is a critical value of the soliton vector potential at which the longitudinal momentum P^1 of the electron fluid becomes singular and the "breaking" of the soliton can take place. One can assume that this process is accompanied by heating and acceleration of some electrons falling at the field maximum.

The possibility of breaking points out an instability of solitons with overcritical amplitudes.

3. To test the solution and to observe possible breaking we carried out numerical simulations using a one-dimensional PIC code. In simulations whose results are presented below, the typical number of particles per cell was 50 and the cell size was $c/(32\pi\omega_{pe})$.

Fig.2 demonstrates the excellent stability of the soliton given by (7), (9). The insignificant damped oscillation in the vicinity of the soliton is a result of the error in the initial electron distribution. This error can be abated by increasing the number of particles per cell and increasing the grid resolution.

In the present calculation the maximum soliton amplitude is $A_0 = 1.5$ and its frequency $\omega = \sqrt{2\sqrt{1+A_0^2}-2}/A_0 \approx 0.8447$. The soliton preserves its shape and amplitude forever

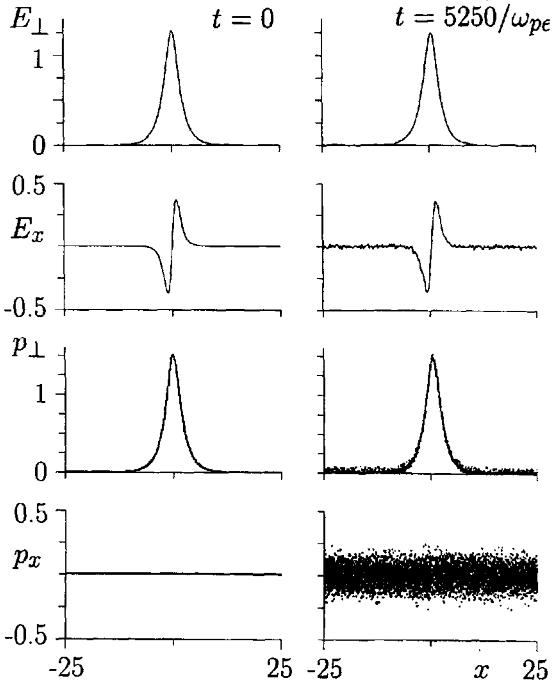

Fig.2. Stability of a soliton of amplitude $A_0 = 1.5$, given by (7), (9). Transverse and longitudinal electric fields are shown as functions of time along with the transverse and longitudinal momentum of the electrons. The steady-state electron temperature is $T_e \sim (2\text{–}4) \cdot 10^{-3}mc^2$

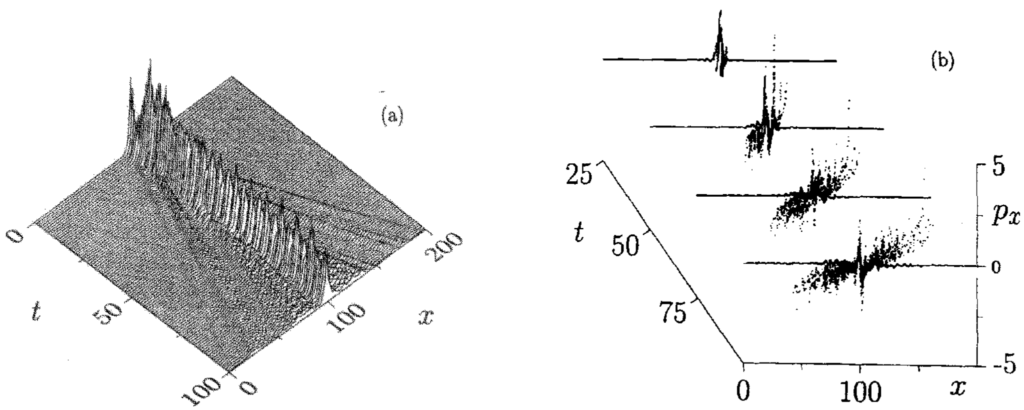

Fig.3. Breaking of a soliton with overcritical amplitude $A_0 = 3$ and acceleration of the electrons. (a) Evolution of the electron transverse momentum averaged over the cell, $p_\perp$. (b) Phase plane (x, p_x) in time

(for longer than the code precision permits following it). The error in the initial electron distribution results in plasma heating to temperatures of the order of $(2\text{–}4) \cdot 10^{-3}m_ec^2$.

In the simulation of a soliton with amplitude $A_0 = 3$, which is greater than the critical value, one can observe heating and acceleration of electrons falling at the maximum field region up to energies $(3-5)m_ec^2$. This is a demonstration of soliton breaking (Fig.3).

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