Pis'ma v ZhETF, vol.68, iss.1, pp.78 - 83

LEVEL STATISTICS INSIDE THE CORE OF A SUPERCONDUCTIVE VORTEX

M.A.Skvortsov, V.E.Kravtsov*+, M.V.Feigel'man**

**L.D.Landau Institute for Theoretical Physics, 117940 Moscow, Russia*

+*International Center for Theoretical Physics, 34100 Trieste, Italy*

Submitted 22 May 1998

Resubmitted 8 June 1998

Microscopic theory of the type of Efetov's supermatrix sigma-model is constructed for the low-lying electron states in a mixed superconductive-normal system with disorder. This technique is used for the study of the localized states in the core of a vortex in a moderately clean superconductor with $\tau^{-1} \gg \omega_0 \sim \Delta^2/E_F$. At low energies $\epsilon \ll \ll \omega_{Th} \sim (\omega_0/\tau)^{1/2}$, the energy level statistics is described by the "zero-dimensional" limit of this supermatrix theory, and the result for the density of states is equivalent to that obtained within Altland-Zirnbauer random matrix model. Nonzero modes of the sigma-model increase the mean interlevel distance by the relative amount $[2\ln(1/\omega_0\tau)]^{-1}$.

PACS: 73.23.-b, 74.40.+k

There is a great deal of activity directed to the study of electron energy levels and wavefunctions in disordered normal metals [1], where they govern low-temperature transport properties. In (s-wave) superconductors, disorder is usually of less importance, since the excitation spectrum has a nonzero gap and single-electron states are almost empty at $T \ll \Delta$. The situation is quite different in mixed superconductive-normal systems (for recent review see [2]) where the gap in the excitation spectrum can be: i) very low compared to the bulk Δ, or ii) just zero. The example of the first case is presented by an S-N-S sandwich with the thickness of the N region $L_N \gg \xi$, l. At generic values of the phase difference φ between superconductors the gap in the electron spectrum in the N region is of the order of the Thouless energy $E_{Th} = D/L_N^2 \ll \Delta$. To calculate the density of states (DoS) fluctuations at $\epsilon > E_{Th}$, and other mesoscopic effects in such systems, a field theory was developed [3] which is an extended version of Efetov's supermatrix σ-model. Qualitatively different is the second case, ii), which is realized, e.g. in the same S-N-S sandwich at $\varphi = \pi$ [4], or in a variety of situations where an external magnetic field is present. Now the DoS is non-zero at arbitrary low energies, and quantum interference due to Andreev scattering strongly affects even the average DoS $\langle\rho(\epsilon)\rangle$. General approach to this kind of systems was initiated by Altland and Zirnbauer (AZ) [5], who employed a generalized random-matrix (RM) approach. The particle-hole symmetry of the Bogolyubov-De Gennes (BdG) Hamiltonian leads to the constraints to be imposed on the RM Hamiltonians. Precise form of the constraint depends on the presence or absence of time inversion and spin rotation symmetries. Thus AZ identified 4 new classes of RM ensembles appropriate for the description of this kind of S-N-S systems. Crossover between such classes has been considered in [6] using the *space-independent* supermatrix sigma-model. While the AZ approach is highly suggestive, it has the same limitation as any *ad hoc* RM theory, i.e. the limits of its applicability to some real physical system are left undetermined.

In the present Letter we develop a microscopic field-theory approach to an example of the ii) type of systems, namely, to the core of a superconductive vortex. It is known since Caroli, De Gennes and Matricon (CdGM) [7] that the BdG equations near the vortex possess localized solutions with energies well below the bulk Δ. The spacing between these localized levels, ω_0, is of the order of $\Delta/(k_F\xi)$ and disappears in the quasiclassical limit $k_F\xi \to \infty$. Thus it was tempting to consider the vortex core as a kind of a "normal tube" inside a superconductor, and in many cases such a simplified picture was found [8] to be at least qualitatively correct. Later it was demonstrated [9] that the presence of a quasi-continuum spectrum branch localized on the vortex follows from general topological arguments. However, it is not always possible to consider the chiral branch as a continuos one. It was shown recently [10, 11], that the discreteness of the localized energy levels becomes of real importance in layered superconductors at sufficiently low temperatures. In the previous paper [10] we employed the AZ approach to find low-current nonlinearities in the current-voltage relation in a mixed state of a moderately clean superconductor (the mean free path $l \gg \xi$, but $l \ll \xi(k_F\xi)$). In such a case the inverse elastic scattering time $1/\tau$ is much larger than interlevel spacing ω_0, therefore the applicability of an appropriate RM model (which is, in fact, class C of the AZ classification) seems natural. Another, qualitatively different, limiting case of a super-clean superconductor with extremely low concentration of impurities ($l \gg k_F\xi^2$) was considered recently by Larkin and co-workers [11]. In the present Letter we again consider a moderately clean limit $\omega_0 \ll 1/\tau \ll \Delta$, now within microscopic approach starting from the BdG equations in the presence of Gaussian random potential. We derive the conditions under which the AZ class C statistics is indeed realized in the vortex core, and estimate the scale of non-universal corrections to it. We consider here purely 2D superconductor, which is a good approximation for the case of a strong layered anisotropy (cf. [10] for more details).

Below we briefly present our method and results (cf. [12] for the details). In the present problem even the calculation of the average single-particle quantities is not trivial and cannot be done within the quasiclassical theory, as long as low energies $\epsilon \sim \omega_0$ are considered. Thus our goal is to derive a field-theory technique for the calculation of the average DoS. To average the Green function over disorder, we use a standard trick [1] of representing it as the functional integral over both Grassmann (χ) and usual complex (S) fields which combine into the superfield Φ. The most direct way would be to work with real-space-dependent superfield $\Phi(\mathbf{r})$; in this way we would obtain a field theory in terms of $Q(\mathbf{r})$ supermatrix. On the other hand, low-lying states of the chiral branch depend upon a single quantum number only, as well as for a generic 1D problem. Therefore, in the basis of such states the BdG Hamiltonian can be represented as a random $N \times N$ Hermitean matrix (where $N \sim \Delta/\omega_0$ is the total number of the localized states in the core) of the certain structure and symmetry which we will discuss below. In the clean limit, $1/\tau \ll \Delta$, the admixture of delocalized ($\epsilon > \Delta$) states to the low-lying ones can be neglected. Thus it is convenient first to reduce the full 2D problem to a sort of RM problem that can be further reduced to the 1D field theory, explicitly containing the chiral spectrum branch only.

In the basis of the CdGM states $\Psi_\mu(\mathbf{r}) = A(J_{\mu-1/2}(k_F r), J_{\mu+1/2}(k_F r))^T e^{i\mu\theta} e^{-K(r)}$ determined in [7] (here $A \sim \sqrt{k_F/\xi}$ is the normalization constant, θ is the azimuthal angle in the real space, $\mu \in [-N/2, N/2]$ is the angular momentum that takes half-integer values, and $K(r) = (1/\hbar v_F)\int_0^r \Delta(r')dr'$, the full Hamiltonian takes the form $\langle\mu|\hat{H}|\mu'\rangle = = \omega_0\mu\delta_{\mu,\mu'} + \langle\mu|\hat{V}|\mu'\rangle$ where the second term is due to the random white-noise impurity potential $U(\mathbf{r})$ with the variance $\langle U(\mathbf{r})U(\mathbf{r}')\rangle = \delta(\mathbf{r}-\mathbf{r}')/(2\pi\nu\tau)$; correspondingly, in the functional integral one should use μ-dependent supervector Φ_μ instead of the superfield $\Phi(\mathbf{r})$. This Hamiltonian obeys the symmetry

$$\hat{H} = -\hat{\gamma}\hat{H}^T\hat{\gamma}^T; \quad \langle\mu|\hat{\gamma}|\mu'\rangle = (-1)^{\mu+1/2}\delta_{\mu+\mu'}, \tag{1}$$

which follows from an identity $\Psi_{-\mu}(\mathbf{r}) = (-1)^{\mu+\frac{1}{2}} i\tau_y\Psi^*_\mu(\mathbf{r})$ that reflects the basic symmetry property of the Hamiltonian; τ_y is the Pauli matrix in the Nambu space.

The standard way to solve a complicated random matrix problem is to represent it in a form of the effective field theory. In order to reduce the RM problem given by Eq. (1) to the 1D field theory we make a continuous Fourier transform (considering N as very large) from the momentum variable μ to the "angle" $\phi \in [0, 2\pi)$, so our superfield will be defined as $\Phi(\phi) = \sum_\mu \Phi_\mu e^{-i(\mu-1/2)\phi} \equiv \Phi_\phi$. Now we can write down an expression for the 'partition function' ($\epsilon_+ \equiv \epsilon + i\delta$):

$$Z^R(\epsilon) = \int \exp i \int \frac{d\phi}{2\pi}\left\{\Phi^*_\phi\left(\epsilon_+ - i\omega_0\frac{\partial}{\partial\phi} - \frac{\omega_0}{2}\right)\Phi_\phi - \int\frac{d\phi'}{2\pi}\Phi^*_\phi V(\phi,\phi')\Phi_{\phi'}\right\}D^2\Phi_\phi \tag{2}$$

Matrix elements $V(\phi,\phi')$ of the random potential in the ϕ-space obey the symmetry relationship that follows from Eq. (1) and is given by $V(\phi,\phi') = -e^{i(\phi-\phi')}V^*(\phi+\pi,\phi'+ +\pi) = A^2\int d^2\mathbf{r}\, w_{\phi\phi'}(r,\theta)U(\mathbf{r})e^{-2K(r)}$. The function $w_{\phi\phi'}$ can be presented, using the Bessel function's summation formulae, as

$$w_{\phi\phi'}(r,\theta) = \left(1 - e^{i(\phi-\phi')}\right)\exp\left\{-ik_F r\left[(\sin\phi - \sin\phi')\cos\theta + (\cos\phi - \cos\phi')\sin\theta\right]\right\}. \tag{3}$$

All features of the theory are encoded in the pair correlator $\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4) = = \langle V(\phi_1,\phi_2)V(\phi_3,\phi_4)\rangle$ where the averaging is performed over the Gaussian distribution of the random potential $U(\mathbf{r})$. Since the typical value of $k_F r \sim k_F\xi \gg 1$, the correlator $\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4)$ is essentially non-zero only when the oscillating exponents in Eq. (3) nearly cancel each other, i.e. when its arguments ϕ_i are pair-wise coinciding [12]:

$$\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4) = \frac{g\omega_0^2}{\pi}T(\phi_{12}+\pi)(2\pi)^2\left[\delta(\phi_{14})\delta(\phi_{23}) - e^{i\phi_{12}}\delta(\phi_{13}+\pi)\delta(\phi_{24}+\pi)\right], \tag{4}$$

where $\phi_{kl} \equiv \phi_k - \phi_l$, $g = 2A^4/\pi\nu\tau\omega_0^2k_F^2 \sim 1/\omega_0\tau \gg 1$ and the kernel T is given by

$$T(\phi) = \frac{\pi}{2}\left|\cot\frac{\phi}{2}\right|, \quad \text{if } |\phi| > \frac{1}{\sqrt{N}}; \qquad T(\phi) \approx \sqrt{N}, \quad \text{if } |\phi| < \frac{1}{\sqrt{N}}. \tag{5}$$

The δ-function approximation (4) for the correlator $\mathcal{W}$ is valid as long as the scale of the angular variations of the field $\Phi(\phi)$ (below it will be seen to be $\ell = [g\ln(N/g)]^{-1}$) is longer than the actual [12] width $w(\phi_{12}) \sim |N\sin(\phi_{12})|^{-1}$ of those δ-functions. Thus the following derivation is strictly valid under the condition $w(\ell) \ll \ell$ which is equivalent to:

$$\tau\sqrt{\omega_0\Delta} \gg \ln\Delta\tau. \tag{6}$$

Below we will assume that the inequality (6) is fulfilled.

The next step of the σ-model derivation is to average the partition function (2) using Eqs. (4), (5). Before doing that we need to take explicitly into account the symmetry (1) which amounts to the doubling of the number of components of the supervector $\Phi(\phi)$. Thus we introduce (cf. with a similar procedure in [6]) an additional 2×2 "particle-hole" (PH) space and define a 4-dimensional superfield $\psi(\phi) = 2^{-1/2}\big(\Phi(\phi), e^{i\phi}\Phi^*(\phi+\pi)\big)^T$. Next we define the bar-conjugated superfield as $\bar{\psi}(\phi) = \psi^+\sigma_z = [C(\phi)\psi(\phi+\pi)]^T$ with $C(\phi) = -e^{-i\phi}\sigma_z C_0$, where σ_z is the Pauli matrix in the PH space, and 4×4 matrix C_0 consists of the blocks $C_0^{pp} = C_0^{hh} = 0$, $C_0^{ph} = 1$, and $C_0^{hp} = k$, where $k = \mathrm{diag}\,(1,-1)$ acting in the Fermi-Bose space. After the averaging over disorder the effective action $\mathcal{A}\{\psi\}$ for the retarded Green function $\mathcal{G}^R(\epsilon) = -i\int \Phi^f(\Phi^f)^+ \exp[\mathcal{A}\{\psi\}]\mathcal{D}\psi^*\mathcal{D}\psi$ (where Φ^f means the fermionic component of Φ) can be written as (we denote $\psi_j \equiv \psi(\phi_j)$):

$$\mathcal{A} = i\int \frac{d\phi_1}{2\pi}\bar{\psi}_1\left(\epsilon_+\sigma_z - i\omega_0\frac{\partial}{\partial\phi_1} - \frac{\omega_0}{2}\right)\psi_1 - \frac{g\omega_0^2}{\pi}\iint \frac{d\phi_1 d\phi_2}{(2\pi)^2} T(\phi_{12}+\pi)\bar{\psi}_1\psi_2\bar{\psi}_2\psi_1. \quad (7)$$

The second term in the action (7) is similar to that of the 1D tight-binding model with off-diagonal random matrix elements with variance decaying as $1/|x|$, as long as we are interested in the scale $|x| \equiv |\phi_1 - \phi_2 + \pi| \ll \pi$. Thus the usual 1D localization is absent in our problem because of the long-range nature of the off-diagonal disorder (cf. [13]).

There is also another way of considering this term, which helps to gain some intuition about its effect. Namely, one can think of the variable ϕ as an angle associated with the 2D quasiparticle momentum $p = k_F\{\cos\phi, \sin\phi\}$. Then the last term in Eq. (7) corresponds to a 2D *particle-hole* scattering strongly enhanced in the forward direction. For such a singular scattering one has to define two scattering lengths ℓ and $\ell_{tr} \gg \ell$ (cf. with a similar situation discussed in [14]): $1/\ell \propto g\int d\phi\ \sigma(\phi) = g\ln(N/g)$, and $1/\ell_{tr} \propto g\int d\phi\ \sigma(\phi)\ (1-\cos\phi) = g \gg 1$, where $\sigma(\phi)$ is the differential cross-section and $\phi = \phi_1 - \phi_2 + \pi$. For the careful evaluation of the logarithmically divergent scattering rate $1/\ell$, one should use the self-consistent Born approximation (SCBA) which takes into account both terms in Eq. (4). It is equivalent to taking into account the "non-crossing" diagrams that can be generated by a perturbative expansion of $\exp[\mathcal{A}\{\psi\}]$ in powers of g. For $\epsilon/\omega_0 \ll 1/\ell$, the "crossing diagrams" of the same order in g turn out to be small by the parameter $\ell/\ell_{tr} = 1/\ln(N/g)$. It stands for the usual quasiclassical parameter $(k_F\ell_{tr})^{1-d}$ in this effectively 1D problem.

The existence of the small parameter $1/\ln(N/g) = 1/\ln\Delta\tau$ that allows to neglect the "crossing diagrams", implies that one can derive an effective field theory (nonlinear sigma-model) which describes the low-energy behavior of the averaged Green function $G^R(\epsilon)$ for $\epsilon/\omega_0 \ll 1/\ell = g\ln(N/g)$. This can be done in a standard way [1] by the Hubbard-Stratonovich decoupling of the quartic term in Eq. (7) and a further saddle point approximation controlled by the parameter $1/\ln(N/g)$. Because of the symmetry relationship (1) and the corresponding relationship between $\bar{\psi}$ and ψ, one has to perform both the local decoupling containing $P(\phi)\,\psi(\phi)\otimes\bar{\psi}(\phi)$ and the non-local one containing $R(\phi_1,\phi_2)\,\psi(\phi_1)\otimes\psi^T(\phi_2)$. Under the condition $\ell \gg 1/\sqrt{N}$ given by Eq. (6), both decouplings are important in order to obtain a correct form for the imaginary part of the Green function in a saddle-point approximation $P(\phi) = P_0$, $R(\phi_1,\phi_2) = R_0(\phi_1-\phi_2)$ which is equivalent to the SCBA:

$$G_\epsilon(\phi) = -\frac{2\pi i}{\omega_0}\sigma_z\theta(-\sigma_z\phi)e^{-|\phi|/\ell}e^{-i\frac{\epsilon}{\omega_0}\phi}, \quad P_0 = \frac{T_0}{\omega_0}\sigma_z, \quad R_0(\phi) = \frac{i}{\pi}T(\phi)G_\epsilon(-\phi), \quad (8)$$

where $T_0 = \int_0^{2\pi} T(\phi)\frac{d\phi}{2\pi} \approx \frac{1}{2}\ln N$. In general, m-th Fourier harmonics of the kernel $T(\phi)$ is given by $T_m \approx \ln\frac{\sqrt{N}}{|m|}$ for $1 \ll m \ll \sqrt{N}$.

Mesoscopic fluctuations are known [1] to be described by the slow rotations of the saddle-point solution, which are represented in our case as $P(\phi) = U^{-1}(\phi)P_0U(\phi)$, $R(\phi,\phi') = U^{-1}(\phi)R_0(\phi-\phi')U(\phi')$. The corresponding action that describes the low-energy spectral properties of the CdGM levels, reads:

$$\mathcal{A}_\sigma[Q,U] = -\frac{\pi g}{4}T_0^2 \iint \frac{d\phi_1 d\phi_2}{(2\pi)^2} T^{-1}(\phi_1-\phi_2+\pi)\,\mathrm{Str}\,Q(\phi_1)Q(\phi_2) - \\ -\frac{\pi i}{2}\int \frac{d\phi}{2\pi}\,\mathrm{Str}\left(\frac{\epsilon}{\omega_0}\sigma_z Q(\phi) - i\sigma_z U(\phi)\frac{\partial U^{-1}(\phi)}{\partial\phi}\right), \qquad (9)$$

where $Q(\phi) = U^{-1}(\phi)\sigma_z U(\phi)$, and $U(\phi)$ is a π-periodic, pseudo-unitary ($U^{-1}(\phi) = \bar{U}(\phi)$) matrix. The action (9) is valid for the energies $\epsilon \ll \omega_0/\ell = \tau^{-1}\ln\Delta\tau$.

The supermatrix Q can be represented in the form $Q(\phi) = \sigma_z[1 + W(\phi) + \frac{1}{2}W^2(\phi) + O(W^3)]$ with the supermatrix W being purely off-diagonal in the PH space. Then the symmetry $Q = \bar{Q}$ and convergence arguments lead to the following form for the W_{ph} and W_{hp} blocks:

$$W_{ph}(\phi) = \begin{pmatrix} iz(\phi) & \alpha_1(\phi) \\ \alpha_1(\phi) & 0 \end{pmatrix}_{fb}, \qquad W_{hp}(\phi) = \begin{pmatrix} iz^*(\phi) & \alpha_2(\phi) \\ -\alpha_2(\phi) & 0 \end{pmatrix}_{fb}. \qquad (10)$$

Here z is a complex number and α_i are Grassmann numbers. Expanding over $W(\phi)$, we obtain in the quadratic approximation

$$\mathcal{A}_2[W_m] = \frac{\pi}{4}\,\mathrm{Str}\sum_m \left\{ 2g\left(\sum_{k=0}^{|m|-1}\frac{1}{2k+1}\right) + i\left(m\sigma_z - \frac{\epsilon}{\omega_0}\right)\right\} W_{2m}W_{-2m}, \qquad (11)$$

where W_m is the m-th harmonics of the field $W(\phi)$. Note that in Eq. (11) only even harmonics enter; odd harmonics, as well as the "longitudinal" modes, have a larger gap of the order of ω_0/ℓ and are excluded from the sigma-model action.

Eq. (11) sets a characteristic scale $L = (g\ln g)^{-1}$ for the angular variations of matrices $U(\phi)$. This scale should be larger than the scattering length ℓ. Only in this case one can neglect higher terms of the gradient expansion in powers of $\partial U/\partial\phi$ as it was done in deriving Eq. (9). Comparing to $\ell = [g\ln(N/g)]^{-1}$ we see that the parameter of the gradient expansion, $\ell/L = \ln g/\ln(N/g)$, is small if the condition (6) is fulfilled. The length L determines the angular size of the elementary propagator corresponding to the sigma-model (9). In this respect it is analogous to the system size in the usual weak-localization problem. The fact that $\ell/L \ll 1$ in our problem tells us that the problem is essentially not ballistic, though it is not diffusive either, since $\ell_{tr}/L = \ln g \gg 1$.

An important property of the action (9) is that it takes a universal form if U is independent of ϕ. At low energies the main contribution comes from the zero harmonics of $Q(\phi)$, i.e. the problem reduces to the 0D σ-model. The uniform supermatrix Q is parametrized by 2 real variables (one of which appears to be cyclic) and 2 Grassmann variables, so the final expression for the average DoS is

$$\langle\rho(\epsilon)\rangle = \frac{1}{4\tilde{\omega}_0}\Re\int_0^\pi d\theta \int d\eta d\zeta \frac{\sin\theta}{1-\cos\theta}\left[(1+\cos\theta) + 2\eta\zeta(1-\cos\theta)\right]e^{\pi i\frac{\epsilon}{\tilde{\omega}_0}(1-\cos\theta)} = \\ = \frac{1}{\tilde{\omega}_0}\left(1 - \frac{\sin(2\pi\epsilon/\tilde{\omega}_0)}{2\pi\epsilon/\tilde{\omega}_0}\right). \qquad (12)$$

The functional form of this result coincides with the result of the AZ phenomenological approach [5]. However, it is expressed via the renormalized mean level spacing $\tilde{\omega}_0 = = \omega_0 \left(1 + \frac{1}{2\ln g}\right)$. The renormalization is due to the contributions of higher $W_{m\geq 2}$ modes which lead [12] to the decrease of the DoS for $\epsilon \leq \omega_0/L = g\omega_0 \ln g$ by the relative amount of $\delta\omega_0/\omega_0 = 1/(2\ln g) \ll 1$. At higher energies this correction decreases as $\delta\omega_0/\omega_0 \propto (g\omega_0 \ln g)^2/\epsilon^2$. This correction can be found using a general approach [15], in which the perturbative treatment of the non-zero modes leads to the "induced" terms in the 0D action. It is given by the *single-cooperon* diagram which is absent in usual normal-metal problems [1],[15]; from the formal point of view, the difference stems from the absence of the BB block in the parametrization (10). A usual [15] *two-cooperon* diagram leads to the "induced" term $\propto (\epsilon/\omega_0)^2(g\ln g)^{-1}$ (cf. [3]). The possibility to neglect this term determines the upper limit of energies where purely 0D description is valid: $\epsilon \leq \omega_{Th} = \omega_0\sqrt{g\ln g}$.

To conclude, we have derived microscopically the supersymmetric field theory for the statistics of the localized electron levels inside the vortex in a moderately clean superconductor. Our supermatrix σ-model, Eq. (9) was *derived* in the main order in the quasiclassical parameter $1/\ln\Delta\tau$. Previously proposed approach [5] is shown to be valid in the low energy range $\epsilon \leq \omega_{Th} = [(\omega_0/\tau)\ln(1/\omega_0\tau)]^{1/2}$ where 0D σ-model is applicable. Mixing between zero- and higher modes leads to the decrease of the DoS by the relative amount of $[2\ln(1/\omega_0\tau)]^{-1}$ at the energies $\epsilon \leq \tau^{-1}\ln(1/\omega_0\tau)$.

Useful discussions with Ya.M.Blanter, K.B.Efetov, V.I.Fal'ko, Yan V.Fyodorov, N.B.Kopnin, A.I.Larkin, V.V.Lebedev, A.D.Mirlin, Yu.V.Nazarov and G.E.Volovik are gratefully acknowledged. The support from the Swiss NSF grant # 7SUP J048531, INTAS-RFBR grant # 95-0302, RFBR grant # 98-02-19252 (M.A.S. and M.V.F.), DGA grant # 94-1189 (M.V.F.), INTAS-RFBR grant # 95-0675, RFBR grant # 96-02-17133 (V.E.K.) and from the U.S. Civilian Research and Development Foundation (CRDF) under Awards # RP1-209 (V.E.K.) and # RP1-273 (M.A.S.) is gratefully acknowledged.

---

1. K.B.Efetov, *Supersymmetry in Disorder and Chaos*, Cambridge Univ. Press, New York, 1997.
2. C.W.J.Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
3. A.Altland, B.Simons, and D.Taras-Semchuk, Pis'ma v ZhETF **67**, 21 (1998).
4. F.Zhou, P.Charlat, B.Spivak, and B.Pannetier, cond-mat/9707056.
5. A. Altland and M.R.Zirnbauer, Phys. Rev. **B55**, 11 42 (1997).
6. K.M.Frahm, P.W.Brouwer, J.A.Melsen, and C.W.J.Beenakker, Phys. Rev. Lett. **76**, 2981 (1996).
7. C.Caroli, P.G.De Gennes, and J.Matricon, Phys. Lett. **9**, 307 (1964).
8. N.B.Kopnin and V.E.Kravtsov, Pis'ma v ZhETF **23**, 631 (1976); ZhETF **71**, 1644 (1976).
9. G.E.Volovik, Pis'ma v ZhETF **57**, 233 (1993); ZhETF **104**, 3070 (1993).
10. M.V.Feigel'man and M.A.Skvortsov, Phys. Rev. Lett. **78**, 2640 (1997).
11. A.I.Larkin and Yu.N.Ovchinnikov, Phys. Rev. **B57**, 5457 (1998); A.I.Larkin and A.A.Koulakov, cond-mat/9802002.
12. M.A.Skvortsov, V.E.Kravtsov, and M.V.Feigel'man, in preparation.
13. A.D.Mirlin, Yan V.Fyodorov, F.-M.Dittes et al., Phys. Rev. **E54**, 3221 (1996).
14. A.G.Aronov, A.D.Mirlin, and P.Wölfle, Phys. Rev. **B49**, 16609 (1994).
15. V.E.Kravtsov and A.D.Mirlin, Pis'ma v ZhETF **60**, 645 (1994).