

## SCALING PROPERTIES OF TRANSVERSE FLOW IN BJORKEN'S SCENARIO FOR HEAVY ION COLLISIONS

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Submitted 3 July 1998

We report a simple analytic solution for the velocity  $u$  of the transverse flow of quark-gluon plasma at a hadronization front in Bjorken's scenario. We establish scaling properties of the transverse flow as a function of the expansion time. We present simple scaling formula for the expansion velocity distribution.

PACS: 24.10.Nz

Landau's hydrodynamic stage [1] is a part of all scenarios for the evolution of the hot and dense matter (quark-gluon plasma – QGP) formed in ultrarelativistic heavy ion collisions [2]. It is now well understood that Landau's complete stopping of Lorentz-contracted colliding nuclei is not feasible because of the Landau – Pomeranchuk – Migdal (LPM) effect, i.e., the finite proper formation time  $\tau_0$  ([3], for the modern modern Quantum Chromodynamics (QCD) approach to the LPM effect see [4], the early works on LPM phenomenology of nuclear collisions are reviewed in [5]), although evaluations of  $\tau_0$  and of the initial energy density  $\epsilon_{max}$  remain controversial [2]. The corollary of the LPM effect in conjunction with the approximate central rapidity plateau is the rapidity-boost invariance of initial conditions. The corresponding solution for a longitudinal expansion in an 1+1-dimensional approximation, neglecting the transverse flow, was found by Bjorken ([6], see also [7]). There is some experimental evidence [8, 9], although a disputed one [10, 11], for a transverse flow which must develop if the lifetime of the hydrodynamical stage is sufficiently long.

In this communication we present a simple solution of the Euler – Landau equation for the velocity of transverse expansion,  $u$ , gained in the hydrodynamic expansion of QGP before the hadronization phase transition. Our solution shows that for the usually considered lifetime  $\tau_B$  of QGP the transverse flow is non-relativistic. It is only marginally sensitive to properties of the hot stage and offers a reliable determination of  $\tau_B$  if the radial profile of the initial energy density is known. We find that the  $u$ -distribution is a scaling function of  $u/u_m$ , where  $u_m$  is a maximal velocity of expansion. We also comment on the impact of the LPM effect on the build-up of the initial energy/matter density.

We start with the familiar Landau relativistic hydrodynamics equations

$$\partial_\mu T_{\mu\nu} = 0, \quad (1)$$

for the energy-momentum tensor  $T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - p\delta_{\mu\nu}$ , where  $\epsilon$  and  $p$  are the energy density and pressure in the comoving frame, and  $u_\mu$  is the 4-velocity of the element of

the fluid [1, 12]. The initial state is formed from subcollisions of constituents (nucleons, constituent quarks and/or partons) of colliding nuclei and is glue dominated at early stages. The LPM effect implies that for a subcollision at the origin,  $x = (t, z, \mathbf{r}) = 0$ , the secondary particle formation vertices lie on a hyperbole of constant proper time  $\tau$ ,  $\tau^2 = t^2 - z^2 \sim \tau_0^2$ , and  $\epsilon, p$  do not depend on the space-time rapidity  $\eta = \frac{1}{2} \ln \left( \frac{t+z}{t-z} \right)$  of the comoving reference frame [6, 7]. In the 1+1-dimensional approximation, this leads to the celebrated Bjorken equation

$$\frac{\partial \epsilon}{\partial \tau} + \frac{\epsilon + p}{\tau} = 0. \quad (2)$$

According to the lattice QCD studies, the familiar [12]  $c_s^2 = \frac{1}{3}$  holds for a velocity of sound  $c_s$  in the QGP excepting a negligible narrow region of the hadronization transition temperature  $T_h \sim 160$  MeV and energy density  $\epsilon_h \sim 1.5$  GeV/fm<sup>3</sup> [13, 14]. With the equation of state  $p = c_s^2 \epsilon$ , the Bjorken equation has a solution

$$\epsilon \propto \tau^{-(1+c_s^2)}. \quad (3)$$

Widely varying estimates for  $\epsilon_{max}$  and the proper time  $\tau_0$  are found in the literature [2, 14–16]. However, as Bjorken has argued [6],  $\epsilon_{max} \propto 1/\tau_0$  and the actual dependence of the Bjorken lifetime  $\tau_B$  on  $\epsilon_{max}$  is rather weak:

$$\tau_B = \tau_0 \left[ \left( \frac{T_{max}}{T_h} \right)^{4/(1+c_s^2)} - 1 \right] = \frac{\tau_0 \epsilon_{max}}{\epsilon_h} \left[ \left( \frac{\epsilon_{max}}{\epsilon_h} \right)^{c_s^2/(1+c_s^2)} - \frac{\epsilon_h}{\epsilon_{max}} \right]. \quad (4)$$

For central Pb+Pb collisions, for which there is some experimental evidence for the QGP formation [2], the typical estimates are  $\tau_B \approx 3$  f/c at SPS [14, 17] and  $\tau_B \approx 6$  f/c at RHIC [2], which are much larger than the standard estimate  $\tau_0 \sim 0.5$  f/c.

We first comment briefly on the two sources of a substantial dilution of the initial energy density: the LPM effect and the finite Lorentz-contracted thickness of colliding nuclei. Because of the LPM effect, the density of produced matter vanishes at  $\tau = 0$  and builds up on the time scale  $\tau \sim \tau_0$ . The build-up of energy density  $E(\tau)$  from a superposition of elementary subcollisions in an absence of expansion can be modeled by the source term  $f(\tau) = dE(\tau)/d\tau$ , which must be put in the r.h.s. of the Bjorken equation (2), which then has a solution

$$\epsilon(\tau) = \tau^{-(1+c_s^2)} \int_0^\tau dt t^{(1+c_s^2)} \frac{dE}{dt}. \quad (5)$$

For the plausible parameterization  $E(\tau) = E_0[1 - \exp(-\tau/\tau_0)]$  such an interplay of the formation and expansion of the hadronic matter dilutes the energy density substantially at short times  $\tau \sim \tau_0$ . Furthermore, at finite energies the Lorentz-contracted thickness of colliding nuclei  $d = 4R_A(m_p/W)$  can not be neglected and leads to a further dilution of the attainable energy density:

$$\langle \epsilon(\tau) \rangle = \int_0^d dt w(t) \epsilon(\tau - t), \quad (6)$$

where the temporal distribution of subcollision vertices for central collisions can be estimated as  $w(t) \approx (1/d) (1 - |2t - d|/d) \theta(d - t) \theta(t)$ . The result of temporal smearing (6)

only depends on  $\gamma = d/\tau_0$ . For instance, in central lead-on-lead fixed target collision at SPS energy  $E/A = 160$  GeV we have  $d \approx 1.3$  fm, which is not small compared to the standard estimate  $\tau_0 = 0.5$  fm. The both dilution (5) and smearing (6) are short time effects and have only a marginal impact on the dynamics of expansion at  $\tau \gtrsim \max\{\tau_0, d\}$  and are negligible at RHIC energies. Their impact at energy densities close to the threshold for QGP formation can be substantial, though, which may be the case for the SPS.

Now we turn to the major theme of collective transverse expansion, which is driven by radial gradient of pressure. As we shall see, the radial flow is nonrelativistic. Then, to the first order in radial velocity  $u_r$ , the radial projection of (1) gives the Euler – Landau equation

$$(\epsilon + p) \frac{\partial u_r}{\partial \tau} + u_r \left( \frac{\partial(\epsilon + p)}{\partial \tau} + \frac{(\epsilon + p)}{\tau} \right) + \frac{\partial p}{\partial r} = 0, \quad (7)$$

in which we can use the Bjorken's solution for  $\epsilon$  and  $p = c_s^2 \epsilon$ . Then the Euler – Landau equation can be cast in a simple form

$$\frac{\partial u_r}{\partial \tau} - \frac{c_s^2}{\tau} u_r = - \frac{c_s^2}{(1 + c_s^2)} \frac{\partial \log p}{\partial r}. \quad (8)$$

The important point is that the transverse expansion of the QGP fireball can be neglected which we can justify *a posteriori*. For this reason the time dependence of the logarithmic derivative  $D(r, \tau) = \partial \log p / \partial r$  can be neglected, it is completely determined by the initial density profile and depends neither on the temperature nor fugacities of quarks and gluons, which substantially reduces the model-dependence of the transverse velocity. Then the solution of (7) subject to the boundary condition  $u_r(\tau_0) = 0$  is

$$u_r(r, \tau) = \frac{c_s^2 \tau c_s^2}{1 - c_s^4} \int_{\tau_0}^{\tau} dt t^{-c_s^2} D(r, t) \approx \frac{c_s^2 \tau D(r, 0)}{1 - c_s^4} \left[ 1 - \left( \frac{\tau_0}{\tau} \right)^{1 - c_s^2} \right]. \quad (9)$$

The model-independent estimates for the initial density/pressure profile are as yet lacking. At RHIC and higher energies of the initial state is expected to be formed by semihard parton-parton interactions for which nuclear shadowing effects can be neglected [2, 15]. Then for central collisions  $\epsilon(r, \tau_0) \propto T_A^k(r)$ , where  $k = 2$  and  $T_A(r) = \int dz n_A(\sqrt{z^2 + r^2})$  is the density of constituents,  $T_A(r) \sim \exp(-r^2/R_A^2)$ , where  $R_A \approx 1.1A^{-1/3}$  fm is the nuclear radius. In another extreme scenario of strong shadowing and of strong LPM effect the soft particle production is not proportional to the multiplicity of collisions of fast partons [4, 5] and  $k = 1$  is more appropriate. Hereafter we take  $k = 2$ . In any case, the logarithmic pressure gradient is approximately linear,  $D(r, t) \approx 2kr/R_A^2$ , and according to the solution (9) the displacement of the fluid element  $\Delta r$  is proportional to the radius,  $\Delta r \propto r\tau^2$ . Consequently, we have the Hubble-type radial rescaling

$$\lambda(\tau) \approx 1 + \frac{\Delta r}{r} \approx 1 + \left( \frac{\tau}{\tau_T} \right)^2, \quad (10)$$

where

$$\tau_T \approx R_A / \sqrt{k} c_s. \quad (11)$$

has a meaning of the *lifetime against transverse expansion*. For central Pb+Pb collisions eq. (11) gives  $\tau_T \approx 10k^{-0.5} f/c$  which is larger than the above cited estimates of  $\tau_B$  and at SPS and RHIC energies the transverse expansion of the fireball can be neglected.

In a quasi-uniform plasma the hydrodynamic expansion lasts until  $\epsilon = \epsilon_h$ . For long-lived QGP and  $\tau \gg \tau_0$  we can use the Bjorken's solution

$$\epsilon(r, \tau) = \epsilon_h \left( \frac{T_A(r)}{T_A(0)} \right)^k \left( \frac{\tau_B}{\tau} \right)^{1+c_s^2}, \quad (12)$$

which gives the position of the hadronization front

$$r_h(\tau) = R_A \sqrt{\frac{1+c_s^2}{k} \ln \frac{\tau_B}{\tau}} \quad (13)$$

and the radial velocity at the hadronization front

$$u(\tau) = u_r(r_h(\tau), \tau) = \frac{c_s^2 \tau}{R_A(1-c_s^4)} \sqrt{4k(1+c_s^2) \ln \frac{\tau_B}{\tau}} \left[ 1 - \left( \frac{\tau_0}{\tau} \right)^{1-c_s^2} \right]. \quad (14)$$

For the usually discussed  $\tau_B$  and  $\tau_0$  we have  $\tau_B \gg \tau_0$ . For such a long-lived QGP,  $\tau_B \gg \tau_0$ , the radial velocity takes the maximal value  $u_m$  at  $t \approx 1/\sqrt{e}$ ,

$$u_m = \frac{c_s^2 \tau_B}{R_A(1-c_s^4)} \sqrt{\frac{2k(1+c_s^2)}{e}} \left[ 1 - \left( \frac{\tau_0 \sqrt{e}}{\tau_B} \right)^{1-c_s^2} \right]. \quad (15)$$

It is remarkable that the average radial acceleration  $u_m/\tau_B$  is approximately constant. The solid line in fig. 1 shows the maximal velocity  $u_m$  evaluated from (14). The large- $\tau_B$  approximation (15), shown by the dashed line, reproduces these results to better than  $\sim 4\%$  at  $\tau_B = 3$  f/c and better than  $\sim 1\%$  at  $\tau_B = 8$  f/c. For the above cited estimates for  $\tau_B$  in central Pb+Pb collisions we find  $u_m(\text{SPS}) \approx 0.13$  and  $u_m(\text{RHIC}) \approx 0.28$ , consequently the nonrelativistic expansion approximation is justified very well. Now notice, that for a long-lived QGP the hadronization front (13), shown in fig. 2, and  $u(\tau)/u_m$  depend only on the scaling variable  $t = \tau/\tau_B$ , with an obvious exception of the short-time region  $\tau \sim \tau_0$ . This scaling property is clearly seen in fig. 3 where we show the  $u/u_m$  as a function of  $t = \tau/\tau_B$ . Notice a convergence to a universal curve with the increasing  $\tau_B$ .

The most interesting quantity is a radial velocity distribution which can be evaluated experimentally from the Doppler modifications of the thermal spectrum. In order to test our results one needs particles which are radiated from the hadronization front. The standard scenario is that the hadronization transition is followed by an expanding mixed phase which, however, does not contribute to the transverse velocity because in the mixed phase  $c_s^2 = 0$ . The mixed phase is followed by a hydrodynamic expansion and post-acceleration of strongly interacting pions and baryons until the hadronic freeze-out temperature  $T_f < T_h$  is reached [18]. This post-acceleration is negligible for weakly interacting  $K^+$  and  $\phi$ -mesons, which gives the desired access to the radial flow at the hadronization transition. In the evaluations of the modification of the thermal spectrum and one needs to know the  $u$ -distribution weighted with the particle multiplicity. For the of constant hadronization temperature contribution of the hadronization surface  $r_h(\tau)$  to the particle multiplicity is

$$dw \propto r_h(\tau) d\tau. \quad (16)$$

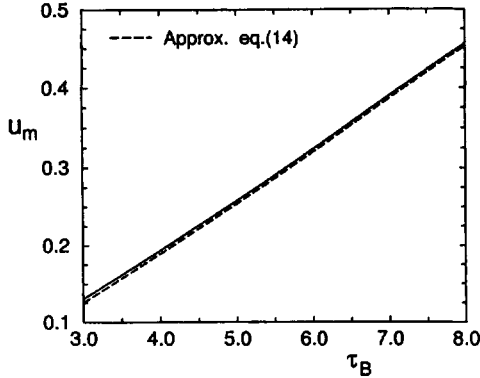


Fig.1. The maximal velocity of radial expansion  $u_m$  for central Pb+Pb collisions as a function of the expansion time  $\tau_B$ . Shown by the dotted line is the large- $\tau_B$  formula (17).

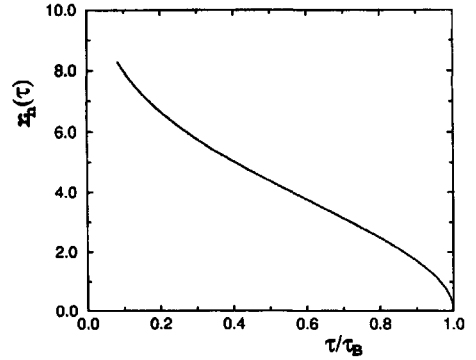


Fig.2. The time dependence of the hadronization front for central Pb+Pb collisions

Making use of the solutions (13) and (14), it can readily be transformed into  $dw/du \propto \tau_h (du/d\tau)^{-1}$ . The important point is that because of the above discussed scaling properties of the transverse flow, the velocity distribution is a scaling function of  $x = u/u_m$ :

$$\frac{dw}{du} = \frac{1}{u_m} \frac{f(x, \tau_B)}{\sqrt{1-x}}, \quad (17)$$

where for a long-lived QGP  $f(x, \tau_B)$  does not depend on  $\tau_B$ . The square-root singularity at  $x = 1$  is a trivial consequence of the vanishing derivative  $du(\tau)/d\tau$  at  $\tau \approx \tau_B/\sqrt{e}$ . In Fig. 4 we show the scaling function  $f(x, \tau_B)$  for  $\tau_B = 6$  f/c. We don't show  $f(x, \tau_B)$  for other values of  $\tau_B$ , because the variations from  $\tau_B = 6$  f/c to 3 f/c to 9 f/c do not exceed several per cent and are confined to a narrow region of  $x \lesssim 0.2$ . The approximation  $f(x) = 0.5$  is good for all the practical purposes.

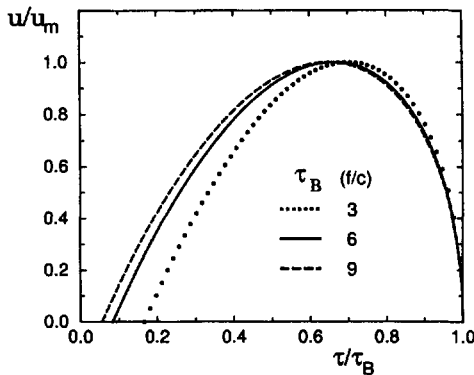


Fig.3. The convergence to the scaling behaviour of the time dependence of the relative radial velocity  $u/u_m$  for central Pb+Pb collisions

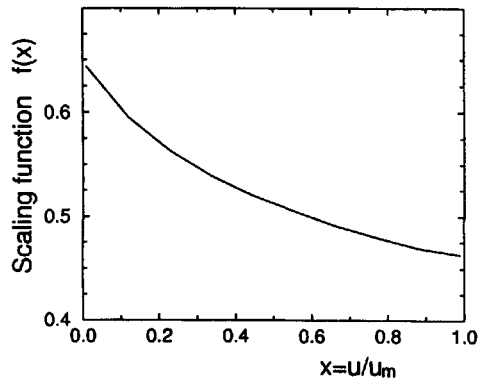


Fig.4. The scaling function  $f(x, \tau_B)$  of. Eq. (17) is shown for  $\tau_B = 6$

In conclusion, we would like to argue that the shape of the velocity distribution is to a large extent the model independent one. The generic origin of the square-root singularity

at  $x = 1$  has already been emphasized, the fact that  $f(0) \neq 0$  is due to a radiation from the surface  $r_h \sim R_A$  at early stages of expansion. Above we assumed that hydrodynamic expansion continues until the hadronization transition. Following Pomeranchuk [19] one can argue that in the non-uniform plasma the hydrodynamic expansion stops when the mean free path

$$l_{int} = 1/n(r_c, \tau)\sigma_t \quad (18)$$

defined in terms of the transport cross section  $\sigma_t$ , is larger than the GQP density variation length  $D(\tau, 0)^{-1}$ . In the partially equilibrated QGP  $l_{int} \propto T^{-1}$ . Then the Pomeranchuk condition gives the temperature  $T_P$  at which the hydrodynamic expansion stops,  $T_P \propto r/R_A^2$ . The possibility remains open that at early stages  $T_P > T_h$ , in which case  $dw \propto r_h(\tau)(T_P/T_h)^3 d\tau$ . This enhanced radiation at early stages at slow radial expansion but at higher temperatures  $T_P$  would mimic radiation at a lower temperature and higher radial velocity. This may result in the apparent depletion of  $f(0)$ ; in order to explore this possibility one needs better understanding of  $l_{int}$  near the hadronization transition.

The NA49 fits to the proton, kaon and pion transverse mass  $m_T$  distribution in central Pb+Pb collisions at SPS assume identical freeze-out temperature for all particle species [8]. For positive particles NA49 finds  $T_f = 140 \pm 7$  MeV and the transverse velocity  $\langle u \rangle = 0.41 \pm 0.11$ . However, for the  $K^+$  one must take the higher freeze-out temperature  $T_f = T_h \approx 160$  MeV given by the lattice QCD. Because of the anti-correlation between the local temperature  $T_f$  and  $\langle u_T \rangle$ , see Fig. 7 in [9], such a fit with larger  $T_f$  to the same  $m_T$  distribution shall yield smaller  $\langle u \rangle$ .

This work was partly supported by the INTAS grant 96-597 and the Grant 94-02-05203 from the Russian Fund for Fundamental Research.

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