Pis'ma v ZhETF, vol.68, iss.5, pp.415 - 419

LACK OF UNIVERSAL ONE-PARAMETER SCALING IN THE TWO-DIMENSIONAL METALLIC REGIME

V.M.Pudalov[+*1)], *G.Brunthaler*[□], *A.Prinz*[□], *G.Bauer*[□]

[+] *P.N.Lebedev Research Centre in Physics*
117924 Moscow, Russia

[*] *High Pressure Physics Institute*
142092 Troitsk, Moscow reg., Russia

[□] *Institut für Halbleiterphysik, Johannes Kepler Universität,*
A-4040 Linz, Austria

Submitted 28 July 1998

We studied the two-dimensional metallic state in a number of Si-MOS structures with peak mobilities varying by a factor of 8.5. The data reveal density and disorder dependences of the major features of the scaling function, and thus lack of the universal one-parameter scaling over wide density range in the metallic regime.

PACS: 71.30.+h, 72.15.Rn

The unexpected appearance of a metal-insulator transition (MIT) in two-dimensional (2D) Si-MOS structures [1] and its confirmation in other works on different samples [2] and materials [3–6] have generated much excitement because the general believe has been so far that all states in 2D are localized. A one-parameter scaling theory (OPST) [7] developed in 1979 for non-interacting particles, predicted that no true metallic behavior is possible at $T = 0$ in 2D system due to quantum interference. On the other hand, the theory developed by Finkel'stein [8] for strong Coulomb interaction, predicted the occurrence of a metallic state in 2D, due to divergence of an interaction parameter in the renormalization procedure as $T \to 0$. The new experimental findings have initiated a debate over the nature of the 2D metallic state [9–11].

Recently, a phenomenologic modification of the OPST has been suggested [12], to treat the case of an interacting system, where the scaling function in 2D systems, $\beta = d\ln G/d\ln L$, changes sign at the critical value of conductivity, G_c. In view of testing the validity of the OPST, we summarize here a number of generic features of the MIT in 2D as experimentally observed in different materials and samples:

(i) The resistivity develops exponentially at temperatures below $T_0 \sim 0.3E_F/k$, with E_F the Fermi energy [1, 10]:

$$\rho = \begin{cases} \rho_0 + \rho_1 \exp(-(T_0/T)^p) & \text{at } n > n_c, \\ \rho_1^* \exp((T_0/T)^p) & \text{at } n < n_c, \end{cases} \tag{1}$$

where n_c is the critical density, the parameter T_0 is sample and density dependent and $p \approx 1$. The empirical Eq. (1) was recently supported theoretically [14, 13].

(ii) For lower temperatures, when $\rho_1 \exp(-T_0/T) \ll \rho_0$, the resistivity crosses over to a logarithmic temperature dependence [14, 13], $C\ln T$. The crossover point corresponds to the point of maxima, $G_m = 1/\rho_0$, of the scaling function [13] $\beta(G)$.

[1)] e-mail: pudal@east.ru

(iii) In the range of the exponential behavior, according to Eq. (1), the resistivity can be scaled by T/T_0 into a metallic and an insulating branches. The single scaling parameter, $T_0 \propto |n - n_c|^q$, demonstrates a critical behavior [1–3, 6, 10] around n_c, whereas the exponent q has the meaning of the product, $q = z\nu \sim 1$, of the dynamical exponent, z, and the correlation length exponent [15, 12], ν.

(iv) The metallic state is destroyed by magnetic fields applied parallel to the 2D plane [16, 2] which points to the importance of spin effects.

In this paper we present experimental data which clearly demonstrate the lack of universality in the modified one-parameter scaling description of the conductivity of the 2D metal. In order to test the role of disorder and interaction, we have performed systematic measurements on a number of Si-MOS samples. Our measurements show that no common scaling function exists through the following facts: (i) the critical conductivity values G_c for various samples differ from each other and show no tendency to converge towards a limiting value; (ii) the "crossover" value of the conductivity, $G_m = \rho_0^{-1}$, is not universal; and (iii) the separatrix, which divides the metallic from the insulating phase in the $\rho - T$-plane (Fig. 1a) is "tilted", i.e. temperature (length scale) dependent.

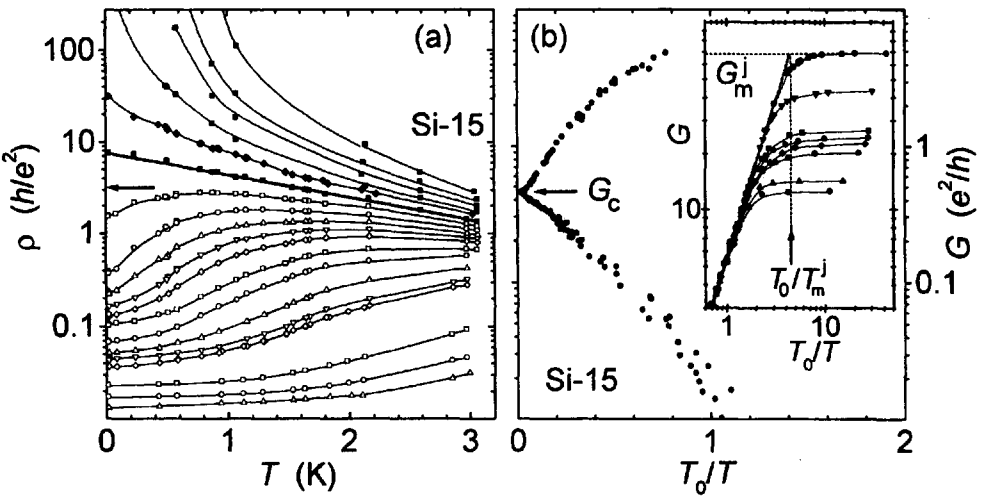

Fig.1. a) $\rho(T)$ for the sample Si-15 over the range 0.016 to 3 K. Different symbols correspond to n from 0.449 to 0.989 (in steps of 0.054), and further 1.1, 1.2, 1.42, 1.64, 1.74, 2.82, 3.9, $4.98 \cdot 10^{11}\,\mathrm{cm}^{-2}$. b) G vs $1/T$ scaled with a single parameter (T_0/T). The inset in b) blows up the metallic part of the scaling plot in the range of low T. Carrier densities (from top to bottom) are 4.96, 3.1, 1.84, 1.74, 1.63, 1.52, 1.31, and $1.2 \cdot 10^{11}$ cm^{-2}. Horizontal arrows show the critical values, ρ_c and G_c

We have performed studies in the metallic regime, i.e. for high conductivities $G > > G_c \sim 1$, where G is given in units of e^2/h. We made measurements on a number of Si-MOS samples with peak mobilities[2] varying by a factor of 8.5: $\mu = 41,000\,\mathrm{cm}^2/\mathrm{Vs}$ (Si-15a), $\mu = 40,000$ (Si-5), $\mu = 39,000$, $\mu = 36,200$ (Si-62), $\mu = 29,000$ (Si-22), $\mu = 24,200$ (Si-2), $\mu = 19,600$ (Si-43), $\mu = 9,300$ (NS-2), and $\mu = 4,800\,\mathrm{cm}^2/\mathrm{Vs}$ (Si-39). The data were taken by a 4-terminal AC-technique in the temperature range from 0.29 K to 4 K (for all samples), and 0.016 K to 15 K (for a few samples). The resistivity for all samples exhibited the characteristic temperature dependence, similar to that shown in Fig. 1, with

[2] We characterize the samples with peak mobility taken at $T = 20$ to 300 mK, because in the high μ samples the resistivity drops exponentially at $T < 2$ K.

a critical density n_c, which separates the metallic ($d\rho/dT > 0$, $n > n_c$) from the insulating ($d\rho/dT < 0$, $n < n_c$) region, in the limit $T \to 0$.

For samples with lower mobility, the magnitude of the drop, ρ_1, is less, whereas the transition shifts to higher densities. In Si-15a the drop $\rho_1/\rho_0 \approx 6$, in Si-62 $\rho_1/\rho_0 \approx$ ≈ 4.5, whereas for Si-39 the decrease in $\rho(T)$ is within a few percent. By scaling the measured conductivities to the metallic and insulting branches, we examine below the major features of the scaling function, namely: the critical G_c – value (at which $\beta(G_c) =$ $= 0$), the slope $z\nu$ of its steep part in the vicinity of G_c, and the conductivity G_m where $\beta(G_m)$ reaches maximum [13]. As an example, the rescaled conductivities for Si-15a are shown in Fig. 1b on a T_0/T- scale. The scaling procedure is straight forward for moderate mobility samples with a temperature independent ("horizontal") $\rho(n_c)$ line and with well-pronounced exponential dependence Eq. (1), such as shown in Fig.2a for the sample Si-62. For the highest mobility sample Si-15a, where $\rho(n_c)$ is strongly temperature dependent ("tilted"), we limited the scaling analysis to $T \leq 0.1E_F/k_B$, in order to decrease the influence of "tilting".

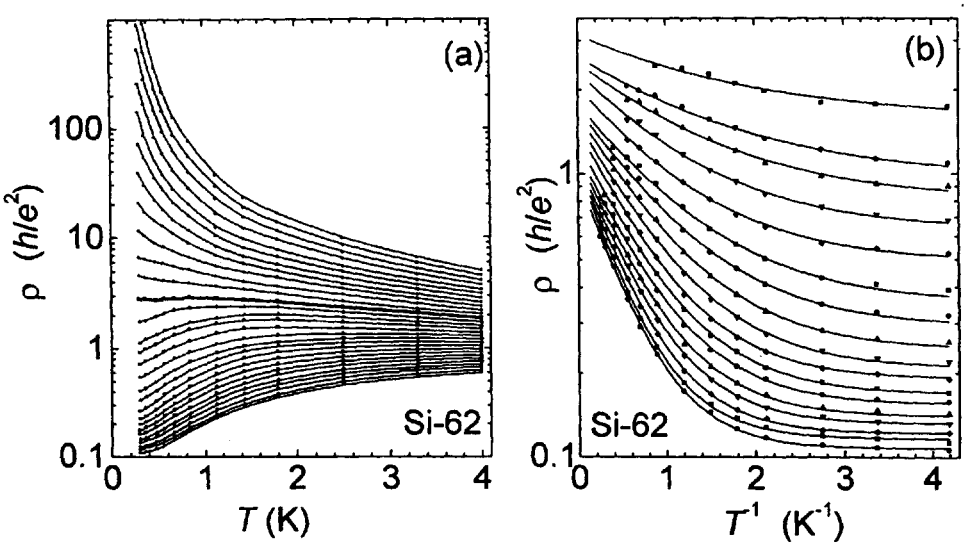

Fig.2. a) $\rho(T)$ for the sample Si-62 ($n_c = 0.956$); b) fit to Eq. (1) with 3 parameters, ρ_0, ρ_1 and T_0 for each density in the metallic range. $n_j = 0.978$ to 1.326 in steps of 0.0218 (from top to bottom) and in unites of 10^{11}cm^{-2}

In Fig.3a, the G_c- and $z\nu$-values for different samples are summarized and plotted as a function of n_c. The latter is roughly proportional to the concentration of scattering centers at the interface [17], and thus may serve as a measure of the disorder. For the most disordered sample Si-39 the value of $G_c \approx 4$ is not well defined due to the weak change of $\rho(T)$ in the metallic state and is thus not shown in Fig. 3. The values of G_c show a systematic dependence as a function of disorder (i.e. n_c) and do not exhibit a trend to saturate at a universal value.

Even in one and the same sample there is clear evidence for the lack of the universal G_c-value. Fig. 2b shows fitting of the $\rho(T)$ data for the sample Si-62 using Eq. (1) with ρ_0, ρ_1, and T_0, the fitting parameters and with $p = 1$. The critical conductivity values $G_c = 1/\rho_1$ found from this fit for each density, are shown in Fig. 3b. Only in a narrow

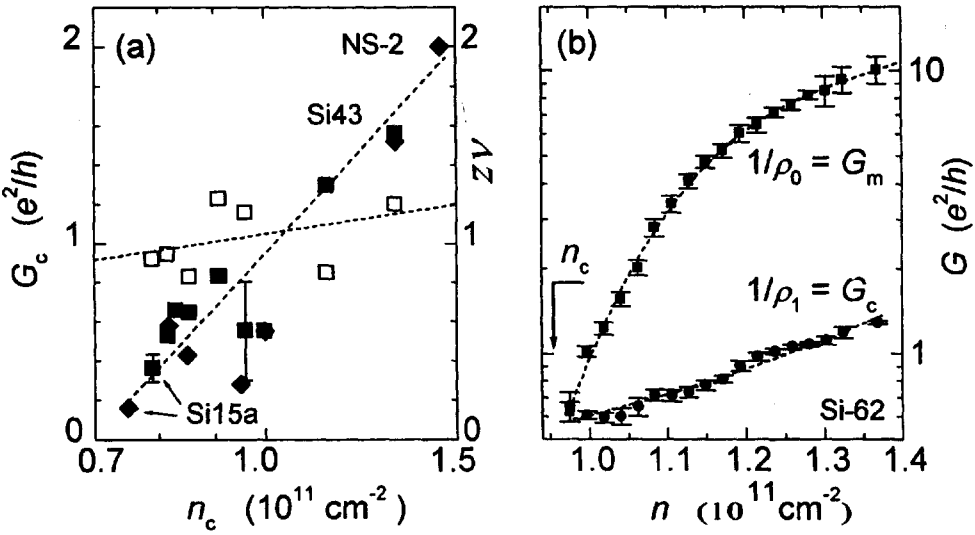

Fig.3. a) Critical conductivity G_c (closed symbols) and the critical index $z\nu$ (open symbols) vs critical density n_c for different samples. Squares are obtained from temperature scaling, diamonds, for comparison, show the onset of the insulating state from non-Ohmic transport measurements [17]. b) G_c vs density, obtained from fitting to Eq. (1). The dashed curves are guides to the eye

range of densities $n/n_c \leq 1.1$, the G_c may be considered as a constant, and further it raises with density.

Lack of the one-parameter scaling can also be seen at higher values of the conductivity (shown in the inset of Fig. 1b), where the individual curves for different electron densities n_j deviate clearly from the common scaling branch. This saturation was found not to be caused by electron overheating [18] and is intrinsic to all studied samples. Different conductivity curves "saturate" at different values $G_m^j = 1/\rho_0(n_j)$. The similar result is also demonstrated in Fig. 3b, where the fitting parameter $1/\rho_0$ is plotted as a function of the density. It does not exhibit a tendency to converge to a universal value.

The absence of a universal scaling behavior also follows from the "tilted" separatrix $\rho(T)$ (the straight bold line in Fig. 1a). The separatrix corresponds to the density at which the curvature, $d^2\rho/dT^2$ vanishes. This separatrix line is almost horizontal for moderate mobility samples (Si-43, Si-2, Si-22), and coincides with n_c in agreement with earlier reports [1], but is tilted for high mobility (less disordered) samples as shown in Fig. 1a. Due to the tilting of $\rho_c(T)$, different G_c values may be obtained by performing the scaling analysis in different temperature ranges. The error bar in Fig. 2 for Si-15a incorporates the results obtained from the scaling over the range $T < 0.1E_F$ and $T < 0.15E_F$ (where $E_F \approx 5.8\,\mathrm{K}$ at $n = n_c$).

The onset of the metallic state occurs, roughly, when the influence of the disordering (localizing) potential is compensated by a corresponding interaction energy, $E_{int}\tau \sim 1$. Fig.3a demonstrates that *higher disorder corresponds to higher n_c and G_c values.* Since the E_{ee}/E_F ratio decreases with density as $n^{-1/2}$, this result indicates that another mechanism, whose ***strength increases with density***, exists apart from the pure Coulomb interaction. The correlation between n_c, G_c and disorder holds also for the p–GaAs/AlGaAs system, as follows from comparison of the data in Refs. [5, 6]. The sample mobility in

Ref. [6] is about 25 times higher, nevertheless, the transition is much weaker and its features are more similar to those displayed by our most disordered sample Si-39 where the transition occurs at seven times higher density. This again confirms our conclusion on the involvement of another mechanism apart from the purely Coulomb interaction. We presume, this interaction is caused by spin- or exchange effects. Fig.3a shows also the exponents $z\nu$ for different samples. Although these data are scattered and less disorder dependent than G_c, there is a trend for $z\nu$ to increase from $z\nu \approx 0.9 \pm 0.1$ for the least disordered sample, to $z\nu \approx 1.2$ for the sample Si-43.

In summary, as we demonstrated above, no universality either in G_c, in $z\nu$, or in G_m can be found for the Si-MOS samples and hence, no universal one-parameter β function exists. However, due to the rather weak disorder dependence of $z\nu$, the slope of the logarithmic derivative $d\ln G/d\ln T$ does not vary much. This explains why the individual conductivity curves for each sample may be scaled in a limited temperature range, $E_F \gg T > T_0/\ln(\rho_1/\rho_0)$ (where the term ρ_0 in Eq. (1) can be neglected), and in a limited density range in the vicinity of n_c. The conductivity, thus may be described in the framework of a particular scaling function [12] $\beta^j = d\ln G/d\ln L$, which should change sign at a certain critical $G_c^j(n_j)$ -value, with β^j being, however, *sample and density dependent* [13].

V.P. acknowledges discussions with M.Baranov, V.Kravtsov, M.Skvortsov, I.Suslov. The work was supported by RFBR (97-02-17387), by the Programs on "Physics of solid-state nanostructures" and "Statistical physics", by INTAS (96-0580, 96-0250), by NWO, and by FWF Vienna, ÖNB (6333) and GME Austria.

---

1. S.V.Kravchenko, G.V.Kravchenko, J.E.Furneaux et al., Phys. Rev. **B50**, 8039 (1994). S.V.Kravchenko, W.E.Mason, G.E.Bowker et al., Phys. Rev. **B51**, 7038 (1995).
2. D.Popović, A.B.Fowler, and S.Washburn, Phys. Rev. Lett. **79**, 1543 (1997); K.P.Li, D.Popović, and S.Washburn, in Proc.: EP2DS-12, Tokyo (1997).
3. P.T.Coleridge, R.L.Williams, Y.Feng, and P.Zawadzki, Phys. Rev. **B56**, R12764 (1997). M.D'Iorio, D.Brown, and H.Lafontain, Cond-mat/9708201.
4. K.Ismail, J.O.Chu, D.Popović et al., Cond-mat/9707061.
5. Y.Hanein, U.Meirav, D.Shahar et al., Phys. Rev. Lett. **80**, 1288 (1998).
6. M.Y.Simmons, A.R.Hamilton, M.Pepper et al., Phys. Rev. Lett. **80**, 1292 (1998).
7. E.Abrahams, P.W.Anderson, D.C.Licciardello, and T.V.Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
8. A.M.Finkel'stein, Z.Phys. **B56**, 189 (1984). Sov. Sci. Reviews/section A- Physics Reviews, Ed. I.M.Khalatnikov, **14**, 3 (1990).
9. P.Phillips and Y.Wan, Cond-mat/9704200. P.Phillips, Y.Wan., I.Martin et al., Cond-mat/9709168. D.Belitz and T.R.Kirkpatrick, Cond-mat/9705023. M.A.Skvortsov, Cond-mat/9712135v2. Y.Lyanda-Geller, Phys. Rev. Lett. **80**, 4273 (1998). S.Chakravarty, S.Kivelson, C.Nayak, and K.Völker, Cond-mat/9805383. S.Cakravarty, L.Yin, and E.Abrahams, Cond-mat/9712217.
10. V.M.Pudalov, Cond-mat/9707076v2; JETP Lett. **66**, 170 (1997).
11. Song He and X.C.Xie, Cond-mat/9711195.
12. V.Dobrosavljević, E.Abrahams, E.Miranda, and S.Chakravarty, Phys. Rev. Lett. **79**, 455 (1997).
13. V.M.Pudalov, G.Brunthaler, A.Prinz, and G.Bauer, in: Proc. of the 10^{th} Winter School "Frontiers in Condensed Matter Physics", Mauterndorf, 1998; Physica E 2 Oct. (1998), in press.
14. V.M.Pudalov, G.Brunthaler, A.Prinz, and G.Bauer, Cond-mat/9801077.
15. S.V.Kravchenko, D.Simonian, M.P.Sarachik et al., Phys. Rev. Lett. **77**, 4938 (1996).
16. V.M.Pudalov, G.Brunthaler, A.Prinz, and G.Bauer, JETP Lett. **65** 932 (1997). D.Simonian, S.V.Kravchenko, M.P.Sarachik, and V.M.Pudalov, Phys. Rev. Lett. **79**, 2304 (1997).
17. V.M.Pudalov, M.D'Iorio, J.W.Campbell, and S.V.Kravchenko, Phys. Rev. Lett. **70**, 1866 (1993).
18. R.Fletcher, V.M.Pudalov, Y.Feng et al., Phys. Rev. **B56**, 12422 (1997). M.D'Iorio, V.M.Pudalov, and S.G.Semenchinsky, Phys. Rev. **B46**, 15992 (1992).