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VORTEX STRUCTURE IN THE PRESENCE OF TILTED COLUMNAR DEFECTS

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It is argued that in layered superconductors, vortices will be trapped by tilted columnar defects even when external magnetic field is oriented along c-axis. For such tilted trapped vortices, the interaction at long distance becomes attractive in some directions. This must result in the formation of vortex chains with an intervortex distance of the order of London penetration depth.

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The defects produced by ion irradiation are the columnar damage tracks with thickness of the order of $2R \sim 50 - 100$ Å. These columnar defects (CD) affect dramatically the vortex pinning and can increase the critical current many times [1, 2]. The investigation of the properties of superconductors with CD attracted much attention recently. Now in high-T_c superconducting oxides, it is possible to introduce the tilted columnar defects by controlling the orientation of a heavy-ion beam (see for example [3] and references cited their).

In the present paper, we consider theoretically the vortices in the layered superconductors with parallel CD slightly tilted in respect to c-axis while the external field is being parallel to the c-axis. Taking in mind a very high degree of anisotropy of high-T_c superconductors (like Bi-2212 or Tl-2201 for example) we may neglect Josephson interaction between layers and consider only electromagnetic one. The vortex line in strongly anisotropic quasi-2d superconductor is in fact a chain of pancakes vortices [4, 5]. For a vortex line parallel to c-axis the situation is the same as in the standard case, and vortex energy per unit length is given by the classic formula $E_v = (\phi_0/4\pi\lambda)^2 (\ln(\lambda/\xi) + 0.12)$[6], where for a layered superconductor $\lambda^{-2} = d_0/\lambda_{\parallel}^2 d$, with d – the distance between superconducting layers, d_0 – its thickness and $\lambda_{\parallel}$ – an in-plane London penetration depth of a single layer. When a vortex is on the CD (which direction coincides with c-axis) the only difference in the vortex energy calculation is the cutoff at the CD radius R rather than ξ (see for example [7, 8]), i.e. vortex energy being $E_v^{CD} = (\phi_0/4\pi\lambda)^2 \ln(\lambda/R)$ and the pinning energy (per unit length) may be estimated as $E_{pin} = (\phi_0/4\pi\lambda)^2 \ln(\lambda/\xi)$.

Namely this large pinning energy (comparable with the vortex energy itself) may stabilize the orientation of vortex along tilted CD and not along the vertical magnetic field. To find out at what angles θ between CD axis l and c-axis a vortex prefers to be oriented along CD, we need to calculate the energy of a tilted vortex in the framework of electromagnetic coupling model. The general expression for the energy of an arbitrary configuration of pancakes vortices has been derived in [4]:

$$E = \frac{1}{8\pi\lambda_{eff}} \sum_n \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left\{ |\mathbf{\Phi}_n(\mathbf{k})|^2 - \mathbf{\Phi}_n(-\mathbf{k}) \times \right.$$

$$\left. \times \sum_m \mathbf{\Phi}_m(\mathbf{k}) \frac{\mathrm{sh}(kd)}{2\lambda_{eff}k} \frac{\left(G_k - \sqrt{G_k^2-1}\right)^{|n-m|}}{\sqrt{G_k^2-1}} \right\}, \qquad (1)$$

where $\lambda_{eff} = \lambda_{\|}^2/d_0$, $\mathbf{\Phi}_n(\mathbf{k})$ – is a Fourier transform of the total London vector of n-th layer $\mathbf{\Phi}_n(\rho) = \sum_\alpha \mathbf{\Phi}_L(\rho - \mathbf{R}_{n,\alpha})$ and summation is performed over all pancakes α in the n-th layer, $\mathbf{\Phi}_L(\mathbf{k}) = i\phi_0 [\mathbf{k}, \mathbf{z}]/k^2$, the function $G_k = ch(kd) + \mathrm{sh}(kd)/2\lambda_{eff}k$.

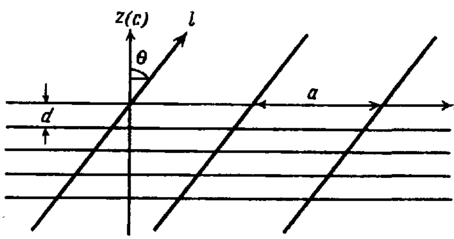

Fig.1

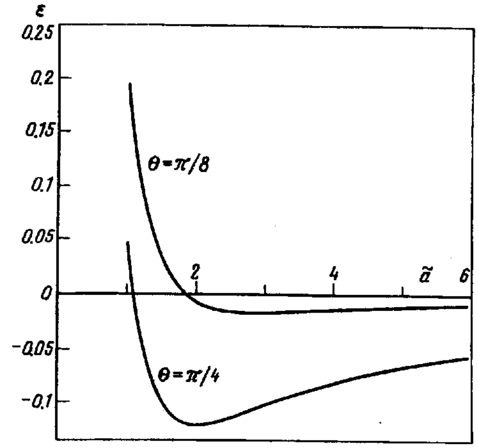

Fig.2

Let us choose x-axis in the plane defined by $\mathbf{z}$(c-axis) and CD axis $\mathbf{l}$ – see Fig.1, then the vector $\mathbf{\Phi}_n(\mathbf{k})$ is $\mathbf{\Phi}_n(\mathbf{k}) = \exp\left[-ind(\mathbf{k}\cdot\hat{\mathbf{x}})\tan\theta\right]\mathbf{\Phi}_L(\mathbf{k})$. Performing summation in the formula (1) over m, we obtain the energy of the vortex segment between two adjacent layers:

$$E_s(\theta) = \frac{d}{8\pi\lambda^2} \int \frac{d^2\mathbf{k}}{(2\pi)^2} |\mathbf{\Phi}_L(\mathbf{k})|^2 \left(1 - \frac{\mathrm{sh}(kd)}{2\lambda_{eff}k} \frac{1}{G_k - \cos((\mathbf{k}\cdot\hat{\mathbf{x}})d\tan\theta)}\right). \qquad (2)$$

As the main contribution to the integral (2) comes from the region of small wavevectors $k \ll d^{-1}$, we may use the expansion of G_k for small k and finally obtain a very convenient expression for the energy difference for tilted and perpendicular vortices:

$$E_s(\theta) - E_s(0) = \frac{d\phi_0^2}{8\pi\lambda^2} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left(\frac{1}{(1/\lambda^2)(k^2/(k^2 + k_x^2\mathrm{tg}^2\theta)) + k^2} - \frac{1}{1/\lambda^2 + k^2} \right) =$$

$$= d\left(\frac{\phi_0}{4\pi\lambda}\right)^2 \ln\left(\frac{1+\cos\theta}{2\cos\theta}\right). \qquad (3)$$

Naturally the energy of the tilted vortex is larger than the perpendicular one, and it is only the additional pinning energy (when vortex axis coincides with the CD axis) which could stabilize it. For the tilted CD, superconductivity is destroyed in the elliptic region on **a-b** plane with semi-axis R and $R/\cos\theta$, and correspondingly the energy of a perpendicular vortex with such an elliptic core is [8]: $E_s^{el} = d\,(\phi_0/4\pi\lambda)^2 \ln(2\lambda\cos\theta/R(1+\cos\theta))$. Taking this into account we see that the decrease of the energy due to a larger core surface compensates its increase related with tilting, and finally the total energy of the vortex on the tilted columnar defect is:

$$E_s^{CD}(\theta) = d\left(\frac{\phi_0}{4\pi\lambda}\right)^2 \ln(\frac{\lambda}{R}) \tag{4}$$

and does not depend on θ. Comparing this with the energy of a perpendicular vortex (which has no gain due to the pinning along the whole length of CD) we conclude that a vortex will prefer to be always oriented along CD. In fact, for large values of θ, the CD may not be elliptic in $(\mathbf{a}, \mathbf{b})$ plane, and the above mentioned compensation will disappear. However in practice, for $\theta < 60°$, the vortices would penetrate along CD when the magnetic field is parallel to the **c**-axis.

The condition that the single vortex penetrates into a sample is $E_s^{CD} - \phi_0 H d/4\pi = 0$, this gives us the first critical field

$$H_{c1} = \frac{\phi_0^2}{4\pi\lambda^2} \ln\left(\frac{\lambda}{R}\right) \tag{5}$$

which is smaller than the critical field for a vortex oriented along the **c**-axis $H_{c1}^0 = (\phi_0^2/4\pi\lambda^2)\ln(\lambda/\xi)$.

The previous analysis is certainly applicable for the well separated vortices when it is possible to neglect an intervortex interaction. This case corresponds to the field close to H_{c1} and a very small concentration of CD. On the other hand, the situation when the interaction between tilted vortices comes into a play is very special and may qualitatively change the process of vortex penetration. In fact, in the framework of an anisotropic London model, an attraction between the tilted vortices appears in (**c**, **l**) plane [9 – 11]. Such attraction decreases exponentially at long distances and leads to the formation of vortex chains [9], which has been observed subsequently on the experiment (see for example [12] and references cited there). An anisotropic London model is not appropriate for extremely quasi-2D compounds like Bi-2212, then the question of the tilted vortex interaction must be treated in the framework of quasi-2D model with an electromagnetic interaction.

With the help of a general expression (1) one may demonstrate that as in the case of an anisotropic London model [9], the attraction between two tilted vortices is maximal in (**c**, **l**) plane, and at long distance the interaction energy varies as

$$E_{int} \sim -\sin^2(\theta)/D^2,$$

where D is the distance between the vortex lines. Such a long range attraction is quite different from an exponentially decreasing attraction in London model [9 – 11]. If the sample contains many CD available for vortex occupation, then vortices will occupy CD forming a chain in (**c**, **l**) plane. The first critical field will correspond to the appearance of the vortex chain, not a single vortex.

To calculate the characteristics of such vortex chain we may write with the help of (1) – (3) the energy of vortex in chain as:

$$E_s^{chain}(\theta) = \frac{d}{8\pi\lambda^2} \int \frac{dk_y}{(2\pi)} \frac{1}{a} \sum_{k_s=2\pi n/a} \left\{ |\Phi_L(\mathbf{k})|^2 \left(1 - \frac{1}{1+\lambda^2 k^2 + \lambda^2 k_x^2 \tan^2\theta}\right)\right\}, \quad (6)$$

where a is the distance between the adjacent vortices in plane (xy) - see Fig.1.

To avoid all the complications related with a cut-off, it is convenient to calculate the difference of the energy of the vortex in chain and a solitary one:

$$E_s^{chain}(\theta) - E_s^{CD}(\theta) = \frac{d\phi_0^2}{8\pi\lambda^2} \left\{ \frac{1}{a} \sum_{k_s=2\pi n/a} \int \frac{dq}{(2\pi)} K(q,Q) - \int \frac{dqdQ}{(2\pi)^2} K(q,Q) \right\}, \quad (7)$$

where $Q = k_x$, $q = k_y$ and the function

$$K(q,Q) = \left(1 - \frac{1}{1+\lambda^2(Q^2+q^2) + \lambda^2 Q^2 \tan^2\theta}\right) \frac{1}{(Q^2+q^2)}. \quad (8)$$

Performing summation and integration over Q in (7) we finally obtain

$$E_s^{chain}(\theta) - E_s^{CD}(\theta) = \frac{d\phi_0^2}{8\pi\lambda^2} \int_0^\infty \frac{du}{(2\pi)} \left\{ \frac{1-\coth(u\,\tilde{a})}{1-u^2\tan^2\theta} u \tan^2\theta + \right.$$

$$\left. + \frac{1}{\cos\theta\sqrt{1+u^2}} \frac{\coth(\tilde{a}\cos\theta\sqrt{1+u^2}) - 1}{1-u^2\tan^2\theta} \right\}, \quad (9)$$

where $\tilde{a} = a/(2\lambda)$ and for $a >> \lambda$

$$E_{int}(\theta) = E_s^{chain}(\theta) - E_s^{CD}(\theta) = -\frac{d\phi_0^2}{4\pi^2 a^2} \tan^2\theta \int_0^\infty [\coth(u) - 1]\, u du, \quad (10)$$

i.e. at long distance there is always an attraction between the vortices located on the tilted CD.

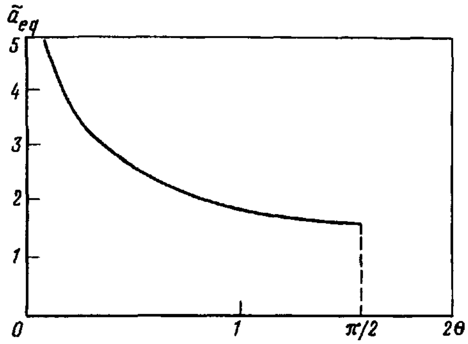

Fig.3

The interaction energy as a function of distance $\tilde{a}$ is presented in Fig.2 for several tilting angles θ: at short distance, vortices repeal each other while at long distance, the interaction becomes attractive, and the minimum of E_{int} corresponds to the equilibrium distance between the vortices in chain. This equilibrium distance $\tilde{a}_{eq}$ as a function of angle θ is presented in Fig.3, and we see that it is rather large $a_{eq} \sim (3-8)\lambda$ (note that at angles θ very close to $\pi/2$ even the weak Josephson interlayer interaction is needed to be taken into account, and our model is no longer valid). For typical concentration of CD (fluence $B_{\Phi} \sim (1-3)$T), the mean distance between them is much smaller than a_{eq} , and at low field vortices could freely choose CD to feel in, forming thus the vortex chains.

In the conclusion, we stress that in the presence of tilted CD, at low magnetic field oriented along c-axis, vortices prefer to be trapped by CD provoking thus magnetization perpendicular to the external field, i.e. torque. The first critical field will correspond to the appearance of a vortex chain and not a single vortex. The vortex lattice at low field will represent the well separating vortex chains, and the increase of field will decrease the distance between chains leaving an inter-vortex distance in chain basically the same one and equal to a_{eq}. It might be interesting to perform magnetization and/or torque measurements, as well as decoration experiments on the samples of high-T_c superconductors with tilted CD.

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