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ENERGY DEPENDENCE OF RATIOS OF MULTIPLICITIES AND THEIR SLOPES FOR GLUON AND QUARK JETS

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The difference between the ratio of multiplicities and the ratio of their derivatives on energy for gluon and quark jets is calculated up to next-to-next-to leading order of perturbative QCD. Its non-zero value is uniquely defined by the running property of the QCD coupling constant. It is shown that this difference is rather small compared to values which can be obtained from experimental data. This disagreement can be ascribed either to strong non-perturbative terms or to experimental problems with a scale choice, jets separation and inadequate assignement of soft particles to jets.

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The ratio of multiplicities in gluon and quark jets has been of much debate during last years. The lowest order prediction of the perturbative QCD drastically overvalues experimental results for this ratio. Therefore, the higher-order corrections up to the next-to-next-to leading terms of the perturbative QCD have been calculated [1, 2], and the ratio has been written as

$$\frac{\langle n_G \rangle}{\langle n_F \rangle} = r(y) = \frac{C_A}{C_F}(1 - r_1\gamma_0(y) - r_2\gamma_0^2(y)) + O(\gamma_0^3), \tag{1}$$

where the scale of the process is defined by $y = \ln Q/Q_0$, Q is a virtuality of a jet, Q_0=const,

$$\gamma_0 = \left(\frac{2C_A\alpha_S}{\pi}\right)^{1/2}, \tag{2}$$

α_S is the running coupling constant, $C_A = 3$, $C_F = 4/3$ are Casimir operators, and

$$r_1 = 2\left[h_1 + \frac{n_f}{12N_c}\left(1 - \frac{2C_F}{N_c}\right)\right] - \frac{3}{4}, \tag{3}$$

$$r_2 = \frac{r_1}{6}\left(\frac{25}{8} - \frac{3}{4}\frac{n_f}{N_c} - \frac{C_F}{N_c}\frac{n_f}{N_c}\right) + \frac{7}{8} - h_2 - \frac{C_F}{N_c}h_3 + \frac{n_f}{12N_c}\frac{C_F}{N_c}h_4, \tag{4}$$

$$h_1 = \frac{11}{24},\ h_2 = \frac{67 - 6\pi^2}{36},\ h_3 = \frac{4\pi^2 - 15}{24},\ h_4 = \frac{13}{3}. \tag{5}$$

Here, n_f is the number of active flavours, N_c=3 the number of colours. Let us note that only the first term of r_2 was obtained in [1] by Feynman graphs technique. Other terms come from higher derivatives of the generating functions [2] when the solution of the QCD equations for generating functions is found out by Taylor series expansion. They take into account the energy conservation in three-parton vertices.

Asymptotically, the ratio $r(y)$ (1) tends to a constant C_A/C_F=9/4 which shows that the gluon jet bremsstrahlung is stronger than for quark jets. However, the correction terms are still noticeable at presently accessible energies.

The energy dependence of average multiplicities of gluon and quark jets used to be expressed in terms of the anomalous dimension $\gamma(y)$ as

$$\langle n_G(y)\rangle = Ke^{\int^y \gamma(y')dy'}, \; \langle n_F(y)\rangle = \langle n_G(y)\rangle / r(y). \tag{6}$$

Then it is easy to get the ratio of their derivatives as

$$\frac{\langle n_G(y)\rangle'}{\langle n_F(y)\rangle'} = r(y)\left(1 + \frac{r'(y)}{\gamma(y)r(y) - r'(y)}\right), \tag{7}$$

or for its difference with the ratio of multiplicities one gets

$$D(y) = \frac{\langle n_G(y)\rangle'}{\langle n_F(y)\rangle'} - \frac{\langle n_G(y)\rangle}{\langle n_F(y)\rangle} = \frac{rr'}{\gamma r - r'}. \tag{8}$$

The anomalous dimension in terms of the running coupling constant looks like

$$\gamma = \gamma_0(1 - a_1\gamma_0) + O(\gamma_0^3). \tag{9}$$

Here

$$a_1 = h_1 + \frac{n_f}{12N_c}\left(1 - \frac{2C_F}{N_c}\right) - \frac{B}{2}, \quad B = \frac{11N_c - 2n_f}{24N_c}. \tag{10}$$

The higher order terms have been omitted in our treatment. Taking into account that

$$\gamma_0' = -h_1\gamma_0^3 + O(\gamma_0^5), \tag{11}$$

one estimates that the second term in the denominator of (8) can be neglected and gets finally

$$D(y) \approx \frac{r'(y)}{\gamma(y)} \approx \frac{N_c}{C_F} r_1 h_1 \gamma_0^2 (1 + a_1\gamma_0)\left(1 + \frac{2r_2\gamma_0}{r_1}\right). \tag{12}$$

The formulas (8) and (12) are the main result of the paper.

It is important to stress that the difference $D(y)$ between the ratios of derivatives and multiplicities directly demonstrates the running property of the QCD coupling constant. It is identically equal to zero for fixed coupling because $r' = 0$ in that case. For the running coupling, this difference is always positive and proportional to the value of the coupling constant. Consequently, it tends to zero in asymptotics.

The corrections due to the anomalous dimension (9) and due to the multiplicity ratio (1) have been specially left as separate factors in brackets in (12) to show their relative importance. One easily notices that the last correction in $D(y)$ is stronger than in the ratio (1) itself because r' in the numerators of (8) and (12) acquires the factors n when differentiating the subsequent terms of γ_0^n in (1).

Asymptotically the terms in brackets tend to 1 since $\gamma_0 \to 0$, but at present energies ($\gamma_0 \approx 0.45 - 0.5$ at Z^0) the corrections are rather large. In the energy range near Z^0, the factor in front of the brackets in (12) is about 0.05–0.06. The anomalous dimension correction in the first bracket enlarges the value of D by about 15%. More important is the expression in the second bracket. With values of r_1 and r_2 given by (3), (4) and inserted in (12), one estimates it as about 3.2, i.e. the linear in γ_0 term (NNLO correction) contributes more than 1 (MLLA term), and therefore next terms should be evaluated. Anyway taking this expression for granted, one would estimate D as

$$D \approx 0.16 - 0.20. \tag{13}$$

Thus the predicted value of D is comparatively small.

Let us note that the first term from (4) only as calculated first in [1] contributes to the second bracket the value about 0.25, and the corresponding contribution to D is equal approximately 0.06 - 0.08.

We have considered the one-scale problem without imposing any limitations on the transverse momentum and considering the jet evolution with energy in the total phase space. In experiment, the more severe restrictions are often introduced due to specifics of installations or due to some special criteria. In particular, one can consider jet evolution as a function of some internal scale (e.g., similar to the jet transverse momentum) at a given energy (usually chosen at Z^0 peak because of larger statistics). Such a procedure has been exploited in Refs [3, 4]. It was claimed that the proportionality of the two scales favours comparison of theoretical predictions with experimental data on the evolution of above ratios. The scale chosen in [4] was

$$\kappa = E_{jet} \sin\frac{\theta}{2}, \tag{14}$$

where E_{jet} is the energy of the jet and θ the angle with respect to the closest jet. In [3], the similar scale of the geometric mean of such scales of the gluon jet with respect to both quark jets in three-jet events has been used. From the data presented in [4] one can estimate that the difference $D(\kappa)$ ranges approximately from 0.95 at κ= 7 GeV to 0.7 at κ=28 GeV. These values are much larger than predicted above.

However, both theoretical and experimental approaches to the problem should be further analysed. The large correction in Eq. (12) implies that next terms of the perturbative expansion can be essential in the ratio of derivatives. They are enlarged in (12) as discussed above and support earlier conclusions (see, e.g., Ref. [5]) that subleading effects are quantitatively important at experimentally accessible energies. Our experience tells us also that power corrections due to conservation laws related to recoil effects can become essential as well [6].

Experimental procedures raise even more questions. There exists the scale dependence in the final result if one compares the data of [3] with that of [4]. The problem of the scale choice has recently been discussed in Ref. [7]. Besides, it has been shown [7] that different algorithms of the jet selection and ascribing soft particles to a jet give rise to somewhat different conclusions concerning jet multiplicities and their slopes. Further study is necessary.

Another problem raised by the procedure used in Ref. [4] is the role of non-perturbative effects in average multiplicities of gluon and quark jets. It has been claimed [4] that the hadronization of quark jets leads to some constant (an assumed ansatz!) excess over gluon jets equal to about two or three particles. It compensates somewhat the perturbative QCD excess of the gluon jet multiplicities. This constant term in energy dependence of multiplicities has been ascribed to a possible non-perturbative contribution. Unfortunately, this statement can hardly be confronted to any reliable theoretical treatment. These terms do not contribute to derivatives of the multiplicities but drastically diminish the multiplicity ratio thus increasing their difference $D(y)$.

At the same time, the theoretical estimate (13) seems quite reasonable qualitatively if one accepts the values of the ratio of derivatives as about 2.1 given in [4] and the value of the ratio of multiplicities about 1.84 predicted theoretically [5] with the experimental result ~ 1.6 claimed in [8].

In conclusion, we have evaluated the difference between the ratio of slopes of average multiplicities and the ratio of multiplicities in gluon and quark jets in the next-to-next-to leading approximation of the perturbative QCD and confronted it to some experimental data.

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