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OVERLAPPING IDENTICAL RESONANCES AND DOUBLE RADIATIVE INTERFERENCE EFFECTS IN RECOMBINATION OF HEAVY MULTICHARGED IONS

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The purely QED effect of double radiative interference in the recombination of electrons with heavy multicharged ions is discussed. Numerical calculations of the corresponding cross sections in the vicinities of $KL_{12}M_{12}$ and $KM_{12}M_{12}$ dielectronic recombination resonances of heliumlike uranium have been performed. The possibility of near-future experimental observation of the effect with the Super-EBIT facility is suggested.

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Overlapping resonances have been thoroughly investigated in nuclear and particle physics. The most interesting case is the overlap of identical resonances, that is, resonances with identical quantum numbers, when the interference terms survive not only in the differential but also in the total cross section of processes after integration over angles. As a consequence, these terms lead to some special interference effects, for example quantum beats, which are well known in neutral K-mesons [1] and the ^{8}Be nucleus [2, 3].

In atomic physics, a similar situation was investigated theoretically [4] and observed experimentally [5] in the decay of coherently excited $2s$ and $2p$ states of the hydrogen atom in an external electric field. The electric field mixes even- and odd-parity states so that the resulting combinations have identical quantum numbers. Though the excited levels do not overlap in this case due to the repulsion in the electric field, they are close enough to be excited coherently and to give the interference effect. Overlap of resonances may arise, in principle, if an external magnetic field is also added [6].

Unlike the case of neutral atoms, where the radiative overlap of identical resonances is very rare, it can easily take place in the spectra of highly charged heavy ions, in particular in heliumlike uranium. The magnitude of these effects can be qualitatively estimated by the magnitude of the radiative broadening compared to the energy interval between the levels of a multiplet with identical parity and total angular momentum quantum numbers. If there are no special exclusions, the interference effect turns out to be of order $(\alpha Z)^3$ [7], that is, about Z times as large as the effect of nonresonant levels on the line shape [8]. From this estimate, one can see that the overlap in the spectra of multicharged heavy ions arises because the radiative shifts and widths become comparable with the interelectron interaction corrections at very high Z values. It means also that this phenomenon can

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only be described within QED theory, where all the radiative corrections are treated in a consistent manner. We use here the same technique as in Refs. [7, 9–15] based on the S-matrix or on the Green's function approaches.

The theory of radiative decay of overlapping identical levels in multicharged ions has been developed in Refs. [9–12]. The process of recombination of an electron with a hydrogenlike heavy ion provides one possible practical way of preparing the situation under investigation [14–16]. The total cross section of the recombination process generally includes resonant dielectronic-recombination (DR) and nonresonant radiative-recombination (RR) cross sections, and terms which describe the interference between DR and RR channels. Interference terms due to radiative overlap of identical DR resonances will also be present but their corresponding effects usually turn out to be masked by DR-RR interference [14–16]. The latter has been investigated in the vicinity of the KLL resonances theoretically [14–20] and recently observed experimentally [21].

It should be noted that in contrast with direct cross-section measurements the technique of recording the photon energy and electron beam energy for every observed event developed in Refs. [21, 22] allows for separation of different x-ray transitions from the dielectronic capture resonances. This way of doing the experiment provides a good means for observing the double radiative interference effects first discussed in Ref. [7]. By these effects we mean the radiative interference in the recombination process on groups of mutually overlapping identical, for example doubly excited, levels. Such situations can give the largest radiative interference effect, since not only initial but also final recombined states overlap in this case. Also it seems to be possible to observe the pure radiative interference effects because, in contrast with them, the DR-RR interference terms in the cross section are suppressed by at least a factor of $1/Z$. In lowest-order perturbation theory, the amplitudes of excitation of the group of double-excited levels in the process of direct radiative capture of an electron by a hydrogenlike ion in the ground state with emission of only one photon vanish due to the orthogonality of the wave functions. These amplitudes only contribute to the cross section if one takes into account higher-order graphs, i.e. either in at least two-photon processes or after improvement of the operator for the emission of a photon by the interelectron interaction corrections of the order of $1/Z$. The RR process manifests itself only as a background in this case. The characteristic x-rays emitted by RR can be used to calibrate the total cross sections [21, 22]. In experiment [21] the combined spectra of a mixture of highly charged uranium ions have been observed. However, the resonances of the He-like ion are shifted in energy compared to those from other recombined ions. The situation discussed in this Letter is quite general and should also apply to other few-electron multicharged heavy ions.

We shall consider the DR process of an electron with a hydrogenlike multicharged ion $A^{(Z-1)+}$ in its ground state, which may be schematically represented as

$$A^{(Z-1)+}(1s_{1/2}) + e^-(\varepsilon) \to A^{(Z-2)+}(d)^{**} \to A^{(Z-2)+}(s)^{**} + \gamma(\omega) \to \ldots,$$

where e^- denotes the incident electron with energy ε and γ is the emitted photon with frequency ω. The labels d and s merely serve to identify two-electron states and do not here refer to any particular value of orbital angular momentum. The d and s states are assumed to be groups of the doubly-excited mutually overlapping levels with identical quantum numbers. Then the resonance condition is $\varepsilon + E_{1s_{1/2}} \simeq E_d$ where $E_{1s_{1/2}}$ and E_d are the energies of the hydrogen- and heliumlike ions, respectively. In addition, only those photons with frequency in the region of $\omega \simeq E_d - E_s$ are supposed to be measured in all directions of emission.

The amplitude of the DR process in the case of radiative channels of decay was obtained in Ref. [7] within the resonance approximation. Then the expression for the cross section can be written as follows (in relativistic units)

$$\sigma_{\rm DR}(\varepsilon) = \frac{\pi^2}{2p^2} \sum_{j,l} \sum_{J,M} \sum_{s} \sum_{d,d'} \frac{W_{dd',s} \langle d_L | \hat{I} | i \rangle \langle d'_L | \hat{I} | i \rangle^*}{(\varepsilon + E_{1s_{1/2}} - \mathcal{E}_d)(\varepsilon + E_{1s_{1/2}} - \mathcal{E}^*_{d'})}. \tag{1}$$

Here $p^2 = \varepsilon^2 - m^2$, and the initial state $|i\rangle$ of the system (one-electron ion in its ground $1s_{1/2}$ state plus continuum electron) depends on the set of quantum numbers $|1s_{1/2}\varepsilon l j JM\rangle$, where J is the total angular momentum of the system, M is the projection of J, and l,j are the orbital and total angular momenta of the incoming electron. The continuum wave functions are normalized to δ-functions in the energy ε. The matrix elements of the operator

$$\hat{I}(r_{12}; E_d^{(0)}) = \alpha \frac{(1 - \alpha_1 \cdot \alpha_2)}{2r_{12}} \{\exp(i|\varepsilon_{d_1} - \varepsilon_{1s_{1/2}}|r_{12}) + \exp(i|\varepsilon_{d_2} - \varepsilon_{1s_{1/2}}|r_{12})\} \tag{2}$$

describe the excitation process of the states d by radiationless capture, taking into account the retardation effect. In Eq. (2), $r_{12} = |\mathbf{x}_1 - \mathbf{x}_2|$, α_i are the Dirac matrices and the fine-structure constant $\alpha = e^2$. The zero-approximation energy $E_d^{(0)}$ of the two-electron states d is defined by the sum of the corresponding one-electron Sommerfeld energies ε_{d_i}, $(i = 1, 2)$. Note that the one-electron energy $E_{1s_{1/2}}$ includes the Lamb shift corrections while $\varepsilon_{1s_{1/2}}$ does not. By $\mathcal{E}_d = E_d - i\Gamma_d/2$ we denote the complex eigenvalues of the non-Hermitian operator $\hat{\mathcal{H}} = E_d^{(0)} \cdot \hat{1} + V_d(E_d^{(0)})$ acting in the corresponding subspace of the unperturbed d states [13–15]. The quasipotential V_d, defined in the lowest approximation, contains contributions due to all $1/Z$-order corrections. The right $|d_R\rangle$ and left $\langle d_L|$ eigenvectors of $\hat{\mathcal{H}}$ [13–15] (see also Ref. [9] where only right vectors have been employed) are normalized by the condition $\langle d_L | d'_R\rangle = \delta_{dd'}$. For states with different quantum numbers there is no difference between the left $|d_L\rangle$ and right $|d_R\rangle$ vectors. The nondiagonal partial widths $W_{dd',s}$ for radiative transitions between d and s states are defined by the multipolar expansion of the expression

$$W_{dd',s} = 2\pi\omega^2 \sum_{\mathbf{e}} \int d\Omega \langle s_R | \hat{R}_\gamma | d_R \rangle \langle s_L | \hat{R}_\gamma | d'_R \rangle^*, \tag{3}$$

where $d\Omega$ means integration over the directions of the photon emission, and the operator for the emission of a photon with polarization e and momentum $\mathbf{k}$ is given by

$$\hat{R}_\gamma = e \sum_{n=1}^{2} \frac{(\alpha \cdot \mathbf{e}^*)}{2\pi\sqrt{\omega}} e^{-i\mathbf{k}\cdot\mathbf{x}_n}.$$

Note that Eq. (3) differs from the definition given in Ref. [7] and generalizes the corresponding expressions in Refs. [14–16]. However, with the present choice of the matrix elements involved in $W_{dd',s}$, our expression for $\sigma_{\rm DR}$ looks similar to the formula for the DR part of cross section published in Refs. [14–16]. Moreover, definition (3) keeps the Bell–Steinberger equality in its conventional form

$$\sum_s W_{dd',s} = i(\mathcal{E}_d - \mathcal{E}^*_{d'}) \langle d'_R | d_R \rangle, \tag{4}$$

but now the final states s may also have identical quantum numbers. Note that, in contrast with Eq. (1), in Eq. (4) the summation over s includes *all* possible low-lying states.

The particular case when $d = d'$ in the sum of Eq. (1) corresponds to the superposition of Lorentz shapes of the DR process. The terms $\sigma_{\rm DR}$ with $d \neq d'$ describe the radiative interference due to overlap between the upper as well as between the lower states, and lead to the asymmetry of the summarized shape. It should be noted that the definition of the "pure" Lorentz shapes is not unique in different basis sets. This means that even if terms $d \neq d'$ are not taken into account in Eq. (1) the radiative interference turns out to be partially involved in $\sigma_{\rm DR}$ in the biorthogonal basis through the complex mixing coefficients of the identical states. Nevertheless, it is usual to use the orthogonal basis set and in this case the Lorentz terms do not include the radiative interference. Asymmetry of the shapes can be numerically characterized, for example, by Low's parameter [8]. We use for this purpose the nonorthogonality integrals $\langle d_R | d'_R \rangle$. In orthogonal basis, such parameter can be chosen to be the ratio of nondiagonal widths to energy intervals between the overlapping levels.

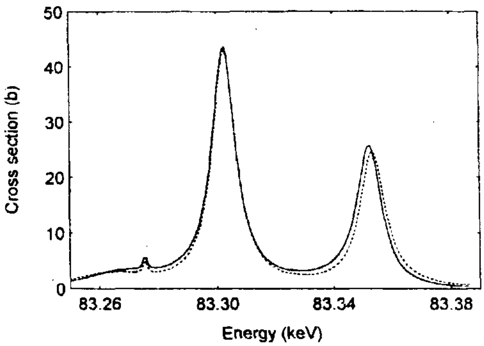

Fig.1. Total DR cross section for U^{91+} in the vicinity of $KL_{12}M_{12}$ resonances as a function of the incident electron energy (solid curve). The dashed curve corresponds to the Lorentz terms in orthogonal basis

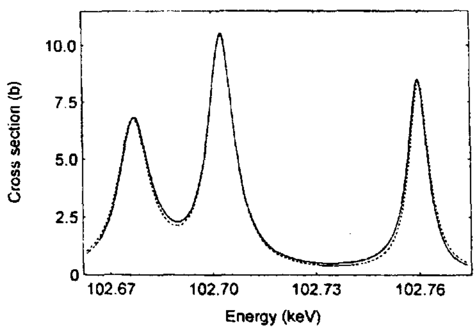

Fig.2. Total DR cross section for U^{91+} in the vicinity of $KM_{12}M_{12}$ resonances (solid curve); dashed curve – with no radiative interference included

Numerical calculations of the DR cross sections have been performed using Eq. (1) in the vicinities of $KL_{12}M_{12}$ and $KM_{12}M_{12}$ resonances of heliumlike uranium. The energies and rates were calculated in the framework of the $1/Z$ expansion. The energies of the levels have the radiative [23–26] (electron self-energy and vacuum polarization) and the exact one-photon interelectron interaction corrections included. The finite nuclear size was taken into account directly in the Dirac wave functions. In the case of $KL_{12}M_{12}$ DR resonances, there are four pairs of identical levels. As final recombined levels in this case, the doubly-excited $[2s^2_{1/2}]_0$, $[2p^2_{1/2}]_0$, and $[2s_{1/2}2p_{1/2}]_{0,1}$ states were chosen. The first pair has the same parity and zero total angular momentum, but the second pair has a different total angular momentum and does not mix. The results of the corresponding DR cross section calculations are shown in Fig. 1. The second example is given by $KM_{12}M_{12}$ resonances. In this case, there are only two identical upper states, $[3s^2_{1/2}]_0$ and $[3p^2_{1/2}]_0$, and the cross section is resolved with regard to $[2l_{1/2}3l'_{1/2}]_{0,1}$ configurations (see Fig. 2). As can be seen in both examples considered the interference effects originate from the radiative overlap of the upper and lower groups of doubly-excited states with identical quantum numbers. The values of nonorthogonality integrals $|\langle d_R|d'_R\rangle|$ are 0.18 and 0.1 for $KL_{12}M_{12}$ and $KM_{12}M_{12}$ resonances, respectively. The main error in the calculations is due to the omission of $1/Z$-order corrections in the evaluation of the radiative widths which are as a consequence uncertain to about $2-3\%$. The magnitude of the effect looks large enough to be observable on the experiments. We should note that in work [16] the differential cross section was measured. To observe the effect we discuss, one should measure the total cross section, that is, to detect the photons emitted in all directions. The last problem is not principal one for Super-EBIT experiments. The cross sections obtained may serve as a focus for near-future experimental investigations.

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