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ON THE ORIGIN OF QUANTUM OSCILLATIONS IN THE MIXED STATE OF ANISOTROPIC SUPERCONDUCTOR

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For the vortex lattice in an anisotropic superconductor with well-separated cores ($H_{c1} \ll B \ll H_{c2}$) it is shown that sizeable de Haas – van Alphen oscillations are caused by the levels crossing of the energy threshold separating localized and extended states of excitations moving in the average magnetic field, B.

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Recent experiments [1,2] show that the de Haas – van Alphen (dHvA) effect persists in the superconducting (SC) state for the magnetic field, B, as low as $B \sim 0.3 \div 0.4 H_{c2}$. The effective SC-gap, Δ, at such fields is large enough to preclude motion of an electron along a closed Larmor orbit with some radius, $r_L \sim v_F/\omega_c$, exceedingly larger than the coherence length, ξ_0, and the intervortex distance, d [3]. The dHvA-signal is expected to weaken exponentially as $\exp(-\Delta/\omega_c)$ [4]. In that which follows we suggest a new mechanism for the quantum oscillations in the mixed SC-state.

In normal state the dHvA-effect is brought about by levels crossing the chemical potential, μ, with the field variation. The oscillations are periodic in B^{-1} because a minor field change, $\Delta B/B \sim \omega_c/\mu$, is enough to push a level across μ.

Electron- or hole-like character of SC-excitations depends on the extent their energy exceeds the gap. Even for a "d-wave" superconductor [5] levels cannot cross the chemical potential. In this sense, there is no difference between a "d-wave" or any other anisotropic SC.

It is shown below that a new energy threshold takes over the role of the chemical potential in the SC-state. Consider, for example, an anisotropic superconductor with a spectrum, $\varepsilon(\mathbf{p}) = \sqrt{v_F^2(p-p_F)^2 + |\Delta(\mathbf{p})|^2}$. Assume, for simplicity, that the gap, $\Delta(\mathbf{p})$, has only one maximum, Δ_{max}, and one minimum, Δ_{min}, along the Fermi Surface (FS). Excitations with $\varepsilon(\mathbf{p}) > \Delta_{max}$ have itinerant behavior, while at $\Delta_{max} > \varepsilon(\mathbf{p}) > \Delta_{min}$ this is only true for excitations with a proper $\mathbf{p}$. The latter become localized in a magnetic field, for the Lorentz force changes the $\mathbf{p}$-direction. Excitations with the energy larger than Δ_{max}, may move along an extended Larmor orbit.

To pose this phenomenon as a theoretical problem, consider the limit of well separated vortices $d \gg \xi_0 (H_{c1} \ll B \ll H_{c2})$. The vortex cores occupying only a minor fraction of the volume may be neglected. A typical electron trajectory would run across the "bulk", where the gap amplitude is saturated:

$$\Delta(\mathbf{p},\mathbf{r}) \cong \Delta(\mathbf{p}) \exp(i\varphi(\mathbf{r})). \tag{1}$$

The method [5] to treat the problem is based on averaging the Gor'kov system of Green functions over quasiclassical trajectories [6] (all notations below from ref. [5]). The Green functions are presented in terms of the position, φ, of an electron along the FS. The core of the method is given by eqs. (23) – (28) in ref.[5]. The Gor'kov matrix being diagonalized, the whole problem reduces to solving the following Schrödinger equation:

$$-\omega_c^2 y'' + [\Delta^2(\varphi) - \omega_c \Delta'(\varphi)]y = E^2 y. \tag{2}$$

The term, $-\omega_c \Delta'(\varphi)$, plays no role and will be omitted. In (2) we have also left out the term $h(\varphi)$ of eq. (31) in ref.[5]. The Doppler shift (31) in ref.[5] is essential for the magnitude of the effect, and will be taken into account later.

The eigen functions are given by solutions of (2) satisfying the periodicity condition ($\varphi \to \varphi + 2\pi$) for

$$y(\varphi)e^{-i\kappa\varphi} \tag{3}$$

where $\kappa = \bar{\mu}/\omega_c$ comes from presenting the chemical potential in the form [5]

$$\mu = \omega_c N_o + \bar{\mu}. \tag{4}$$

With μ being large, $N_0 \gg 1$, and the specific N_0 does not affect the pattern of periodic (in B^{-1}) oscillations of the magnetization which depends on κ in the interval (0,1). Eq. (2) and (3) become the problem of finding the band structure for a particle moving in the periodic potential $\Delta^2(\varphi)$ with κ as a quasimomentum.

Re-write (2) in the form

$$-\omega_c^2 y'' + (\Delta^2(\varphi) - \Delta_{max}^2)y = (E^2 - \Delta_{max}^2)y \tag{2'}$$

and consider (2′) first in the quasiclassical WKB-approximation ($\omega_c \ll \Delta$). At $|E| \gg \Delta$ the periodicity of (3) leads to the spectrum of free electrons in the magnetic field: $E_n = \omega_c n + \bar{\mu}$. At $E^2 - \Delta_{max}^2 < 0$ the attractive potential in (2′) has many ($\Delta/\omega_c \gg 1$) "localized" levels (tunneling across the barrier is neglected). The boundary separating "extended" and "localized" states in the WKB-sense lies at Δ_{max}. Introduce in (2′):

$$E^2 - \Delta_{max}^2 \simeq 2\Delta_{max}(-\varepsilon) \tag{5}$$

for $|E|$ close to Δ_{max}. The WKB-solutions are [7]:

$$y_\pm(\varphi) = A(S'(\varphi))^{1/2} \exp[\pm i \int_0^\varphi S'(\varphi)d\varphi] \tag{6}$$

with

$$\omega_c S'(\varphi) = \left[2\Delta_{max}(-\varepsilon) + \Delta_{max}^2 - \Delta^2(\varphi)\right]^{1/2} \tag{7}$$

(A is the normalization coefficient). The BCS-factors, $u(\varphi)$ and $v(\varphi)$, in (23) in ref.[5] are to be normalized together: $\overline{|u^2|} + \overline{|v^2|} = 1$ (the bar in $\overline{(\ldots)}$ means the normalization integral: $(2\pi)^{-1}\int_0^{2\pi}(\ldots)d\varphi$). Two auxiliary expressions which follow from eqs. (26 – 28) in ref.[5]:

$$\overline{|u^2|} = \frac{1}{2}\left\{\overline{|y|^2} + (i\omega_c/2E)\overline{(y^*y' - yy^{*\prime})}\right\}, \quad \overline{|v^2|} = \frac{1}{2}\left\{\overline{|y|^2} - (i\omega_c/2E)\overline{(y^*y' - yy^{*\prime})}\right\} \tag{8}$$

immediately show that $\overline{|y|^2} = 1$.

Expression (6) in ref.[5], containing oscillatory effects

$$M = -\frac{\mu e}{\pi c}\sum_\lambda \overline{|u_\lambda(\varphi)|^2} \tag{9}$$

(λ enumerates the eigenvalues, factor 2 added to (6) in ref.[5], accounts for spins, and $n(E_\lambda) \equiv 1$ at $E_\lambda < 0$ and $T = 0$), becomes an integral over λ with the use of the Poisson formula:

$$\sum_{-\infty}^{+\infty} \delta(\lambda - n) = \sum_{k=-\infty}^{+\infty} e^{2\pi i K\lambda}. \tag{10}$$

Integration by parts transforms M_{osc} into [5]:

$$M_{osc} = \frac{i\mu e}{2\pi^2 c} \sum_K \frac{1}{K} \int_{-\infty}^{+\infty} e^{2i\pi K\lambda} \frac{d}{d\lambda} \left(\overline{|u_\lambda(\varphi)|^2} \right) d\lambda. \tag{10'}$$

Although integration over λ acquires the meaning only after the connection between λ and the energy is established, the threshold separating "localized" and "extended" (in the WKB-sense (6), (7)) states is already seen in eq. (10'): for "localized" states, $|E| < \Delta_{max}$, the wave functions are real, and from (8) $\overline{|u_\lambda^2|} = 1/2$. For "extended" states (6) we have:

$$\overline{|u_\lambda|^2} = \frac{1}{2} \left\{ 1 \pm \frac{\omega_c}{E_\lambda} |A_\lambda|^2 \right\} \tag{11}$$

with $|A_\lambda|^2$-a function of energy (see (6)). The derivative in (10') thus *eliminates states below* Δ_{max} with $\overline{|u_\lambda|^2} = 1/2$ being energy independent.

Returning to summation over λ, eqs. (10) and (10'), we need to construct such the function:

$$\lambda(E) = \Phi(E)/2\pi \tag{12}$$

that the provision:

$$\Phi(E_n) = 2\pi n \tag{12'}$$

would enumerate all energy levels in the consecutive order. In the WKB-approach $\Phi(E)$ is given by:

$$S(2\pi, -\varepsilon) = (1/\omega_c) \int_0^{2\pi} S'(\varphi, -\varepsilon) d\varphi - 2\pi\kappa \tag{13}$$

which at large energies matches the Landau free electron spectrum. The approach falls short near $(-\varepsilon) = 0$.

Choose $\Delta^2(\varphi)$ near Δ_{max} as

$$\Delta^2(\varphi) = \Delta_{max}^2 (1 - a\varphi^2). \tag{14}$$

Expanding (13) in $(-\varepsilon) > 0$, one obtains:

$$S(2\pi, -\varepsilon) \simeq S(2\pi, 0) - (l/2) \ln(l/\wedge) \tag{15}$$

with the useful notation in (14):

$$l = (-2\varepsilon)/\omega_c a^{1/2} \; ; \quad \wedge = (a^{1/2} \Delta_{max}/\omega_c) \gg 1. \tag{16}$$

Similarly, the factor $|A|^2$ in (6) and (11) at small l is proportional to:

$$|A|^2 \propto [\ln(l/\wedge)]^{-1}. \tag{15'}$$

Because of the singularity (15) the "numbering" function $\Phi(E)$ cannot be comprised of the two WKB-branches, the one that is given by (13) (at $(-\varepsilon) > 0$), and the other which counts "localized" states ($(-\varepsilon) < 0$).

Note that far away from $\varphi = 0$ the WKB-solution

$$y(\varphi) = ay_+(\varphi) + by_-(\varphi) \tag{17}$$

is still correct. With the use of (5), (14), (16), eq. (2′) can be solved near $\varphi = 0$ in terms of the parabolic cylinder functions. It establishes the matrix relation between coefficients (a, b) in (17) on the R.H.S. of $\varphi = 0_+$, and the other set (a', b') on its L.H.S., $\varphi = 0_-$:

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \tag{18}$$

Beginning at $\varphi = 0_+$, moving along with eq. (17) toward $(2\pi)_-$ and using (18), the periodicity condition for (3) provides the equation:

$$R(l) \equiv |\alpha|(e^{i\tilde{S}} + e^{-i\tilde{S}}) = 2\cos 2\pi\kappa. \tag{19}$$

Abbreviations in eq. (19) are:

$$\tilde{S} = S(2\pi, -\varepsilon) - \theta, \ \ \alpha = |\alpha| \exp(i\theta). \tag{19′}$$

Of the two solutions of eq. (19) we chose

$$e^{i\tilde{S}} = \frac{1}{|\alpha|}\left\{\cos 2\pi\kappa + i\sqrt{|\alpha|^2 - \cos^2 2\pi\kappa}\right\} \equiv \rho(l) \tag{20}$$

because at $(-\varepsilon)$, i.e. l, large and positive $|\alpha| \Rightarrow 1$, $\theta \Rightarrow 0$, and we return in this limit to (13). On the real axis of l the function

$$\Phi(l) = S(2\pi, l) - \theta(l) - \frac{1}{i}\ln\rho(l) \tag{21}$$

is positive, with $d\Phi/dl > 0$, and matches asymptotically (at $l \to \infty$) the free electron spectrum. With the help of (12) and (21) the oscillatory magnetization is

$$M_{osc} = \frac{i\mu e}{2\pi^2 c}\sum_K \frac{1}{K}\int_{-\infty}^{+\infty} e^{iK\Phi(l)}\frac{d}{dl}\left(\overline{|u_l(\varphi)|^2}\right) dl. \tag{22}$$

$\Phi(l)$ being continued analytically into the complex l-plane, the integration may be shifted into the upper half-plane (from (11), (15′) one has the behavior of $d(\overline{|u_l|^2})/dl$ at large $|l|$). Consider singularities in (22). Thus from the expression for $|\alpha|$ [1)]

$$|\alpha| = (1 + e^{-\pi l})^{1/2} \tag{23}$$

we conclude that branching points in (23) lie at

$$l_m = \pm(2m+1)i. \tag{24}$$

[1)] To the best of the author's knowledge, the complete matrix in (17) was not published, although transmission/reflection processes for a parabolic barrier have been studied (see in ref. [7]). The author thanks V. Pokrovsky for discussion of references.

This is also true for $\theta(l)$ (see $\tilde{S}(2\pi,l)$ below). The definition of $\rho(l)$ together with (23) for $|\alpha|$, leads to the square root singularities at

$$l'_m = \pm(2m+1)i - \frac{1}{\pi}\ln(\sin^2 2\pi\kappa) \equiv l_m + l_0. \qquad (24')$$

For $\Phi(l)$ to be analytical in a strip at the real axis, the branch-cuts in the l-plane, caused by singularities (24, 24′), must be chosen parallel to the imaginary axis.

The integral in (22) may be bent to the contours, C_1 and C_2, each encircling the branch-cuts (24) and (24′) in the upper half-plane. The non-analytical terms of eq. (15) at $|\lambda| \sim 1$ are now absent in $\tilde{S}(2\pi,l)(c \sim 1)$:

$$\tilde{S}(2\pi,l) \simeq S(2\pi,0) + \frac{1}{2}l[\ln(\wedge\cdot c)] - \frac{1}{2i}\ln\left[\Gamma(\frac{1}{2}+\frac{il}{2})/\Gamma(\frac{1}{2}-\frac{il}{2})\right]. \qquad (25)$$

Both integrals (along C_1 and C_2) rapidly converge.

It is necessary to normalize $\overline{|u_l(\varphi)|^2}$ with the accuracy better than that given by the WKB-approximation in (6), (11). Fortunately the properties of the Bloch functions in a one-dimensional periodic potential are well-studied. With the help of eq. (4.18) in ref. [8] and our eqs. (8) we derive:

$$\overline{|u_l(\varphi)|^2} = \frac{1}{2} - \pi a^{1/2}\sin 2\pi\kappa\left[\frac{dR}{dl}\right]^{-1} \qquad (26)$$

(here $R(l)$ is the R.H.S. of (19)). After differentiation on l in (22) had eliminated 1/2 in (26), one may use for the rest the rapid convergence of the integrals to integrate back by parts in (22). Single terms under the sum symbol (22) become:

$$I_K = \frac{i\pi a^{1/2}\sin 2\pi\kappa}{2}\int\frac{\exp(iK\Phi(l))dl}{(\sin^2 2\pi\kappa + e^{-\pi l})^{1/2}} \qquad (27)$$

with integrals running along C_1, C_2. Assume that $\ln\wedge \gg 1$ in (25). First, the term in (22) with $K = 1$ prevails. In addition, as seen from (25), it is enough to consider the nearest singularities with $m = 0$ in (24), (24′). Defining the branches of the square roots in (27) properly, from (27) one obtains contributions to I_1 from the two contour integrals, over C_1 and C_2, correspondingly:

$$I_1(C_1) = -\frac{(2\pi)^{3/2}a^{1/2}tg2\pi\kappa}{(\wedge c)^{1/2}(\ln\wedge c)^{1/2}}e^{iS(2\pi,0)+\frac{i\pi}{4}} \qquad (28)$$

$$I_1(C_2) = \frac{\pi a^{1/2}\sin 2\pi\kappa e^{\frac{i\pi}{4}}}{(\wedge c)^{1/2}(\ln\wedge c)^{1/2}}\left[\frac{\Gamma(1-\frac{il_0}{2})}{\Gamma(1+\frac{il_0}{2})}\right]^{1/2}\cdot e^{iS(2\pi,0)+\frac{il_0}{2}\ln(\wedge c)} \qquad (28')$$

with l_0 from (24′). (I_1 has the form (28), (28′) at l_0 not too small ($l_0 \gtrsim (\ln\wedge c)^{-1}$.) Otherwise the two contours C_1 and C_2 start to merge. Also, if l_0 becomes large ($\sin^2 2\pi\kappa \to 0$), expression (25) for $\tilde{S}(2\pi,l)$ being correct at $|l| \sim 1$, ceases to be applicable).

Expressions (28) explicitly present the periodic (in B^{-1}) oscillations in the magnetization as κ varies in the interval (0,1). It is notable that the amplitude is of the order of $(\omega_c/\Delta_{max})^{1/2}$, i.e., is not exponentially small. Both (28) and (28′) lead to the large content of the higher harmonics. In principle, M_{osc} could be measured directly as a function of small changes in B. $I_1(C_1)$ discloses a rather regular behavior in ΔB (i.e., κ), while $I_1(C_2)$ rapidly becomes chaotic due to the phase factor in (28′), $\exp[i(l_0/2)\ln(\wedge c)]$, contributing into higher harmonics. At the Fourier analysis of the dHvA-signal a few first harmonics are expected to be seen with the intensity of the order of

$$(\omega_c/\Delta_{max})^{1/2}. \tag{29}$$

Unfortunately, (29) does not take into account scattering of electrons on the flux lines. The term $h(\varphi)$ of eq. (31) in ref.[5], if included, adds to the potential of eq. (2'):

$$2\Delta_{max}h(\varphi). \tag{30}$$

Even though $h(\varphi) \sim v_F/d$ is small compared with Δ_{max}, (30) drastically distorts the potential near $\varphi = 0$. It is a local $\tilde{h}_{max}$ in the vicinity of the maximum in $\Delta(\varphi)$ which now sets in the energy threshold between "localized" and "extended" states. Note that although $h(\varphi)$ is rather irregular (for a given trajectory) and does change typically on the scale of $\delta\varphi \sim (d\omega_c/v_F)$, its local maxima produce potential barriers in (30) which remain impenetrable in the quasiclassical sense. The above analysis of M_{osc} can be performed in exactly the same manner as above around $\tilde{h}_{max}$. There is a change in the scale (29), because the curvature, a, near a maximum in (30) is much higher than in (14). Without going into details, we comment that this only increases the effect, because the potential $h(\varphi)$ near $\tilde{h}_{max}$ comprises a much sharper barrier, if compared with eq. (14).

The major destroying effect comes from the phase factor in (25), $S(2\pi, 0)$. At $h(\varphi) \neq 0$, $S(2\pi, 0)$ may be expanded in $\delta h(\varphi) = h(\varphi) - \tilde{h}_{max}$. (Now $(-\varepsilon) \Rightarrow E- -\Delta_{max} - \tilde{h}_{max}$.) The fluctuating part, $\delta S(2\pi, 0)$, is

$$\delta S(2\pi, 0) = -(\Delta_{max}/\omega_c) \int_0^{2\pi} \delta h(\varphi) \left(\Delta^2_{max} - \Delta^2(\varphi)\right)^{-1/2} d\varphi. \tag{31}$$

Since $\langle \delta h(\varphi) \rangle$ (the average over all trajectories) is obviously zero, fluctuations in $\exp(i\delta S(2\pi, 0))$ lead, as in [5], to an effective Dingle factor of the form

$$\exp(-v_F/d\omega_c) \Rightarrow \exp(-(\Delta/\omega_c)(\xi_0/d)). \tag{32}$$

The exponent in (32) provides much more favorable conditions for observation of the dHvA-effect than previous results [4].

To conclude, in the developed mixed state of an anisotropic superconductor there exists the energy threshold sorting excitations into two categories: the localized and extended ones. Crossing this threshold by the excitations levels at the change of the magnetic field comprises the new mechanism for quantum oscillations. Scattering on the flux lines reduces the dHvA-effect. Nevertheless, the effect remans bigger than anticipated.

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