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PHONON RELAXATION OF SUBGAP LEVELS IN SUPERCONDUCTING QUANTUM POINT CONTACTS

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Superconducting quantum point contacts are known to possess two subgap states per each propagating mode. In this note we compute the low-temperature relaxation rate of the upper subgap state into the lower one with the emission of an acoustic phonon. If the reflection in the contact is small, the relaxation time may become much longer than the characteristic lifetime of a bulk quasiparticle.

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In the present paper we address the question of phonon relaxation of subgap levels in superconducting quantum point contacts (SQPC) [1–3]. We use this term for a class of junctions between two superconductors (of the BCS type), were only a small number of modes propagates and the scattering center of the contact is much shorter than the superconducting coherence length. Under the latter condition, the internal structure of the scatterer is not important, but may be described by a scattering matrix for normal electrons [4, 5]. Thus, in the class of SQPC we include both SNS junctions with a thin normal layer and with quantized propagating modes [6] and mechanically-controllable breakjunctions [7].

Theoretically, SQPCs in different setups are predicted to exhibit various quantum phenomena. The most advanced predictions involve quantum-mechanical evolution of a superconducting island connected to external leads by SQPCs [8, 9]. Other works discuss interference between time evolutions of localized states and/or excitations of localized states by an external electromagnetic field [10–12]. For experimental observation of these predictions, the decay time of excited localized states must be sufficiently long. This decay time directly enters the expressions for the thermal fluctuations of the Josephson current in SQPC [13–16]. Potential use of SQPC in quantum computing devices also crucially depends on their ability to preserve quantum coherence [17].

At low temperatures ($T \ll \Delta$, where Δ is the superconducting gap), the dominant inelastic processes are the transitions between localized subgap levels, without participation of continuum spectrum. There are two major channels of inelastic relaxation. One is the emission or absorption of a phonon, the other is due to the electromagnetic coupling to the external environment. It will depend on the particular experimental setup, which of the two relaxation channels dominates. In the present paper we consider only the phonon relaxation in its simplest form, when the superconducting phases on the contact terminal are assumed to be rigidly fixed.

In several works, the time of decay into phonons was estimated as the characteristic time τ_0 of a bulk quasiparticle [10–12]. The latter lifetime is of order $\tau_0 \sim \Theta_D^2/\Delta^3$ — the same as the relaxation time of a normal electron with energy Δ above the Fermi level [18] (we set $\hbar = 1$ throughout the paper). We want to point out that this assumption is not justified for decay of localized states. In refs. [16, 19] the relaxation of subgap states in SQPC is associated with bulk subgap states appearing due to electron-phonon interaction. This contribution is exponentially small for temperatures much lower that the superconducting gap.

We compute the *direct* matrix element for decay of localized states into phonons. This direct decay leads to a non-vanishing relaxation rate even at zero temperature. Further, we simplify our discussion by setting the temperature much lower than the energy of the subgap level. Then the only allowed process is the transition from the upper to the lower level with the emission of an acoustic phonon. The extension of our result to include thermal phonons is obvious (see eq. (16) below).

The characteristic energy scale of the subgap levels is Δ, thus the wavelength of the phonon is of order $s/\Delta \sim \xi s/v_F \ll \xi$, where s and v_F are sound and Fermi velocities respectively, ξ is the superconducting coherence length. The rate and the angular distribution of the emitted phonon may depend on the particular geometry of the contact (or, more precisely, on the geometry of the wavefunction of the subgap states). In this paper we discuss the simplest setup of a narrow one-dimensional contact. By this we mean that the whole subgap state is localized in a narrow strip of width much smaller than the phonon wavelength. Although very idealistic, this assumption is consistent with the model of adiabatic constriction [2] and gives an upper bound for the actual decay rate.

Each propagating mode in a quasi-one-dimensional contact may be described by the Hamiltonian

$$H = \int_{-\infty}^{+\infty} dx \Big[i\Psi^\dagger_{L\beta}\partial_x\Psi_{L\beta} - i\Psi^\dagger_{R\beta}\partial_x\Psi_{R\beta} + \Delta(x)\big(\Psi^\dagger_{R\uparrow}\Psi^\dagger_{L\downarrow} - \Psi^\dagger_{R\downarrow}\Psi^\dagger_{L\uparrow}\big) +$$

$$+\Delta^*(x)\big(\Psi_{R\downarrow}\Psi_{L\uparrow} - \Psi_{R\uparrow}\Psi_{L\downarrow}\big)\Big] + H_{scatt}, \tag{1}$$

where $\Psi^\dagger$ and Ψ are electron operators (L and R subscripts denote left-and right-movers, $\beta = \uparrow, \downarrow$ is the spin index), $\Delta(x)$ is the superconducting gap with the following x-dependence:

$$\Delta(x) = \begin{cases} \Delta, & x < 0 \\ \Delta e^{i\alpha}, & x > 0 \end{cases} \tag{2}$$

[It will be convenient for us to use the units with Fermi velocity equal to one throughout the paper]. The scattering term H_{scatt} expresses elastic scattering at $x = 0$ and may be described by a scattering matrix [5]. Diagonalizing the Hamiltonian (1) gives the subgap state operators $\gamma^\dagger_\uparrow$ and $\gamma^\dagger_\downarrow$ raising energy by

$$E(\alpha) = \pm\Delta\sqrt{1 - t\sin^2\frac{\alpha}{2}} \tag{3}$$

each (t is the normal transparency of the contact)[4, 20]. The two levels below continuum are the ground state $|0\rangle$ and the first excited state $|1\rangle = \gamma^\dagger_\uparrow\gamma^\dagger_\downarrow|0\rangle$. The decay of the state $|1\rangle$ to the state $|0\rangle$ with the emission of a phonon depends on the density matrix element

$$\langle 0|n(x)|1\rangle = \langle 0|\Psi^\dagger_\beta(x)\Psi_\beta(x)|1\rangle. \tag{4}$$

Since $\gamma^\dagger_\uparrow$ and $\gamma^\dagger_\downarrow$ are linear in electron operators, the matrix element (4) may be computed by commuting density operator with them:

$$\langle 0|n(x)|1\rangle = \left\{ \left[n(x), \gamma^\dagger_\uparrow\right], \gamma^\dagger_\downarrow \right\} = i|b|\kappa e^{-2\kappa|x|}\mathrm{sign}(x), \tag{5}$$

where b is the backscattering amplitude ($|b| = \sqrt{1-t}$), and $\kappa = \sqrt{t}\Delta|\sin(\alpha/2)|$ is the inverse length of the subgap state. The matrix element is purely imaginary if the relative phases of $|0\rangle$ and $|1\rangle$ are chosen according to [5]

$$\langle 0|\frac{\partial}{\partial\alpha}|0\rangle = \langle 1|\frac{\partial}{\partial\alpha}|1\rangle = 0, \qquad \langle 0|\frac{\partial}{\partial\alpha}|1\rangle \text{ is real.} \tag{6}$$

This fact is of no importance for the present calculation, but will be used elsewhere in the discussion of the phonon emission in the presence of dynamics in α.

The electron-phonon interaction is described by the deformation potential:

$$H_{e-ph} = g\int d^3r\,\varphi(r)\Psi^\dagger_\beta(r)\Psi_\beta(r), \tag{7}$$

$$\varphi(r) = \frac{1}{\sqrt{V}}\sum_k\sqrt{\frac{\omega_k}{2}}(b_k e^{i(kr-\omega_k t)} + b_k^\dagger e^{-i(kr-\omega_k t)}), \tag{8}$$

where b_k are phonon operators normalized by $[b_{k_1}, b^\dagger_{k_2}] = \delta_{k_1k_2}$,

$$g^2 = \frac{\pi^2\zeta}{2\varepsilon_F}, \tag{9}$$

(in the units with Fermi velocity equal to one), ζ is the coupling constant of order one.

The transition rate is then given by

$$\tau^{-1} = 2\pi\sum_k\left|\langle 0,k|H_{e-ph}|1\rangle\right|^2\delta(\omega_k - 2E) = \pi^3\zeta\frac{E}{\varepsilon_F^2}\int\frac{d^3k}{(2\pi)^3}\delta(\omega_k-2E)\left|\langle 0|n_k|1\rangle\right|^2. \tag{10}$$

Here E is the energy of the subgap states given by (3) (so that the energy of the emitted phonon is $2E$), ε_F is the Fermi energy, ω_k is the phonon dispersion relation, and n_k is the three-dimensional density operator. Assume the linear isotropic phonon spectrum:

$$\omega_k = s|k| \tag{11}$$

and the narrow contact limit, where the matrix element of n_k depends only on the component of k parallel to the constriction and is given by the Fourier transform of the one-dimensional matrix element (5). Finally, using $k \gg \kappa$, we arrive to the answer for τ^{-1}:

$$\tau^{-1} = \pi^2\zeta\frac{E^2}{(c\varepsilon_F)^2}\int_{-\infty}^{\infty}dx\left|\langle 0|n(x)|1\rangle\right|^2 = \frac{\pi^2\zeta}{2}(1-t)\frac{E^2\kappa}{(s\varepsilon_F)^2}. \tag{12}$$

Returning to the physical units, we find up to a constant factor of order one

$$\tau^{-1} = \sqrt{t}(1-t)\left|\sin\frac{\alpha}{2}\right|(1-t\sin^2\frac{\alpha}{2})\frac{\Delta^3}{\Theta_D^2}, \tag{13}$$

where Θ_D is the Debye temperature.

If compared to the characteristic bulk quasiparticle inverse lifetime $\tau_0^{-1} \sim \Delta^3/\Theta_D^2$ [18], the result (13) is smaller by a factor depending on the backscattering probability $1-t$[1)]

[1)] The actual lifetime of a bulk quasiparticle diverges at the bottom of the quasiparticle band. The "characteristic" lifetime τ_0 corresponds to energies of order Δ above the bottom of the band. There is no *a priori* reason for the relaxation rate τ^{-1} of the localized states to be of order τ_0^{-1}.

In the case of weak backscattering, this factor may contribute up to orders of magnitude to the decay time. This effect is easy to understand: in ideally conducting contact ($t = 1$) the two Andreev states carry opposite momenta equal to the Fermi momentum, and the matrix element (4) contains only a rapidly oscillating with momentum $2k_F$ part[2]:

$$\langle 0|n(x)|1\rangle = \kappa e^{-2\kappa|x|} e^{2ik_F x}. \tag{14}$$

This oscillating part of the matrix element gives the lower bound for the relaxation rate (13) as $t \to 1$:

$$\tau_{t=1}^{-1} \sim \frac{E^3\kappa^4}{s^3\varepsilon_F^6} \sim \left|\cos^3\left(\frac{\alpha}{2}\right)\right| \sin^4\left(\frac{\alpha}{2}\right) \frac{\Delta^7}{\varepsilon_F^3\Theta_D^3}. \tag{15}$$

For realistic values of Δ, Θ_D, and ε_F, this relaxation time is unphysically large. The actual relaxation time at $t = 1$ will be bounded by other factors such as finite thickness of the interface and non-one-dimensionality of the contact. These effects go beyond the simple model of the present paper[3]

Another important feature of the relaxation rate (13) is that it vanishes at $\alpha \to 0$, since in this limit the subgap states become delocalized.

Our assumption of one-dimensionality of the contact also results in overestimating the relaxation rate. If the "tails" of the localized states are smeared in the terminals, they give weaker contribution to phonon emission. Unfortunately, this effect is highly geometry-dependent, and should be considered separately in each experimental realization.

As a direct application of the above result, the decay rate entering the fluctuations of the Josephson current in a single SQPC [13, 14, 16] is given by

$$\gamma = \tau^{-1}(1 + 2n_B(2E(\alpha)) = \tau^{-1} \coth \frac{E(\alpha)}{T}, \tag{16}$$

where $n_B(2E(\alpha))$ is the Bose occupation number for the phonons involved in the transition [15], τ^{-1} is the zero-temperature rate (13). The low-frequency current noise is [13, 16].

$$S(\omega) = S_0 \frac{2\gamma}{\omega^2 + \gamma^2}, \tag{17}$$

where S_0 is the integral low-frequency noise.

To summarize, we calculated the rate of direct relaxation of subgap states in SQPC into acoustic phonons at low temperature in the simplest one-dimensional geometry. The relaxation rate does not vanish in the $T \to 0$ limit, but it is strongly suppressed in the case of a nearly ballistic contact.

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