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BREATHING SOLITONS IN OPTICAL FIBER LINKS

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We introduce the concept of "breathing" solitons to describe optical-pulse dynamics in transmission lines with passive compensation of fiber chromatic dispersion. The "breathing" pulse can be used as the information carrier. The presented theory is complementary to the concept of the guiding-center soliton. It is shown that an average bright soliton can propagate in the system with large variations of dispersion, including segments with high normal dispersion.

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In this Letter we examine optical pulse transmission in cascaded communication systems. It is well known that picosecond optical pulse propagation in lossless fibers is described by the integrable nonlinear Schroedinger equation (NLSE). A powerful method of the inverse scattering transform (IST) has been applied to the NLSE [1], and it was shown that solitons determine the asymptotic states of this system. In long transmission lines, however, the dissipation plays a prominent role in the pulse dynamics. To overcome the decreasing of the soliton amplitude due to attenuation, periodic amplification of the signal is needed. It is not evident that the soliton survives non-integrable modifications of the NLSE, because perturbations (losses and amplification) typically are not small effects in cascaded systems. The basic element of an optical communication system is a piece of transmission fiber followed by an optical amplifier. An optical pulse propagating along such a line must be periodically reproduced at the output of each element. Transformation of the pulse after propagation in the fiber element and amplification can be considered as a mapping of the input pulse into the output pulse. Therefore, the carrier of one bit of information must be a stationary point of such a mapping. The condition for stable information transmission through the whole line leads to the requirement of the stability of this stationary point.

Both of these requirements can be satisfied in modern transmission systems based on low dispersion fibers. In this case, pulse propagation between two amplifiers can be considered as linear and non-dispersive. The main effect is the decreasing of the pulse amplitude due to fiber losses. The linear character of the process and a proper choice of the coefficient of amplification provide the stationarity of the mapping for a pulse of arbitrary shape. As has been shown in [2-5], the slow dynamics of a pulse is again governed by the NLSE. Therefore, the stable soliton solution of NLSE, which has been named the "guiding-center" (or average) soliton, represents the carrier of information in such *dissipative* systems.

Recently, an alternative approach to optical data transmission at 1.55μm, based on dispersion compensation technique, has been come into focus of intensive research. The incorporation of a piece of fiber with high normal dispersion reduces the total dispersion of the fiber span between two amplifiers. This method has many advantages: it is compatible with the present concept of all-optical transparency of the system, it is cascadable, and it allows one to increase the capacity of the existing optical lines based on standard highly dispersive telecommunication fibers. Dispersion-compensating fibers (DCF) have been used in the experiments [6, 7] to overcome fiber chromatic dispersion in optical transmission lines. Numerical simulations of soliton data transmission in short standard monomode fiber (SMF) systems, upgraded by dispersion compensation, have been performed in [8]. Recent technological achievements, such as the design of chirped fiber grating (see the review in [9]), allows dispersion of 500ps/nm or even more to be compensated by a grating fiber of a few decimeters in length. But even using commercially available DCFs leads to a significant upgrading of networks. We also mention that a construction of a dispersion-allocated soliton transmission line using dispersion-shifted fibers has been studied recently in [10]. Cascaded transmission systems based on the direct dispersion compensating technique have great potential due to high-capacity, low-error bit rate, and low cost.

In this Letter we present a theory of optical pulse propagation in transmission systems using dispersion compensation technique. As a result of our research, a new concept of "breathing solitons" has arisen, which complements the center-guiding soliton theory presented in [3, 2], although these two concepts have different bases. We demonstrate that an average bright soliton can propagate along the line even in the presence of segments with high normal dispersion.

The evolution of optical pulses in an optical fiber is described by the modified NLS equation.

$$i\Psi_z + \frac{Z_{NL}}{Z_{dis}} D(z)\Psi_{tt} + |\Psi|^2\Psi = i\, G(z)\Psi \tag{1}$$

where

$$G(z)\Psi = Z_{NL}(-\gamma + (\exp(Z_a\gamma) - 1)\sum_{k=1}^{N}\delta(z - z_k))\Psi.$$

Here, the time is normalized by the initial pulse-width $t = T/t_0$, the envelope of the electric field $E = E(T, Z)$ is normalized by the initial pulse power $|E|^2 = P_0|\Psi|^2$, and the coordinate along the fiber z by the nonlinear length $z = Z/Z_{NL}$. Here γ describes fiber losses, $Z_k = kZ_a$ $(k = 1, .., N)$ are the amplifier locations; Z_a is the amplifier spacing. There are three characteristic scales in this problem: Z_a, $Z_{NL} = 1/(\alpha P_0)$ — the nonlinear length and $Z_{dis} = 2t_0^2/|\beta_2|$ — the dispersion length corresponding to the SMF. (In the system under consideration there exist, in fact, three dispersion scales: the dispersion lengths corresponding to the chromatic dispersion of the DCF, SMF and the dispersion length related to the residual dispersion of each section Z_{DR}). The symbol β_2 denotes the group-velocity dispersion, α is the coefficient of nonlinearity.

Consider a transmission line consisting of periodic alternating fiber sections and point optical amplifiers. We study, without loss of generality, the so-called precompensating scheme: a piece of DCF with the dispersion D_- and length Z_c is followed by a piece of SMF with the dispersion D_+. In the ideal line, the dispersions should be completely compensated; in practice, however, there is always some residual dispersion. Pulse propagation through the transmission line can be described in the first approximation as follows: at first, during propagation through the DCF, pulses broaden dispersively and acquire a positive dispersion-induced frequency chirp; during the evolution through the SMF, pulses compress because the sign of the dispersion has been reversed and the condition for compression is satisfied (see e.g. [11]). Thus, the pulse experiences breather-like oscillations. After pulses propagate through both pieces of fibers, they must be amplified to compensate fiber losses. This entire process, including the amplification, is then repeated. The dispersion and fiber losses are the main acting factors over one cycle of the process. The influence of residual dispersion and the Kerr nonlinearity appears at distances large than Z_a (namely, at Z_{DR} and Z_{NL}). Thus, in the description of a "slow" evolution of a pulse, it is necessary to take into account both the residual dispersion and the nonlinearity.

Let transform, following [2, 3], Ψ to a new function A by eliminating rapid oscillations of the amplitude due to the periodic amplification $\Psi = A(t,z)\exp(\int_0^z G(z')dz')$. The equation for A can be written in Lagrangian form with the Lagrangian L as

$$S = \int L dt dz = \int dt dz \left[\frac{i}{2}(AA_z^* - A^* A_z) + \frac{Z_{NL}}{Z_{dis}} D(z)|A_t|^2 - \frac{c(z)}{2}|A|^4\right]. \tag{2}$$

Here $c(z) \equiv \exp(2\int_0^z G(z')dz')$ can be represented as a sum of rapidly varying and constant parts $c(z) = \langle c(z)\rangle + \tilde{c}(z)$, where $\langle \tilde{c}(z)\rangle = 0$ and $\langle c(z)\rangle = [1-\exp(-2\gamma z_a)]/(2\gamma z_a)$. We also write $D(z) = \langle D(z)\rangle + \tilde{D}(z)$, where $\langle \tilde{D}(z)\rangle = 0$ and $\langle D\rangle = (D_- z_c + D_+(z_a - z_c))/z_a$ is a small perturbation due to the average residual dispersion ($\langle D\rangle \approx Z_{dis}/Z_{DR} \ll 1$).

In the limit $Z_a, Z_{dis} \ll Z_{NL}$, one may treat the nonlinearity as a perturbation. At the lowest order, fast oscillations of the pulse-width are given by a solution of the linear problem $A(z,t) = \int_{-\infty}^{+\infty} d\omega A_\omega \exp(i\omega t - i\omega^2 R(z))$ with $R(z) = \int_0^z \tilde{D}(\xi) d\xi Z_{NL}/Z_{dis}$. Nonlinear effects come into play on a large scale compared to Z_a, namely at distances proportional to Z_{NL}. The concept of the guiding-center soliton considered in [2, 3] corresponds to the limit $Z_a \ll Z_{NL} = Z_{dis}$. In this Letter we study another regime with $Z_{NL} \gg Z_a \approx Z_{dis}$. The existence of small parameters Z_{dis}/Z_{DR} and Z_a/Z_{NL} allows us to introduce fast and slow scales. The fast process corresponds to the oscillations of the amplitude and the shape of the pulse due to the dispersion compensation and periodic amplification, and slow dynamics describes the average changes due to the nonlinearity and residual dispersion. Therefore, we assume that A_ω varies slowly with z and represent the function $A(z,t)$ in the form

$$A(z,t) = \int_{-\infty}^{+\infty} d\omega Q_\omega(z) \exp[i\omega t - i\omega^2 R(z)]. \tag{3}$$

To obtain the equation for the slow average evolution of $A(z,t)$ we substitute (3) into L, and average the Lagrangian L over the interval Z_a. Because the function $Q(\omega, z)$ is assumed to vary slowly on the scale of the amplification distance, it can be placed outside of the averaging integral. After straightforward calculations we obtain the Lagrangian describing the evolution of the slowly-varying envelope.

$$S = \int \tilde{L} dt dz = \int dz d\omega \left[\frac{i}{2}(Q_\omega Q_{\omega z}^* - Q_\omega^* Q_{\omega z}) + \frac{Z_{NL}}{Z_{dis}}\langle D(z)\rangle \omega^2 |Q_\omega|^2\right] -$$
$$\int dz d\omega_1 d\omega_2 d\omega_3 d\omega_4 Q_{\omega_1} Q_{\omega_2} Q_{\omega_3}^* Q_{\omega_4}^* \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) F(\omega_1, \omega_2, \omega_3, \omega_4), \tag{4}$$

where the function F is given by

$$F = \left\{ \frac{1 - \exp(-2\gamma Z_c(1 + igD_-))}{8\gamma\pi Z_a(1 + igD_-)} + \right.$$

$$\left. + \frac{\exp(-2\gamma Z_c - i\gamma g D_+(Z_c - Z_a)) - \exp[-2\gamma Z_a(1 + ig\langle D\rangle)]}{8\gamma\pi Z_a(1 + igD_+)} \right\}, \tag{5}$$

and $g = (\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2)/(2\gamma Z_{dis})$.

Thus, the equation describing the slow evolution of Q_ω can be obtained by taking the variation of the functional S:

$$i\frac{\partial Q_\omega(z)}{\partial z} - \omega^2 \frac{Z_{NL}}{Z_{dis}}\langle D(z)\rangle Q_\omega(z) + 2\int d\omega_1 d\omega_2 d\omega_3 F\delta(\omega_1 + \omega_2 - \omega - \omega_3) Q_{\omega_1} Q_{\omega_2} Q^*_{\omega_3} = 0 \tag{6}$$

Equation (6) governs the average dynamics of "breathing" pulses. It can also be derived by direct averaging of the original equation [12]. As it was shown in [12] Eq. (6) can be further simplified in the case $Z_{DR}/(\gamma Z_{dis} Z_{NL}) \ll 1$. In this limit, in the time domain, at the leading order, the equation describing the average dynamics of breathing solitons is again the NLSE:

$$iU_z + \frac{Z_{NL}}{Z_{dis}} U_{tt}\langle D(z)\rangle + \langle c(z)\rangle |U|^2 U = R_2. \tag{7}$$

Here $U = U(t,z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{i\omega t} Q_\omega(z) d\omega$ is the slowly-varying part of A. The term R_2 describes the higher-order corrections in Z_{dis}/Z_{DR} and Z_a/Z_{NL}.

An alternative approximate approach to describing breathing-pulse propagation is based on the direct use of the variational problem (2). Using the well-known solution of the linear problem we now substitute in the variational problem (2) the following trial function

$$A(z,t) = a(z) f\left[\frac{t}{b(z)}\right] \exp\left[i\lambda(z) + i\frac{\nu(z)}{b(z)}t^2\right], \tag{8}$$

here $f(x)$ is a function corresponding to the initial pulse profile. Using the Ritz optimization procedure we obtain the variational equations with respect to a, a^*, b, λ and ν. After some simple calculations (a similar approach has been used in other problems in [13], where more details of procedure can be found), we obtain the equations describing the oscillations of the pulse width $b(z)$ and the pulse chirp ν/b

$$b_z = \frac{4Z_{NL}}{Z_{dis}} D(z)\nu; \qquad \nu_z = \frac{Z_{NL} D(z) C_1}{Z_{dis} b^3} - \frac{c(z) N^2 C_2}{b^2}. \tag{9}$$

Here $D(z) = D_\mp$, $N^2 = a^2 b = const$, $C_1 = \int_{-\infty}^{+\infty} |f_x|^2 dx / (\int_{-\infty}^{+\infty} x^2 |f|^2 dx)$, $C_2 = \int_{-\infty}^{+\infty} |f|^4 dx / (4\int_{-\infty}^{+\infty} x^2 |f|^2 dx)$. For instance, for the soliton shape $f(x) = \mathrm{sech}(t)$, $C_1 = 2C_2 = 4/\pi^2$. To describe the propagation of the initial pulse in the form $A(0,t) = N\ rmsech(t)$, we fix as initial conditions to Eq. (9) $b|_{z=0} = 0$ and $\nu|_{z=0} = 0$. Equations (9) must be solved in the first $(D = D_-)$ and second $(D = D_+)$ fibers and at $z = z_c$ we require $b_- = b_+$ and $\nu_- = \nu_+$.

In the limit $Z_a, Z_{dis} \ll Z_{NL}$, one may again treat the nonlinearity as a perturbation. At the lowest order, fast oscillations of the pulse amplitude and width are given by the solution of the linear problem

$$\frac{d^2 B_\pm}{dz^2} = \frac{16 Z_{NL}^2 D_\pm^2(z)}{\pi^2 Z_{dis}^2 B_\pm^3}. \tag{10}$$

The solution of this equation has the form $B_\pm^2 = 1 + 16 R_\pm^2(z)/\pi^2$. It follows from here, in particular, that a small change of b over period due to residual dispersion is given by $b_{res} = |4\langle D(z)\rangle Z_a/(\pi Z_{dis})| \geq 0$.

To obtain the equation governing small changes of $b(z)$ due to nonlinearity, we linearize Eq. (9) about the linear solution $B_\pm$ assuming $b = B_\pm + \tilde{b}_\pm$ and $\tilde{b}_\pm \ll B_\pm$

$$\tilde{b}_z = \frac{4 Z_{NL}}{Z_{dis}} D(z)\tilde{\nu}; \qquad \tilde{\nu}_z = -\frac{12 Z_{NL} D(z)}{\pi^2 Z_{dis} B^4}\tilde{b} - \frac{2c(z)N^2}{\pi^2 B^2}. \tag{11}$$

Here and in what follows we drop $\pm$ to avoid complex notation. Initial conditions to Eq. (11) at $z = 0$ are: $\tilde{b} = 0$ and $\tilde{\nu} = 0$. Solution of Eq. (11) with these conditions is found as

$$\tilde{b}_- = -\frac{8 Z_{NL} D_- N^2 B_z}{\pi^2 Z_{dis}} \int_0^z \frac{dy}{B_y^2} \int_0^y \frac{c(x) B_x}{B^2} dx, \tag{12}$$

$$\tilde{b}_+ = r_1 B_z + r_2 B_z \int_{z_c}^z \frac{dy}{B_y^2} - \frac{8 Z_{NL} D_+ N^2 B_z}{\pi^2 Z_{dis}} \int_{z_c}^z \frac{dy}{B_y^2} \int_{z_c}^y \frac{c(x) B_x}{B^2} dx. \tag{13}$$

Coefficients r_1 and r_2 are determined by matching of these solutions at $z = z_c$. Resulting change of the pulse width due to nonlinearity over one period in the main order is given by

$$\tilde{b}_+(z_a) = \frac{8 N^2 Z_{NL}}{\pi^2 Z_{dis}} [D_- \int_0^{z_c} \frac{c(z) z}{B_-^3} dz - D_+ \int_{z_c}^{z_a} \frac{c(z)(z_a - z)}{B_+^3} dz] \tag{14}$$

In the case we consider here $D_- < 0$, $D_+ > 0$ and respectively $b_+(z_a) < 0$. One can see that this compression of a pulse ($b_+(z_a) < 0$) due to nonlinear effects can balance slight dispersive broadening of a pulse due to residual dispersion ($b_{res} > 0$) over one period.

Equations (9) give an approximate description of the breathing dynamics of optical pulses in cascaded optical systems with dispersion compensation. As was shown in [14], variational approach in the nondissipative NLSE is not applicable to describe interaction of soliton with radiation. Therefore, one should be very careful, making quantitative conclusions about soliton interaction with radiation from the developed above method. It should be combined with direct numerical simulations. We would like to note, that on the distances of tens amplifications distances (this case is of importance for Europe networks, for instance), this approximate description of the pulse evolution is in a good agreement with the results of numerical simulations [15]. A comprehensive analysis of the pulse dynamics described by Eqs.(9) will be presented in a subsequent publication [15].

In conclusions, we present a concept of a "breathing" average soliton in optical transmission systems. We have derived an averaged equation for the evolution of the slowly-varying envelope of the optical signal propagating in cascaded transmission lines with periodic amplification and dispersion compensation. In the span between two amplifiers, a pulse experiences strong attenuation and large-width oscillations.

We have presented an approximate variational approach that allows us to describe pulse amplitude and width oscillations without averaging. We have demonstrated that a bright soliton can propagate in the fiber system with segments of high normal dispersion under condition $Z_{DR}/(\gamma Z_{NL} Z_{dis}) \ll 1$.

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