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# ORBITAL MAGNETISM OF 2D CHAOTIC LATTICES

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We study the orbital magnetism of 2D lattices with chaotic motion of electrons within a primitive cell. Using the temperature diagrammatic technique we evaluate the averaged value and rms fluctuation of magnetic response in the diffusive regime within the model of non-interacting electrons. The fluctuations of magnetic susceptibility turn out to be large and at low temperature can be of the order of $\chi_L(k_F l)^{3/2}$, where $k_F$ is the Fermi wavevector, $l$ is the mean free path, and $\chi_L$ is the Landau susceptibility. In the certain region of magnetic fields the paramagnetic contribution to the averaged response is field independent and larger than the absolute value of Landau response.

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In the recent years, the orbital magnetism of disordered electronic systems has attracted much attention [1-5]. Generally speaking, three different situations can be considered: (i) clean systems (ballistic regime); (ii) weak disorder (diffusive regime); (iii) strong disorder (hopping regime). The magnetization in the case of strong disorder is due to the coherent electron tunneling between localized states [5]. The orbital magnetic response in the clean systems and its fluctuations arising from the different shapes of the boundaries have been considered in [4].

We study the orbital magnetism in the case of weak disorder ($k_F l \gg 1$). In the limit of classically weak magnetic field, when the cyclotron radius is larger than the electron mean free path, the structure of energy levels of mesoscopic sample is very sensitive to the impurity configuration. As a result, the fluctuations of magnetic response turn out to be larger than the disorder-averaged value, i.e., the magnetic response of mesoscopic sample has a random sign as a function of impurity configuration. This makes the investigation of sample-specific fluctuations to be of great importance for understanding of the orbital magnetism of mesoscopic systems.

In the work of Oh et al. [1], the most attention was paid to the extreme quantum coherence limit, $L \ll L_H, L_T, L_\phi$. Here $L$ is the linear size of sample, $L_\phi$ is the phase coherent length, $L_H = \sqrt{c\hbar/eH}$ is the magnetic length, $L_T = \sqrt{\hbar D/T}$ is the thermal length, $D$ is diffusion coefficient and $T$ is the temperature. Mesoscopic fluctuations in the limit $L_T < L$ were calculated by Raveh and Shapiro [2]. The averaged magnetic response in different regimes was considered by Altshuler et al. [3]. The authors of these works dealt with the case of random distribution of impurities all over the system.

In this letter, we study the orbital magnetism in 2D chaotic lattices, i.e., in periodic complex systems with chaotic motion of electrons within a primitive cell. As example of such systems, we consider 2D metallic sample (linear size $L$) consists of $N$ parts (linear size $a = L/\sqrt{N}$) with the same random configuration of impurities [7]. We assume that $a \gg l$, so that the motion of electrons within a primitive cell is diffusive and can be described by the diffusion constant $D$. The system under consideration is schematically shown in Fig.1.

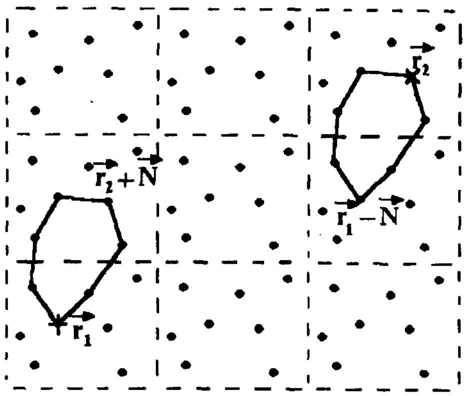


Fig.1. Schematic representation of chaotic lattice and example of the pair of phase-coherent electron paths

We evaluate the mesoscopic fluctuations of magnetic response from the correlation function of the thermodynamic potential $\Omega$. The latter is related to the correlation function of particles number $\mathcal{N}$ by the following expression:

$$\langle \delta\Omega(H_1)\delta\Omega(H_2)\rangle_{lat} = \int_{-\infty}^{\mu}\int_{-\infty}^{\mu} d\mu_1 d\mu_2 \langle \delta\mathcal{N}_1 \delta\mathcal{N}_2\rangle_{lat} , \tag{1}$$

where $\delta\mathcal{N}_i \equiv \delta\mathcal{N}(\mu_i; H_i)$, $\mu$ is the chemical potential, $H$ is the magnetic field. Symbol $\langle\,\rangle_{lat}$ means averaging over the different configurations of impurities which have the long-ranged correlation of their positions.

The correlation function of particles number is related to the correlation function of electron density $\rho(\mathbf{r})$ : $\langle \delta\mathcal{N}_1\delta\mathcal{N}_2\rangle_{lat} = \int_S\int_S d\mathbf{r}_1 d\mathbf{r}_2 \langle\delta\rho_1(\mathbf{r}_1)\delta\rho_2(\mathbf{r}_2)\rangle_{lat}$, where $S = L^2$ is the square of lattice. Scattering contribution to the electron density at some point $\mathbf{r}_1$ is due to the sum over the all possible scattering paths returning to $\mathbf{r}_1$. The correlation function $\langle\delta\rho_1(\mathbf{r}_1)\delta\rho_2(\mathbf{r}_2)\rangle_{lat}$ is determined only by the contribution of the pairs of phase-coherent electron paths (see, e.g. [6]). In the case of lattice, an electron traversing the closed path starting at the point $\mathbf{r}_1$ and passing near the point $\mathbf{r}_2 + \mathbf{N}$ acquires the same phase as an electron traversing the similar, but shifted in space by $\mathbf{N}$, path, as it is shown in Fig. 1. Here $\mathbf{N}$ is 2D vector with components $\{ai, aj\}$, where integers $i, j = 0...\sqrt{N} - 1$ numerate the primitive cells of lattice. Thus, calculating the sum over $\mathbf{N}$ we obtain

$$\langle \delta\mathcal{N}_1\delta\mathcal{N}_2\rangle_{lat} = N\langle\delta\mathcal{N}_1\delta\mathcal{N}_2\rangle_0 , \tag{2}$$

where $\langle\,\rangle_0$ means averaging over the paths shifted by $\mathbf{N} = 0$.

The correlation function $\langle\delta\mathcal{N}_1\delta\mathcal{N}_2\rangle_0$ can be conveniently calculated by the temperature diagrammatic technique. We use the diffusion approximation which is valid when $T > \Delta$, where $\Delta = (a^2\nu)^{-1}$ is the mean spacing between energy bands in chaotic lattice, $\nu$ being the mean density of states [7, 8]. The largest contributions to $\langle\delta\mathcal{N}_1\delta\mathcal{N}_2\rangle_0$ are the double-diffuson and double-Cooperon diagrams. We consider magnetic fields applied perpendicularly to 2D system and satisfying the condition $l \ll L_H$. We also assume that $L_T < L$. For this case the correlation function is

$$\langle\delta\mathcal{N}_1\delta\mathcal{N}_2\rangle_{\bar{0}} = \frac{ST}{\pi^2\hbar D}\sum_{\omega>0}^{1/\tau}\omega\mathrm{Re}\sum_{n\geq 0}\sum_{s=\pm}\omega_s\left(\omega+\hbar\omega_s(n+\frac{1}{2})+i\mu_{12}\right)^{-2}. \quad (3)$$

Here $\omega = 2\pi Tm$, $m$ is an integer number, $\mu_{12} = \mu_1 - \mu_2$, $\omega_\pm = 2De(H_1 \pm H_2)/c\hbar$, $c$ is the speed of light, $e$ is the electron charge, $n$ is the Landau level number. The upper cutoff in (3), $1/\tau$, is due to the validity of the diffusion regime. After substitution of (2) and (3) in (1) and some transformations we arrive at the following expression for the correlation function of thermodynamic potential in the form of $t$-dependent integral:

$$\langle\delta\Omega(H_1)\delta\Omega(H_2)\rangle_{lat} = \frac{NST^2}{\pi\hbar D}\int_0^\infty\frac{dt}{t^2\sinh^2(\pi Tt)}\left(\frac{\omega_+t/2}{\sinh(\omega_+t/2)}+\frac{\omega_-t/2}{\sinh(\omega_-t/2)}\right). \quad (4)$$

To obtain the correlation function of the magnetic moment, one has to successively differentiate (4) on $H_1$ and $H_2$. We would like to note that the integral in (4) converges only after differentiation on $H_1$ and $H_2$. The analytical form of the correlation function can be found in two limiting cases.

1) In the "high temperature" limit, $\hbar\omega_+ \ll T$, we obtain

$$\langle\delta M(H_1)\delta M(H_2)\rangle_{lat} = \frac{28}{5}\frac{\hbar D}{Ta^2}(k_Fl)^2\chi_L^2H_1H_2. \quad (5)$$

Here $\chi_L = -e^2S/12\pi m^*c^2$ is 2D Landau susceptibility, $m^*$ is an effective electron mass. The rms fluctuation of magnetic susceptibility can be directly obtained from (5):

$$\langle\delta\chi^2\rangle_{lat}^{1/2} = \frac{2\sqrt{7}}{\sqrt{5}}\frac{L_T}{a}k_Fl|\chi_L|. \quad (6)$$

If impurity positions are random within entire system (no long-range correlation, i.e., $N = 1$), $a$ should be substituted on $L$. In this case, Eq. (6) coincides (up to a numerical factor) with the result obtained by Raveh and Shapiro [2]. Here we would like to draw attention to the following peculiarity of chaotic lattice. We remind that Eq. (6) was obtained for the case $L_T/L < 1$. At the same time, the quantity $L_T/a = \sqrt{N}L_T/L$, appearing in the Eq. (6), can be large. Indeed, the validity of the perturbative approach is $T > \Delta$ and in the low temperature limit $T \sim \Delta$ we have [1)] $L_T/a \sim (\hbar D/\Delta a^2)^{1/2} \sim (\hbar D\nu)^{1/2} \sim (k_Fl)^{1/2}$. Therefore, the rms fluctuation of magnetic susceptibility in the case of lattice can be of the order of $(k_Fl)^{3/2}|\chi_L|$.

2) In the "high field" limit, $\hbar\omega_+ \gg T$, we have

$$\langle\delta M(H_1)\delta M(H_2)\rangle_{lat} = \frac{162\zeta(3)}{\pi^3}\frac{c\hbar}{ea^2}(k_Fl)^2\chi_L^2(|H_1+H_2|-|H_1-H_2|), \quad (7)$$

where $\zeta(x)$ is Riemann Zeta-Function. Such form of the correlation function indicates that the magnetic moment as well as the amplitude of its oscillations are growing functions of magnetic field $H$. It is also interesting to note that the correlation function $\langle\delta M(H_1)\delta M(H_2)\rangle_{lat}$ depends only on $\min\{H_1, H_2\}$.

[1)] We can consider the temperatures $T \sim \Delta$ when it does not contradict to the condition $L_T < L$ ($T > \Delta k_Fl/N$), i.e., for the lattices with $N > k_Fl \gg 1$.

Now let us calculate the canonically averaged magnetic response $\langle M\rangle_{lat}$. It can be expressed as the sum of the diamagnetic Landau response and a paramagnetic contribution: $\langle M\rangle_{lat} = \chi_L H + \langle M_p\rangle_{lat}$, where

$$\langle M_p\rangle_{lat} = -\frac{1}{2\nu S}\frac{\partial}{\partial H}\langle\delta\mathcal{N}^2\rangle_{lat} . \tag{8}$$

For complete discussion about the difference between canonical and grand-canonical situation see [1, 3] and references therein. Note that in the case of lattice the number of electrons in a primitive cell is fixed. Calculating the correlation function $\langle\delta\mathcal{N}^2\rangle_{lat}$ (Eqs. (2) and (3) at $\mu_1 = \mu_2 = \mu$ and $H_1 = H_2 = H$) we finally obtain

$$\langle M_p\rangle_{lat} = \begin{cases} \frac{1}{4\pi}\frac{\Delta}{T}k_F l|\chi_L|H \text{ , if } \hbar\omega_H \ll T \\ \frac{\ln 2}{2\pi^3}\frac{\Delta S e}{\hbar c} \text{ , if } \hbar\omega_H \gg T. \end{cases} \tag{9}$$

Here $\omega_H = 4DeH/c\hbar$ is the Cooperon cyclotron frequency. In the weak magnetic fields, $\hbar\omega_H \ll T$ ($L_H \gg L_T$) the averaged magnetic response is paramagnetic and grows linearly with $H$. In stronger magnetic fields, $\hbar\omega_H \gg T$ ($L_H \ll L_T$), the paramagnetic contribution $\langle M_p\rangle_{lat}$ is $H$ - independent. As one can see from (9), the paramagnetic contribution $\langle M_p\rangle_{lat}$ is larger than the absolute value of diamagnetic Landau response $|\chi_L H|$ until magnetic fields $\hbar\omega_H \sim \Delta k_F l$ ($L_H \sim a$). We note that in the case of small metallic particles the paramagnetic contribution $\langle M_p\rangle$ becomes $H$-independent only in the fields, where it is small compared to the Landau response [3].

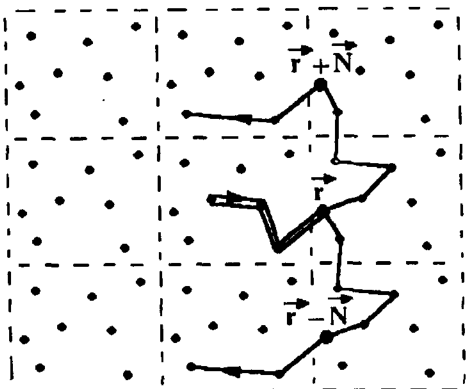


Fig. 2. Fragment of the phase-coherent electron trajectories giving rise a correction to the diffusion coefficient

In conclusion, we discuss the approximation that we used in derivation of (3). In the calculation of diffusion propagators we neglected a contribution from another type of phase-coherent electron paths (fragment of such trajectories is shown in Fig. 2). The contribution from the paths shifted by 2N is of the order of weak localization correction to the diffusion constant

$$\frac{\delta D(2\mathrm{N})}{D} \sim \frac{1}{k_F l}W(\mathrm{N},\omega) ,$$

where $W(\mathrm{R},\omega)$ is the probability for an electron to travel on the distance R during a time $\omega^{-1}$. Because the number of such trajectories is of the order of

$(L_T/a)^2$, the total contribution $\sum_N \delta D(2N)/D \sim (L_T/a)^2/k_F l \sim \Delta/T$. For $T > \Delta$ we can neglect it. We note that this correction depends on the magnetic field. This dependence contains not only a monotonous part, but also a part which oscillates with the period $\Phi_c/\Phi_0$, where $\Phi_c = Ha^2$ is the magnetic flux through a unit primitive cell, $\Phi_0 = hc/e$.

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