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FERMIONIC STRING FROM ABELIAN HIGGS MODEL WITH MONOPOLES AND Θ-TERM

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The four dimensional abelian Higgs model with monopoles and Θ-term is considered in the limit of the large mass of the higgs boson. We show that for $\Theta = 2\pi$ the theory is equivalent, at large distances, to summation over all possible world-sheets of fermionic strings with Dirichlet type boundary conditions on string coordinates.

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There are processes in quantum field theory in which a particle description is not convenient. The examples are QCD at low energies [1] and some astrophysical phenomena in early universe [2]. Due to that it is interesting to study string theories which follow from field theories.

For example, the quantum theory of Abrikosov–Nielsen–Olesen (ANO) strings can be obtained from the abelian Higgs model (AHM) [3–6]. It occurs that for thin ANO strings the theory is local. The effective action for thin ANO strings contains a rigidity term [7] with the negative sign.[2)] The string theory with Nambu–Goto and rigidity term may have problems with unitarity, presence of tachyon in the spectrum and crumpling of the string world-sheet (see e.g. review [8]). But such problems are absent for the Neveu–Schwarz–Ramond (NSR) string in ten dimensions. This string is equivalent (in the stable point of the β-function for the rigidity term) [9] to the string theory with the action containing the rigidity term plus topological Wess–Zumino–Novikov–Witten (WZNW) term. In the present paper we give an example how a similar theory can be obtained from the four-dimensional AHM with monopoles and Θ-term in the limit of the big mass of the Higgs boson.

We start with the following partition function in the Euclidian space time[3)]:

$$Z = \int [\mathcal{D}\tilde{z}_\mu]\mathcal{D}A_\mu\mathcal{D}\Phi \exp\Big\{ -\int d^4x \Big[\frac{1}{4}\Big(F_{\mu\nu} + \bar{F}_{\mu\nu}(\tilde{z})\Big)^2 + \\ +\frac{1}{2}|D_\mu\Phi|^2 + \lambda(|\Phi|^2 - \zeta^2)^2 + \\ +i\frac{\Theta e^2}{32\pi^2}\epsilon_{\mu\nu\alpha\beta}\Big(F_{\mu\nu} + \bar{F}_{\mu\nu}(\tilde{z})\Big)\Big(F_{\alpha\beta} + \bar{F}_{\alpha\beta}(\tilde{z})\Big)\Big]\Big\}, \qquad (1)$$

where

$$D_\mu = \partial_\mu - ieA_\mu - ie\bar{A}_\mu,$$

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[2)]The sign of the rigidity term depends on the type (first or second) of the superconductor [4].

[3)]Below we assume a lattice regularization [3] and, in formula (4), we take the naive continuum limit.

$$\epsilon_{\mu\nu\alpha\beta}\partial_\nu \tilde{F}_{\alpha\beta}(\tilde{z}) = \frac{4\pi}{e}\int_C d\tilde{z}_\mu \delta^{(4)}(x-\tilde{z}), \quad \partial_{[\mu}\tilde{A}_{\nu]} = \tilde{F}_{\mu\nu}, \tag{2}$$

and $j_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\alpha\beta}\partial_\nu \tilde{F}_{\alpha\beta}(\tilde{z})$ is the conserving monopole's current: $\partial_\mu j_\mu = 0$, $\tilde{z}_\mu$ is a position of the monopole; $\int[\mathcal{D}\tilde{z}_\mu]$ is the functional integral over all closed paths, the measure is well known, see e.g. [1]; C are the trajectories of the monopoles defined by $\tilde{z}_\mu$. $\Phi = |\Phi|e^{i\theta}$ is the Higgs field with the standard integration measure: $\mathcal{D}\Phi = \mathcal{D}\mathrm{Re}\Phi\mathcal{D}\mathrm{Im}\Phi = [|\Phi|\mathcal{D}|\Phi|]\mathcal{D}\theta$.

The theory (1) can be considered as the low energy limit of the $SU(2)$ Georgy–Glashow model with the Θ-term and with the additional breaking of the gauge $U(1)$ symmetry. This model is known to have ANO strings and 't Hooft–Polyakov monopoles as the solutions of the classical equations of motion [2]. At the low energy, monopoles can be considered as Wu–Yang type ambiguities in the gauge potential A_μ [1]. In (1) we explicitly write these ambiguities as $\tilde{F}_{\mu\nu}(\tilde{z})$.

Since in the center of the ANO strings $\mathrm{Im}\Phi = \mathrm{Re}\Phi = 0$ the phase θ is singular on the two dimensional surfaces, which are world-sheets of ANO strings. The character of the singularity is:

$$\begin{aligned}\partial_{[\mu,}\partial_{\nu]}\theta^s(x,\tilde{x}) &= 2\pi\epsilon_{\mu\nu\alpha\beta}\Sigma_{\alpha\beta}(x,\tilde{x}),\\ \Sigma_{\alpha\beta}(x,\tilde{x}) &= \int_{\Sigma+\Sigma_C} d^2\sigma\epsilon^{ab}\partial_a\tilde{x}_\alpha\partial_b\tilde{x}_\beta\delta^{(4)}[x-\tilde{x}(\sigma)],\end{aligned} \tag{3}$$

where Σ and Σ_C are collections of all closed surfaces and surfaces opened on monopole's world–lines C.

Using the Bianci identity ($\epsilon_{\mu\nu\alpha\beta}\partial_\nu F_{\alpha\beta} = 0$) and conservation of the monopole current ($\partial_\mu j_\mu = 0$) we can rewrite the Θ-term as $\frac{\Theta e^2}{2\pi} j_\mu A_\mu$ [10], which is the interaction of the electric charge of the dyon with the gauge field.

In eq. (1) $\mathcal{D}\theta$ contains the integration over functions which are singular on two–dimensional manifolds (3), and we subdivide θ into the regular θ^r and the singular θ^s parts: $\theta = \theta^r + \theta^s$; θ^s is defined by eq. (3). To simplify the calculations we consider the London limit ($\lambda >> 1$), in this case the radial part $|\Phi|$ of the Higgs field Φ is fixed $|\Phi| = \zeta$ and $\mathcal{D}\theta = \mathcal{D}\theta^r\mathcal{D}\theta^s$. After the change of variables from θ^s to $\tilde{x}_\mu$ and integration over A_μ and θ^r in (1), we get [6]:

$$\begin{aligned} Z = \mathrm{const}\int[\mathcal{D}\tilde{z}_\mu][\mathcal{D}\tilde{x}]J(\tilde{x})\cdot \\ \cdot\exp\Bigg\{-\int \mathrm{d}^4x\int \mathrm{d}^4y\Bigg[\pi^2\zeta^2\Sigma_{\mu\nu}(x)\mathcal{D}_m^{(4)}(x-y)\Sigma_{\mu\nu}(y) + \\ +\left(\left(\frac{\Theta e}{2\pi}\right)^2 + \frac{1}{4e^2}\right)j_\mu(x)\mathcal{D}_m^{(4)}(x-y)j_\mu(y) + \\ +\frac{\Theta e}{2\pi}j_\mu(x)\mathcal{D}_m^{(4)}(x-y)\partial_\nu\epsilon_{\mu\nu\alpha\beta}\Sigma_{\alpha\beta}(y)\Bigg] + \\ +i\Theta\mathbb{L}(\Sigma,C) + i\Theta\mathbb{L}(\Sigma_C,C)\Bigg\}, \end{aligned} \tag{4}$$

where $\tilde{x}_\mu$ is the position of the string, $[\mathcal{D}\tilde{x}_\mu]$ assumes both integration over all possible positions and summation over all topologies of the string's world-sheets Σ

and Σ_C; $\mathcal{D}_m^{(4)}(x-y)$ is the Green's function: $(\Delta+m^2)\mathcal{D}_m^{(4)}(x-y)=\delta^{(4)}(x-y)$, $m^2 = e^2\zeta^2$ is the mass of the gauge boson; $J(\tilde{x})$ is the Jacobian of the transformation from the field θ^s to the string position $\tilde{x}_\mu$. $J(\tilde{x})$ was estimated in [6] for string with the topology of a sphere or of a disk. This Jacobian contains a term which cancels the conformal anomaly coming from Nambu–Goto action in four dimensions. The mechanism of cancelation is the same as suggested in [11].

Boundary condition for the open strings in theory (4) is: $\partial_\mu\Sigma_{\mu\nu} = j_\nu$, which leads to the Dirichlet condition $\tilde{x}_\mu(s)|_C = \tilde{z}_\mu(s)$.

First three terms in the exponent in eq. (4) describe the interaction and the self interaction of strings and dyons through exchange of the massive gauge bosons. The last two terms: $\mathbb{L}(\Sigma, C)$ and $\mathbb{L}(\Sigma_C, C)$ describe the topological interaction of strings and dyons,

$$\mathbb{L}(\Sigma, C) = \frac{1}{4\pi^2}\int_C d\tilde{z}_\alpha \int_\Sigma d^2\sigma\epsilon^{ab}\partial_a\tilde{x}_\mu\partial_b\tilde{x}_\nu\epsilon_{\mu\nu\alpha\beta}\partial_\beta\frac{1}{|\tilde{x}-\tilde{z}|^2} \tag{5}$$

is the four-dimensional Gauss linking number of the world-sheet of the closed string Σ and of the dyon path C. This term is a four-dimensional analogue of the Aharonov–Bohm interaction of the strings and dyons discussed in [6, 12]. The string behaves like a solenoid which scatter the dyon. The other topological interaction

$$\mathbb{L}(\Sigma_C, C) = \frac{1}{4\pi^2}\int_C d\tilde{z}_\alpha \int_{\Sigma_C} d^2\sigma\epsilon^{ab}\partial_a\tilde{x}_\mu\partial_b\tilde{x}_\nu\epsilon_{\mu\nu\alpha\beta}\partial_\beta\frac{1}{|\tilde{x}-\tilde{z}|^2} \tag{6}$$

is the generalization to the four dimensions of a similar interaction of open paths of the particles in three dimensions [13]. Formally it is equal to zero because there is no linking between open surface and closed path. But since the dyon trajectory C coincides with the boundary of the world-sheet of the ANO string Σ_C, the integral in (6) should be regularized. At each point of the curve C we define a tangent vector $e^1_\mu(s^1) = \partial_{s^1}\tilde{y}_\mu(s^1)$; the vector $e^2_\mu(s^1)$ which is orthogonal to $e^1_\mu(s^1)$ and tangent to the surface Σ_C, and two vectors $n^a_\mu(s^1)$ $a = 1, 2$ which are orthogonal to the $e^a_\mu(s^1)$ [14]. Consider the path C_ϵ which is the shift of the path C along one of the normals (for example[4] $n^1_\mu(s^1)$) from the border of the surface Σ_c by a distance ϵ. Now we define $\mathbb{L}(\Sigma_C, C) = \lim_{\epsilon\to 0}\mathbb{L}(\Sigma_C, C_\epsilon)$, and it is easy to find:

$$\mathbb{L}(\Sigma_C, C) = -\frac{1}{4\pi}\int_0^L ds^1\epsilon_{\mu\nu\alpha\beta}\partial_{s^1}n^1_\alpha(s^1)e^1_\mu(s^1)e^2_\nu(s^1)n^1_\beta(s^1), \tag{7}$$

where L is the length and s^1 is a parametrization of the boundary C. If in (7) we consider the closed surface Σ and the path C is lying on this surface then the expression in the RHS can be easily represented in WZNW form [14]. To consider the open surface Σ_C it is convenient to introduce (as it was done in [15]) the following three form:

$$\Omega_{ijk} = \epsilon_{\mu\nu\alpha\beta}e_\mu\partial_i e_\nu\partial_j e_\alpha\partial_k e_\beta, \tag{8}$$

[4]The choice of normals is unimportant even in the case of open strings [14].

defined on some three dimensional compact manifold B with boundary containing the surface Σ_C. In the last expression e_μ is an extension, to the manifold B, of the vector $e_\mu(s^1,s^2) = \cos(s^2)e^1_\mu(s^1)+\sin(s^2)e^2_\mu(s^1)$, tangent to Σ_C (later we assume the similar extension of the vectors n^a_μ); $\partial_i = \frac{\partial}{\partial s^i}$, $i,j,k = 1,2,3$ and s^3 is the additional to s^a coordinate on B. Since $(e_\mu)^2 = 1$ by construction, the three form (8) is closed, $\partial_{[l}\Omega_{ijk]} = 0$, and one can represent Ω_{ijk} locally as $\partial_{[k}\Lambda_{ij]} = -\Omega_{ijk}$ where Λ_{ij} is a squesymmetric two form. So that $I\!\!L(\Sigma_C, C)$ is:

$$I\!\!L(\Sigma_C, C) = \frac{1}{8\pi^2}\int_{\Sigma_C} d^2s\epsilon^{ab}\Lambda_{ab}, \quad a,b = 1,2 \tag{9}$$

Λ_{ab} is defined up to the transformation $\Lambda_{ab} \to \Lambda_{ab} + \partial_{[a}E_{b]}$. But since Σ_C is an open surface the expression (9) is not invariant under this transformation. This ambiguity affects only a boundary terms of the string: dyon's theory. But our further discussion is independent on such terms.

Consider the part of the string theory (4) corresponding to a surface Σ_C with the topology of the disc. If $e^2 > \lambda$ this string theory is local and contains the rigidity term with the positive sine [4], which is important for the consistency of the quantum string theory [7]:

$$S(\Sigma_C) = \eta\int_{\Sigma_C}\sqrt{g}d^2s + k\int_{\Sigma_C}\sqrt{g}\Big(\Delta(g)\tilde{x}_\mu\Big)^2 d^2s - \\ -i\Theta I\!\!L(\Sigma_C, C) - \ln J(\tilde{x}) + O(\frac{1}{m^2}), \tag{10}$$

here string tension η and rigidity $k > 0$ are some coefficients [4, 6].

The WZNW term $I\!\!L(\Sigma_C, C)$ is defined by (9) for a particular choice of the reference system e^a_μ and n^a_μ (parametrization of the world-sheet Σ_C). To get this term for an arbitrary reference system we can rotate it to any other position by some $SO(4)$ matrix. The last matrix is defined up to $SO(2)$ local rotations on the string world-sheet and up to $SO(2)$ local rotations in the space orthogonal to the world-sheet. The last $SO(2)$ invariance is defined by the following matrix h [9]: consider the gauge field $A^n = n^a_\mu dn^b_\mu M^{ab}$, where M^{ab} is the generator of the $SO(2)$ rotations. The definition of h is the following: $d_+(h^{-1}d_-h) = dA^n + A^n\wedge A^n$. By this way the theory (10) can be represented as a gauged $SO(4)/SO(2)SO(2)$ WZNW model, which is related to the geometrical quantization on the group orbits. In this model the β–function for the rigidity coefficient k acquires IR stable point and the rigidity term is relevant in the IR [9] (see also [16]). For $\Theta = 2\pi$ this point corresponds to $k = 1/2$. So for $\Theta = 2\pi$ we get the fermionic string action considered in [9, 17] for four dimensions and vector representation:

$$S(\Sigma_C) = \eta\int_{\Sigma_C}\sqrt{g}d^2\sigma + \frac{1}{2}\int_{\Sigma_C}\sqrt{g}\Big(\Delta(g)\tilde{x}_\mu\Big)^2 d^2\sigma + \frac{1}{2}\int_{\Sigma_C}(e^a_\mu\partial_a e^b_\mu)^2 d^2\sigma + \\ +\frac{i}{16\pi}\int_{\Sigma_C} tr(h^{-1}dh)^2 d^2\sigma + \frac{i}{24\pi}\int_B tr(h^{-1}dh)^3 d^3s - \ln J(\tilde{x}) + O(\frac{1}{m^2}). \tag{11}$$

Due to the existence of IR stable point there is no crumpling of the string world-sheets in the theory [8]. Probably there is also no tachyon in the theory, if crumpling and existence of tachyon are related. Moreover in the functional integral for the theory (11) there is an integration over $\mathcal{D}\tilde{x}_\mu(\sigma)$ which should be defined by the introduction of the intrinsic metric [1]. Due to the existence of the IR stable point of the rigidity coefficient the obtained string theory does not coincides with the standard Liouville or super Liouville theories [7].

The considered effect of appearance of the spin of the strings from bosonic theory is general for any string with dyon on it's boundary. This phenomenon is the mechanism of fermi-bose transmutation for strings in four dimensions.

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