Pis'ma v ZhETF, vol.64, iss.4, pp.283 - 288

SIGN PHASE TRANSITION AND DIRECTED PATHS IN RANDOM MEDIA

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Submitted 22 July 1996

We consider a lattice model which corresponds to the high temperature expansions of disordered Ising and Heisenberg models and to the deeply localized regime of the disordered Anderson model. The spin correlation functions for the Ising and Heisenberg model and the amplitude of electron tunneling for the Anderson model exhibit a "sign phase transition". At small concentration x of scatterers with a negative scattering amplitude these quantities have predictable signs while at large x their signs are unpredictable.

PACS: 05.20.-y, 82.20.-w

This letter represents a discussion of the following lattice model, which was proposed in [1,2]. Let us consider directed paths labeled by the index Γ on the lattice shown in Fig.1*a*. The amplitude associated with a directed path Γ on the lattice is defined as

$$A_\Gamma = \prod_{k\in\Gamma} \alpha_k, \tag{1}$$

where α_k's are independent random variables associated with each site k

$$\alpha_k = \begin{cases} 1 & \text{with probability} \quad 1-x, \\ M & \text{with probability} \quad x, \end{cases} \tag{2}$$

where M is a real number and $x < 1$ is the concentration of "scatterers". The index k in Eq.(1) runs over lattice sites along the path Γ.

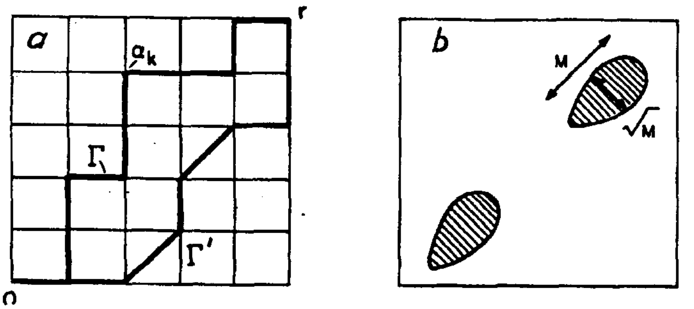


Fig.1. *a*) The lattice model. Γ and Γ' are pathes in the lattice, the index k labels the sites on the lattice. *b*) The qualitative picture on the distribution of signs of $A(\mathbf{r})$, shaded areas correspond to negative sign of $A(\mathbf{r})$

We are interested in the statistical properties of the quantity

$$A = \sum_{\Gamma} A_{\Gamma} \quad (3)$$

where the sum is taken over all directed paths Γ connecting points 0 and r (See Fig.1a).

The amplitude Eq.(3) is directly related with the electron tunneling amplitude in the deep localized regime $\frac{I}{\epsilon_0} \ll 1$ of the Anderson model [1,2] described by the Hamiltonian

$$H = \sum_{k} \epsilon_k a_k^{\dagger} a_k + \sum_{k,k'} I_{k,k'} a_k^{\dagger} a_{k'} \quad (4)$$

Here $a_k^{\dagger}$ and a_k electron are creation and annihilation operators, $I_{k,k'}$ are hopping integrals, different from zero and equal to I only for nearest neighbor sites k, k' and ϵ_k are electron energies, associated with the lattice sites, which are equal to $\epsilon_0 > 0$ and ϵ_0/M with the probabilities $1 - x$ and x respectively. If $I/\epsilon_0 \ll 1$, the paths on the lattice containing loops and returns give an exponentially small contribution to the total probability of tunneling and can be neglected. As a result, the amplitude of probability $A_{0,r}$ for an electron with zero energy $\epsilon = 0$ to tunnel between points 0 and r is $A_{0,r} = (I/\epsilon_0)^n A$. Here n is the number of directed steps on the lattice between points 0 and r. The amplitude Eq.3 appears also when calculating the spin correlation function $K(\mathbf{r_k} - \mathbf{r'_k}) = \langle S(\mathbf{r_k}) S(\mathbf{r'_{k'}}) \rangle$ for the Ising model with the Hamiltonian

$$H = -\sum_{k,k'} J_{k,k'} S(\mathbf{r_k}) S(\mathbf{r_{k'}}) \quad (5)$$

at high temperatures $T \gg max(J, |M|J)$ (See for example [3,4]). Here $S(\mathbf{r_k})$ are spin operators on sites k, $\mathbf{r_k}$ are the coordinates of the sites k and the angle brackets stand for thermodynamic averaging. We assume that exchange integrals $J_{k,k'}$ are nonzero only for nearest neighbor spins and equal J or JM with the probabilities x or $1 - x$ respectively ($J > 0$). In this case the index k in Eq.(2) is runing over bonds along the path Γ and we have $K = (J/T)^n A$. A similar result can be obtained in the case of the Heisenberg model.

The model described above is also related with the directed polymer problem [5-8] and with the interfaces problem in random media [9-11].

Let us introduce probabilities $P_+(\mathbf{r})$ and $P_-(\mathbf{r})$ for the random quantity $A(\mathbf{r})$ to be positive and negative respectively

$$P_+(\mathbf{r}) = \int_0^{\infty} F(A, \mathbf{r}) \mathrm{dA}, \quad (6)$$

$$P_-(\mathbf{r}) = \int_{-\infty}^{0} F(A, \mathbf{r}) \mathrm{dA}, \quad (7)$$

where $F(A, \mathbf{r})$ is the distribution function of $A(\mathbf{r})$. It was suggested in [1,2] that in the case $M < 0$, $d \geq 2$ there exists a "sign phase transition" in the following sense. For $x < x_c$ the sign of the random quantity $A(|\mathbf{r}| = \infty)$ is predictable and $\delta P(\infty) = P_+(\infty) - P_-(\infty) > 0$, while for $x > x_c$ the sign of $A(\infty)$ is unpredictable since $\delta P(\infty) \equiv 0$. Here x_c is a critical concentration of scatterers with $\alpha_k = M$ and d is the dimensions of space. The qualitative picture of the transition in

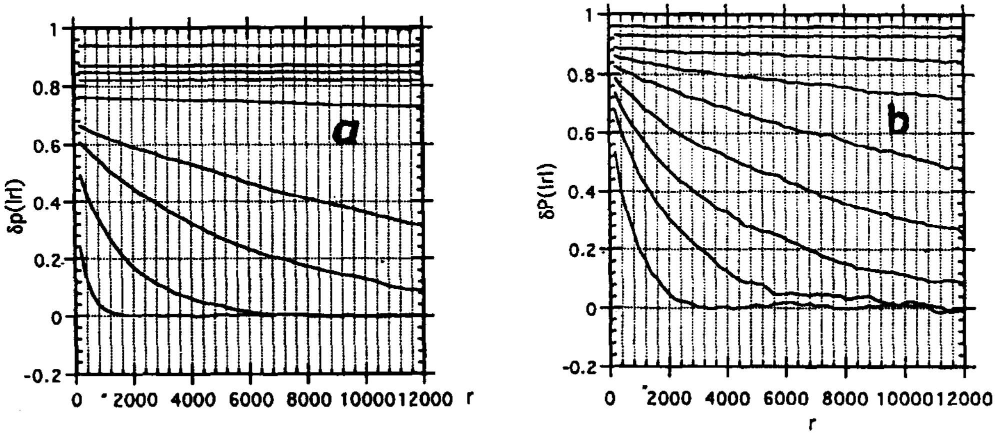


Fig.2. The dependence $\delta P(|\mathbf{r}|)$ in $d=2$ case. *a*) $M=-0.1$, $x=0.03$, 0.06, 0.07, 0.08, 0.1, 0.125, 0.135, 0.15, 0.175 from top to bottom respectively. *b*) $M=-1.5$, $x=0.004$, 0.006, 0.008, 0.01, 0.012, 0.013, 0.015 from to bottom respectively

the case $|M|>1$ is the following.[1,2]. A scatterer with the scattering amplitude $M<0$ creates a cigar shaped region, shown in Fig.1*b* where the sign of $A(\mathbf{r}')$ is negative. The typical lengths of this region are of order $|M|$, the widths of order $\sqrt{|M|}$ and the volume of order of $v=|M|^{(d+1)/2}$. In the case of a small concentration of scatterers, $B=x|M|^{(d+1)/2}<<1$, the regions created by different scatterers do not overlap and occupy only a small fraction of the total volume, which implies that $\delta P(\infty)>0$, while for $B\gg 1$ these regions overlap most of the time and $\delta P(|\mathbf{r}|)$ asymptotically approaches zero as $|\mathbf{r}|\to\infty$.

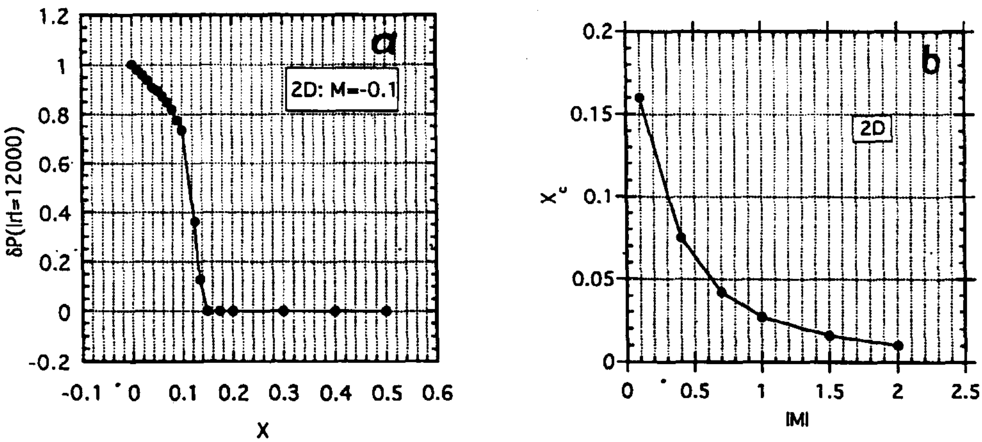
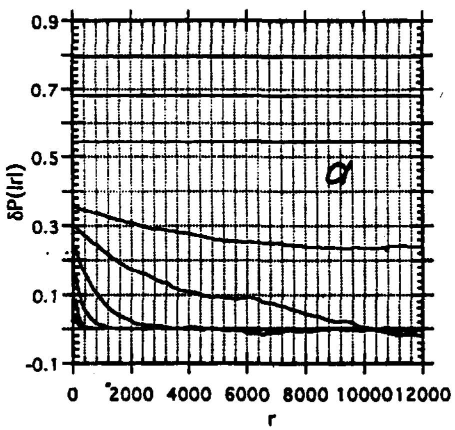


Fig.3. *a*) The dependence $\delta P(\infty,x)$ in $d=2$ case for $M=-0.1$. *b*) The dependence $x_c(|M|)$ in the $d=2$ case

In this paper we present numerical simulations which lend support to the idea of such a "sign phase transition". The results of numerical simulations for $\delta P(|\mathbf{r}|)$ as a function of $|\mathbf{r}|$ in $d=2$ and $d=3$ cases for different values of M and x



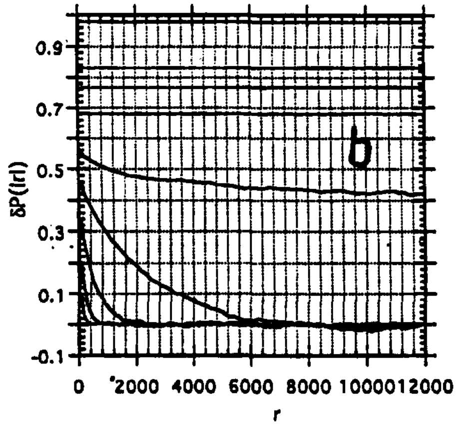
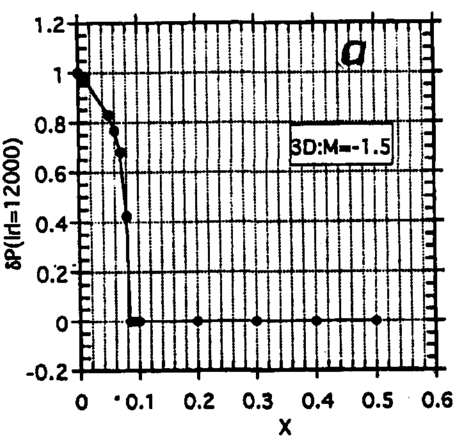


Fig.4. The dependence $\delta P(|\mathbf{r}|)$ in $d=3$ case. *a*) $M=-0.4$, $x=0.1$, 0.15, 0.2, 0.25, 0.26, 0.27, 0.28, 0.29,030 from top to bottom respectively. *b*) $M=-1.5$, $x=0.01$, 0.05, 0.06, 0.07, 0.8, 0.085, 0.09, 0.095, 0.1 from top to bottom respectively

are shown in Figs. 2, 4. It follows from these results that at small enough x, $\delta P(|\mathbf{r}|)$ asymptotically approaches nonzero values $P(\infty, x, M)$, which decrease as x and $|M|$ increase. At large enough x, $\delta P(|\mathbf{r}|)$ asymptotically tends to zero. These features a characteristic for the sigh phase transition.

The example of the dependences $\delta P(|\mathbf{r}|=12000, x)$ for $d=2$, $M=-0.1$ and $d=3$, $M=-0.4$ are shown in Figs.3*a*, 5*a*. We think that $P(|\mathbf{r}|=12000, x)$ is a good approximation for $P(\infty, x)$. The dependences $x_c(|M|)$ for d=2 and d=3 cases are shown in Figs.3*b*, 5*b*. respectively.



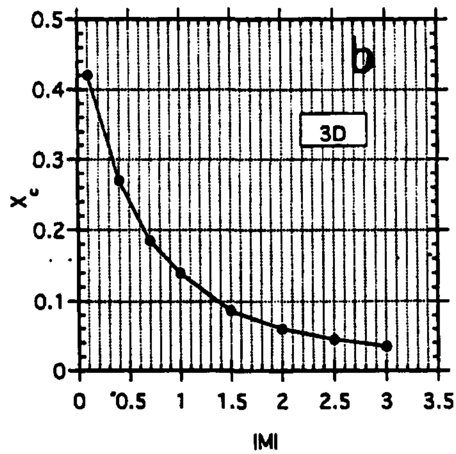


Fig.5. *a*) The dependence $\delta P(\infty, x)$ for the case $M=-0.4$, *b*) The dependence $x_c(|M|)$ in $d=3$ case

The main difference between the numerical results presented here and the qualitative picture of the "phase transition" proposed in [1,2] is that the "sign phase transition" turns out to be of the "first order". $\delta P(\infty)$ has a jump at $x = x_c$ from a finite value $\delta P(\infty, x = x_c - 0) > 0$ to zero. One can see this for example from the absence in Fig.4a (the case d=3 and M=-0.4) curves with nonvanishing at $|\mathbf{r}| \to \infty$ limits at levels below $\delta P(\infty) = 0.52$.

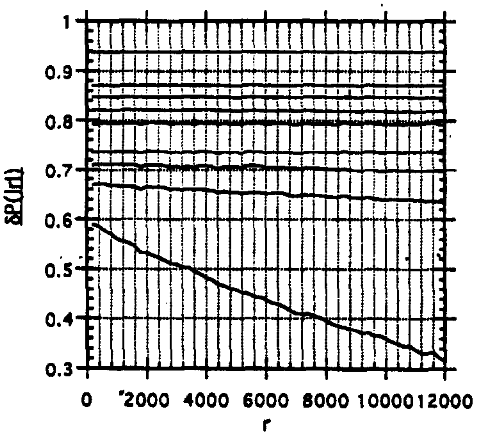


Fig.6. The dependences $\delta P(|\mathbf{r}|)$ in the case $d = 2$, $M = -0.1$ in the model, when pathes Γ' on the lattice Fig.1 go both along bonds and along diagonals; $x = 0.03$, 0.06, 0.07, 0.08, 0.1, 0.125, 0.135, 0.15, 0.175 from top to bottom respectively

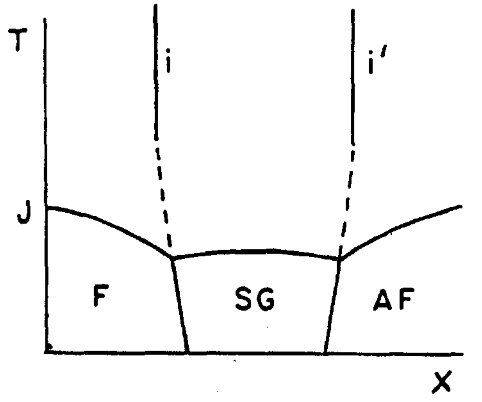


Fig.7. The phase diagram corresponding to the spin Hamiltonian Eq.(5), F, SG, and AF stand for ferromagnetic, spin glass and antiferromagnetic phases respectively

A possible reason why numerical simulations performed in [11-13] did not exhibit the "sign phase transition" is that in the two-dimensional case the values of $(\delta P(\infty, x_c - 0) - 1)$ turn out to be relatively small. For example in the case $d = 2$ and $M = -1.5$ we have $(1 - \delta P(\infty, x_c - 0)) \sim 0.1$.

In order to check the universality of above considered phenomenon we have considered a modification of the model Eq.(1), (3) when directed pathes can go not only along the bonds of the lattice but also along diagonals. An example of such a path Γ' is shown in Fig.1. The results of numerical simulations for $\delta P(|\mathbf{r}|)$ for this case are shown in Fig.6 and we can see the existence of the "sign phase transition" with a different x_c however.

The "sign phase transition" manifests itself in the appearence of the line i on the $(T - x)$ phase diagram for the spin Hamiltonian Eq.(5). This line at $T >> J$ divides "sign ordered" phase (when $\delta P(\infty) > 0$ and the sign of $K(\infty)$ is predictable) from the sign disordered p[hase (when $\delta P(\infty) = 0$ and the sign of $K(\infty)$ is unpredictable)) [14] (see.Fig.7).

In the particular case when $M = -1$ and $x = 1$ the sign of $K(\infty)$ is trivially predictable to be $(-1)^n$. It is convenient to introduce the quantity $K' = (-1)^n K$ and the corresponding line i' in Fig.7 which divides "sign ordered" and "sign disordered" phases for this new quantity $K'(\infty)$. It is known that at low temperatures $T < J$ in $d = 3$ case the system described by the Ising Hamiltonian has three phases: ferromagnetic, spin glass and antiferromagnetic[15]. Since the lines i and i' divide phases with different symmetries they can end only on the axis or on lines separating different phases. Therefore it is natural to assume that the lines i and i' turn into lines which divide ferromagnetic, spin glass and antiferromagnetic phases as shown in Fig.7. In dimentions lower than the lower critical one the spin glass phase doesnot exist [15]. This corresponds to the absence of the horisontal line in Fig.7, which divides spin glass and paramegnetic sign disordered phases.

Another consequence of the sign phase transitions is that the magnetoresistance of strongly localized conductors in variable range hopping regime changes its sign from positive (in sign ordered regime) to negative (in sign disordered regime) [1,2] as the concentration of scatterers increases.

We suspect that the "sign phase transition" also manifests itself in singularities of $F(A)$ at asymptotically large |r|. Another open question is whether such a behavior of the distribution function $F(A)$ also exists in the case $M > 0$.

At last we would like to mention the moments $\ll |A|^m \gg$ with $m \geq 1$ are determined by the asymptotic tail of the distribution function $F(A)$. Here the brackets $\ll\gg$ represent averaging over random realizations of α_k. One can see it from the fact that $\ll A^m \gg$ increase with m much faster than $\exp(m)$ [11,12]. This means that the form of the distribution function $F(A)$ can not be recovered from its momentums [16] and they cannot provide information about the phase transition. In particularly the first moment $\ll A \gg = (1 - 2xM)^n$ does not exhibit any singularities.

The authors would like to acknowledge useful discussions with A.Bulgac and B.I.Shklovskii. This work was partially supported by Division of Material Sciences US National Science Foundation under contract DMR-9205144.

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