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$n-\bar{n}$ TRANSITIONS IN NUCLEI AND MIXING OF NUCLEAR STATES WITH A AND $A-2$[1)]

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The new lower limit on the period of $n-\bar{n}$ oscillations $\tau_{n\bar{n}} > (8 \div 11)\cdot 10^8$ sec which follows from stability of Fe is found. This limit is almost by 10 times larger than previous estimations.

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In this paper I present new lower limit on the period of $n-\bar{n}$ oscillations which follows from stability of nuclei. This limit is considerably larger than previous estimations [1-8]. In Sect. 1 I consider the mixing of sectors with $A=2$ and $A=0$ in deuterium to emphasize once more the difference of $n-\bar{n}$ oscillations in medium and in free space. In Sect. 2 the deuteron decay due to the $n-\bar{n}$ transition is analyzed. The $n-\bar{n}$ oscillations in nuclei are considered in Sect. 3 with a special attention to the binding effect. The new limit on $\tau_{n\bar{n}}$ from stability of Fe is presented.

1. Mixing of sectors with $A=2$ and $A=0$ in deuterium. When the $n-\bar{n}$ transition occurs inside a deuteron, the latter can decay into hadrons (mesons): $d\to h$. The Hamiltonian which describes the two-channel problem $p\to d, d\to h$ and $h\to h$ can be written as follows

$$\hat{H}=\begin{pmatrix} E_1 & V_{12} \\ V_{21} & E_2 \end{pmatrix}. \quad (1)$$

If we take the initial state containing only the $A=2$ component, then the admixture of the $A=0$ component at the time t will be equal to

$$< h(t)|d >= V_{12}\frac{e^{iE_{21}t}-1}{E_{21}} \quad (2)$$

where $E_{21}=E_2-E_1$. The probability to find the h component at the time t is

$$w_{21}(t)=2|V_{12}|^2\frac{1-\cos(E_{21}t)}{E_{21}^2}. \quad (3)$$

It is important that the transition $d\to h$ occurs to continuum with the density of sates $\rho(E_2)$. This is the main reason why the transition probability for large time intervals

$$\gamma_{21}=\int dE_2\rho(E_2)w_{21}(t)=2\pi|V_{12}|^2\rho(E_2=E_1)t \quad (4)$$

is proportional to t (see e.g. [9]).

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This is very different from the $n-\bar{n}$ oscillations in a free space where the transition occurs from discrete to discrete state. If in the latter case we start at $t=0$ from the neutron state the probability to find $\bar{n}$ at the time $t \ll E_{21}^{-1}$ is proportional to t^2

$$w_{21}(t) \approx \varepsilon_{n\bar{n}}^2 t^2 \quad (5)$$

where $\varepsilon_{n\bar{n}} = V_{12}(n \to \bar{n})$.

If we know the lower limit for the life time of deuteron T, we would find the upper limit on $|V_{12}|^2$ using eq. (4)

$$2\pi |V_{12}|^2 \rho(E_2 = E_1) < \frac{1}{T}. \quad (6)$$

The amplitude of the transition $d \to h$ is proportional to $\varepsilon_{n\bar{n}}$

$$V_{12} = D\varepsilon_{n\bar{n}} \quad (7)$$

where D is the constant which is calculated in Sect.2.

Therefore we can find lower limit on the $n-\bar{n}$ oscillation period

$$\tau_{n\bar{n}} = \varepsilon_{n\bar{n}}^{-1} > (T_0 T)^{1/2} = (\frac{T_0}{T})^{1/2} T \quad (8)$$

where $T_0 = 2\pi D\rho(E_2 = E_1)$ is the characteristic time which depends on the $\bar{n}p$ annihilation probability and is of the order of $10^{-22} \div 10^{-23}$ sec.

As $T \geq 10^{31} \div 10^{32} y \approx 10^{38} \div 10^{39}$ sec the parameter $(T_0/T)^{1/2}$ is very small $\leq 10^{31}$ and this small factor describes the suppression of $n-\bar{n}$ oscillations in nuclei [1-8].

Note that in recent paper [10] it was claimed that the probability of $n-\bar{n}$ transition in nuclei is also proportional to t^2. In this case the small factor $(T_0/T)^{1/2}$ in eq. (8) would be absent. However this statement contradicts to eq. (4) which follows from general principles of quantum mecanics. It might happen only in the case when a very narrow meson state with the mass very close the mass of deuteron would exist.

2. $n-\bar{n}$ transition in deuteron. In the first approximation in $\varepsilon_{n\bar{n}}$ the amplitude of the deuteron decay into the final state f containing mesons can be written in the following form

$$M(d \to f) = \varepsilon_{\bar{n}n} \int \frac{d^3\mathbf{n}}{(2\pi)^3} \varphi_d(\mathbf{n}) \frac{1}{B_d + \mathbf{n}^2/m} \frac{1}{\sqrt{m}} A(\bar{n}p \to f) \quad (9)$$

where m is the nucleon mass, $\varphi_d(\mathbf{n})$ is the deuteron wave function in the momentum space, B_d is the binding energy of deuteron and $A(\bar{n}p \to f)$ is the amplitude of the $\bar{n}p$-annihilation into the meson state f.

Introducing the induced $\bar{n}p$ wave function

$$\psi_{\bar{n}}(r) = \int \frac{d^3\mathbf{n}}{(2\bar{n})^3} e^{i\mathbf{n}\mathbf{r}} \varphi_d(\mathbf{n}) \frac{1}{B_d + \mathbf{n}^2/m} \frac{1}{\sqrt{m}} \quad (10)$$

and taking into account that the radius of $\bar{n}p$ annihilation is much smaller than the deuteron radius $r_{ann} \ll r_d$ we can write

$$M(d \to f) = \varepsilon_{n\bar{n}} \psi_{\bar{n}}(\mathbf{r}=0) A(\bar{n}p \to f) \quad (11)$$

The amplitude $M(d \to f)$ should be singular at $B_d \to 0$ when the radius of deuteron becomes very large. To demonstrate this let us take the Hülthen model of the deuteron wave function

$$\psi_d(r) = N \frac{e^{-\alpha r} - e^{-\beta r}}{r} \quad (12)$$

where $\alpha^2 = mB_d$ and $\beta \approx 5.2\alpha$. In this case

$$\psi_{\bar{n}}(r=0) \approx 0.2\sqrt{\frac{m}{\alpha}} \tag{13}$$

which is divergent when $\alpha = \sqrt{mB_D} \to 0$. Note that this divergence is related with the contribution of the first term in eq. (12), i.e. with the contribution of large r. The second term is introduced to have correct behaviour of the wave function at $r \to 0$. Numerically we have for $B_d \approx 2.23\,\text{MeV}$

$$\psi_{\bar{n}}(r=0) \approx 0.9. \tag{14}$$

The total width of the deuteron

$$\Gamma_d = \sum_f |M(d \to f)|^2 \frac{1}{2m_d} d\phi_f \tag{15}$$

can be expressed through the $\bar{n}p$ annihilation cross section

$$\sigma_{\bar{n}p}^{ann} = \sum_f |A(\bar{n}p \to f)|^2 \frac{1}{4p_{c.m.}\sqrt{s}} d\phi_f \tag{16}$$

or through the imaginary part of the $\bar{n}p$ scattering length $a_{\bar{n}p}$

$$\Gamma_d = \varepsilon_{n\bar{n}}^2 |\psi_{\bar{n}}(\mathrm{r}=0)|^2 2p_{c.m.}\sigma_{\bar{n}p}^{ann} = -\varepsilon_{n\bar{n}}^2 |\psi_{\bar{n}}(\mathrm{r}=0)|^2 8\pi \mathrm{Im} a_{\bar{n}p}. \tag{17}$$

Eq.(17) is very different from the formula for the annihilation width Γ_a which was used as ansatz by Dover, Gal and Richard [5]

$$\Gamma_a = -2\int d^3r |\psi(r)|^2 W_{\bar{n}p}(r). \tag{A1}$$

Here $\psi(r)$ is the solution of the inhomogeneous Schrödinger equations with the $\bar{n}p$ complex optical potential

$$V_{\bar{n}p} = U_{\bar{n}p}(r) + iW_{\bar{n}p}(r) \tag{A2}$$

and the source $\varepsilon_{\bar{n}n}\psi_d(r)$ with $\psi_d(r)$ being the deuteron wave function.

This ansatz was also used by other authors (see e.g. [6-8]) and is equivalent to the assumption that $\bar{n}p$ interaction can be described by the one–channel optical approach. The approach which is used here is based on the nonrelativistic diagram technique and takes into account that $\bar{n}p$ interaction is multichannel. In principle within our approach it is possible to express the probabilities for different exclusive channels of deuteron decay through partial widths of $\bar{n}p$ annihilation. The unitarity condition permits to express the total decay width of deuteron through the observable quantity: the imaginary part of $\bar{n}p$ scattering length.

The possibility to avoid the use of optical potential is important because the experimental $\bar{N}N$ scattering length can be described using completely different values for its strength and radius. As we show in Sect. 3 two different approaches give similar numerical values for Γ_a in the deuteron case. However for complex nuclei there are drastic differences: in the approach (A1) the $n\bar{n}$ oscillations occur mainly on the surface of nucleus while in our approach all the volume of nucleus gives contribution.

3. $n - \bar{n}$ transition in nuclei. It is convenient to introduce the amplitude A_1 which describes the three–step process:

i)$n - \bar{n}$ transition in nucleous with the atomic number A;

ii)scattering of antineuteron on the $A - 1$ nuclear cluster;

iii) inverse transition $\bar{n} - n$ with formation of the initial nucleus.

This amplitude can be written in the following form

$$A_1 = \varepsilon_{n\bar{n}} \int \frac{d^3\mathbf{n}_1}{(2\pi)^3}\varphi(\mathbf{n}_1)\frac{1}{B+\mathbf{n}_1^2/2\mu}\frac{1}{\sqrt{2\mu}}\cdot$$

$$\cdot T_{\bar{n}N}(0)S(\mathbf{n}_1-\mathbf{n}_2)\varepsilon_{n\bar{n}}\frac{d^3\mathbf{n}_2}{(2\pi)^3}\varphi(\mathbf{n}_2)\frac{1}{B+\mathbf{n}_2^2/2\mu}\frac{1}{\sqrt{2\mu}} \tag{18}$$

where B is the average binding energy of neutron, μ is the reduced mass of the $n-(A-1)$ system, $T_{\bar{n}N}(0)$ is the forward $\bar{n}N$ scattering amplitude and $S(\mathbf{n}_1-\mathbf{n}_2)$ is the form factor of the $(A-1)$ nuclear cluster.

The imaginary part of this amplitude is related to the decay width of A nucleous as follows

$$-\,\mathrm{Im}A_1 = m\frac{\Gamma}{N} \tag{19}$$

where N is the number of neutrons.

Generally speaking the amplitude $T_{\bar{n}N}(0)$ should be taken off-shell. However taking into account that the binding is small ($B\sim 8$ MeV) we can ignore off–shall effects and express $T_{\bar{n}N}(0)$ through the $\bar{n}N$ absorption probability and the $\bar{n}N$ scattering length

$$-\,\mathrm{Im}T_{\bar{n}N}(0) = 2p_{c.m.}\sqrt{s}\sigma_{\bar{n}N} = -8\pi\sqrt{s}\mathrm{Im}a_{\bar{n}N}. \tag{20}$$

Introducing $T_{\bar{n}N}(0)$ we avoid the use of optical potential (see e.g. refs. [5–8]) and express the decay width of nucleous in terms of observable quantity $\mathrm{Im}a_{\bar{n}N}$.

As in the deuteron case we introduce the induced $\bar{n}$ wave function

$$\phi_{\bar{n}}(r) = \int \frac{d^3\mathbf{n}}{(2\pi)^3}e^{i\mathbf{n}\mathbf{r}}\varphi(\mathbf{n})\frac{1}{B+\mathbf{n}^2/2\mu}\frac{1}{\sqrt{2\mu}} \tag{21}$$

and rewrite the amplitude A_1 in the following form

$$A_1 = \varepsilon_{n\bar{n}}^2 \int d^3\mathbf{r}\rho(\mathbf{r})|\psi_{\bar{n}}(\mathbf{r})|^2 T_{\bar{n}N}(0), \tag{22}$$

where

$$\rho(r) = \int e^{-i\mathbf{n}\mathbf{r}}S(\mathbf{n})\frac{d^3\mathbf{n}}{(2\pi)^3}. \tag{23}$$

For the estimation we take the neutron wave function in the form

$$\varphi(\mathbf{n}) = N[\frac{1}{\mathbf{n}^2+\alpha^2} - \frac{1}{\mathbf{n}^2+\beta^2}] \tag{24}$$

where $\alpha^2 = 2\mu B$,

$$N = (\frac{8\pi\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2})^{1/2},$$

β is the cut-off parameter and we take it to be equal to 0.5 GeV. As we shall see below the result is not very sensitive to the choice of β.

The amplitude A_1 should be singular at $B\to 0$. Indeed for the wave function (24) we get

$$A_1 = \varepsilon_{n\bar{n}}^2 T_{\bar{n}N}(0)D_A\sqrt{\frac{\mu}{B}} \tag{25}$$

and if $B\to 0$, $A_1 \to 1/\sqrt{B}$.

Calculating A_1 for ^{56}Fe with $\rho=\rho_0$ for $R\le R$ and $\rho=0$ for $>R$ with $R=4.1\,\mathrm{fm}$, $B=8.8\,\mathrm{MeV}$ we find $D_{\mathrm{Fe}}=0.3$. Note that the first term in the wave function (26) gives dominant contribution $\sim 90\%$.

Summarising Sect.2 and 3 we can write the decay widths per neutron

$$-\Gamma(\text{Fe})/N = \varepsilon_{n\bar{n}}^2 C_{\text{Fe}}^2 8\pi \text{Im} a_{\bar{n}N} \tag{26}$$

for iron and

$$-\Gamma^{(d)} = \varepsilon_{n\bar{n}}^2 C_d^2 8\pi \text{Im} a_{\bar{n}p} \tag{27}$$

for deuteron with $C_{\text{Fe}} = 3$ and $C_d \approx 0.8$.

The imaginary part of $a_{\bar{p}p}$ is known from the width of $\bar{p}p$ atom. It is in the interval (see e.g. [11])

$$-\text{Im} a_{\bar{p}p} = (0.7 \div 1.2)\text{fm}. \tag{28}$$

The data of OBELIX collaboration [12] show that at $p_{lab} \geq 60$ MeV/c the annihilation and total cross sections of $\bar{p}p$ and $\bar{n}p$ are almost the same within 20%. Therefore we take

$$a_{\bar{n}N} = \frac{Z}{A} a_{\bar{n}p} + \frac{N}{A} a_{\bar{n}n} \approx a_{\bar{p}p}. \tag{29}$$

In this case we get

$$\Gamma_{\text{Fe}}/N = \varepsilon_{\bar{n}n}^2 (3.5 \div 6) \cdot 10^{-22}\text{sec}, \tag{30}$$

$$\Gamma_d = \varepsilon_{\bar{n}n}^2 (0.5 \div 0.8) \cdot 10^{-22}\text{sec}, \tag{31}$$

where $\varepsilon_{n\bar{n}}$ is in sec^{-1}.

It is useful to compare annihilation widths for deuteron, O^{16} and Fe calculated in ref.[5]:

$$\Gamma_d = \varepsilon_{n\bar{n}}^2 (0.36 \div 0.37) \cdot 10^{-22}\text{sec},$$

$$\Gamma(O^{16})/N = \varepsilon_{n\bar{n}}^2 (0.3 \div 2.2) \cdot 10^{-23}\text{sec},$$

$$\Gamma(\text{Fe})/N = \varepsilon_{n\bar{n}}^2 (0.6 \div 0.9) \cdot 10^{-23}\text{sec}.$$

In view of experimental uncertainties for $a_{\bar{n}N}$ our values for Γ_d are not very different. However we predict that the probability for neutron to annihilate in O^{16} or Fe is much higher than in deuteron while the ansatz (A1) leads to the prediction that it is much smaller.

According to the data of Frejus collaboration [13]

$$T_{\text{Fe}} > 6.5 \cdot 10^{31} yr \approx 2 \cdot 10^{39}\text{sec}.$$

Using this limit we find the lower limit for the period of of $n - \bar{n}$ oscillations

$$\tau_{n\bar{n}}^2 = \varepsilon_{n\bar{n}}^{-2} > [(8 \div 11) \cdot 10^8\text{sec}]^2. \tag{32}$$

If for the deuteron $T_d \approx T_{\text{Fe}}$ then

$$\tau_{n\bar{n}}^{(d)} > (3 \div 4) \cdot 10^8\text{sec}. \tag{33}$$

The limit (32) for $\tau_{n\bar{n}}^2$ is by two order of magnitude higher than the limit

$$(\tau_{n\bar{n}}^{(\text{Fe})})^2 > [(0.8 \div 1.0) \cdot 10^8\text{sec}]^2 \tag{34}$$

found by Alberico, De Pace and Pignone [7].

Using the result of KAMIOKANDE for O^{16} [14]

$$T(O^{16}) > 2.4 \cdot 10^{31}\text{yr}.$$

Dover, Gal and Richard [5] found the lower limit for $\tau_{n\bar{n}}^2$

$$(\tau_{n\bar{n}}(O^{16}))^2 > [(1 \pm 0.3) \cdot 10^8\text{sec}]^2 \tag{35}$$

which is very close to the result of Alberico et.al (34) for Fe and is also by two orders of magnitude lower than the result (32).

The most recent lower limit on the $n-\bar{n}$ transition time in a free space reached by the ILL–Grenoble experiment [15] is

$$\tau_{n\bar{n}} > 8.6 \cdot 10^7 \text{sec}. \tag{36}$$

This limit is approximately by 10 times smaller than the lower limit (32) on $\tau_{\bar{n}n}$ deduced from Frejus experiment [13].

As it was pointed out in refs.[16,17] the proposed experiment on the search for free $n-\bar{n}$ oscillations at Oak Ridge can reach $\tau_{n\bar{n}}$ by 10 times larger than the limit given by (32). The limit (32) can also be used to put new restriction on the difference of neutron and antineutron masses [18]: $\delta m/m \lesssim \epsilon_{n\bar{n}}/m < (6 \div 9) \cdot 10^{-34}$.

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