Pis'ma v ZhETF, vol.64, iss.8, pp.532 - 537 © 1996 October 25

ON THE DYNAMICS OF SLIDING ELASTIC MANIFOLDS WITH RANDOM INTERACTION

K.V.Samokhin[1)]

Institut für Theoretische Physik, Universität zu Köln, D-50937 Köln, Germany
and
L.D.Landau Institute for Theoretical Physics RAS
117940 Moscow, Russia

Submitted 24 September 1996

The friction dynamics of contacting D-dimensional disordered elastic manifolds, driven by external forces, is studied, and the existence of a zero-temperature depinning transition below some critical dimensionality is demonstrated for different kinds of elastic response. It is shown, that this model falls into the universality class of single interface depinning in a $2D$-dimensional random medium.

PACS: 46.30.Pa, 68.35.Ja, 81.40.Pq

The phenomenon of dry friction between two solid bodies is described phenomenologically by well-known Coulomb–Amonton laws [1]. Despite of their simplicity and a fairly long age, there is still neither a comprehensive proof of these laws, starting from a reasonable microscopic model, nor complete understanding of the dynamical processes, accompanying dry friction, and of the role of elasticity in these processes. Recently this problem attracted a lot of experimental [2], and theoretical [3, 4] interest due to the progress, achieved in last years in the theory of driven depinning of elastic manifolds, such as domain walls [5] or Abrikosov vortices [6] *etc*, in disordered media (a nice review of many related topics can be found in Ref.[7]). Since dry friction looks like a typical depinning phenomenon, it would be tempting to study some simple microscopic models in spirit of this theory.

In the present article we consider the dynamics of dry friction between two geometrically smooth D-dimensional elastic objects, for example, disordered membranes or adjacent surfaces of solid bodies, which are confined to displace only along themselves. The most natural way to take into account both the randomness of these objects and the interaction between them is to introduce the scalar functions ("quenched charges") $\rho_1(\mathbf{r}_1)$ and $\rho_2(\mathbf{r}_2)$, which are assumed to be independent Gaussian random variables with zero mean and $\langle\rho_i(\mathbf{r}_1)\rho_j(\mathbf{r}_2)\rangle = \delta_{ij}K_i(\mathbf{r}_1-\mathbf{r}_2)$, $K_i(\mathbf{r})$ being short-range functions, vanishing at $R>\sigma$. Lateral displacements of the manifolds are described by the functions $\mathbf{R}_i(\mathbf{r}_i,t)=\mathbf{r}_i+\mathbf{v}_i t+\mathbf{u}_i(\mathbf{r}_i,t)$ $(i=1,2)$, giving the position of $\mathbf{r}_i$-th element in the fixed reference frame. Functions $\mathbf{u}(\mathbf{r},t)$ take into account the elastic deformations, which are assumed to be small.

We consider the driven dynamics of the system at $T=0$ to be purely dissipative:

$$\frac{1}{\Gamma}\frac{\partial\mathbf{R}_i}{\partial t}=\pm\mathbf{F}-\frac{\delta\mathcal{H}}{\delta\mathbf{R}_i}, \qquad (1)$$

where Γ is the mobility coefficient. Without any loss of generality we restrict ourselves by the case of two identical isotropic sliders, driven by the oppositely

[1)] e-mail: samokhin@thp.uni-koeln.de

directed external forces. The hamiltonian looks like this

$$\mathcal{H} = H_{elastic}[\mathbf{u}_1] + H_{elastic}[\mathbf{u}_2] + H_{int}[\mathbf{R}_1, \mathbf{R}_2].$$

Energy of the elastic deformations can be written as follows:

$$H_{elastic} = \int d^D r d^D r' \; D_{\alpha\beta\gamma\delta}(\mathbf{r}-\mathbf{r}')\nabla_\alpha u_\beta(\mathbf{r})\nabla_\gamma u_\delta(\mathbf{r}') =$$

$$= \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}(\gamma_{\|} n_\alpha n_\beta + \gamma_\perp(\delta_{\alpha\beta} - n_\alpha n_\beta))|\mathbf{k}|^n u_\alpha(\mathbf{k}) u_\beta(-\mathbf{k}), \qquad (2)$$

where $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$ and $\gamma_{\|(\perp)}$ are the longitudinal and transverse elastic moduli. If the sliders are some membrane-like objects (as in the real experiments [2], where latex membranes were used), then the kernel $D(\mathbf{R})$ is local, and the elastic hamiltonian has a standart form with $n = 2$. We can also consider a more general form of the elastic hamiltonian, allowing for the possibility of bulk-mediated response. This is a quite reasonable assumption in the solid dry friction problem, in which the sliders are indeed the contacting surfaces of two solid bodies. In this case the kernel in (2) becomes non-local in real space [8] and non-analitical in the momentum representation: $D(\mathbf{k}) \sim |\mathbf{k}|^{-1}$ and $n = 1$.

The most general form of local interaction between the sliders is given by the expression:

$$H_{int} = \int d^D r_1 d^D r_2 \; U(\rho_1(\mathbf{r}_1), \rho_2(\mathbf{r}_2))\delta(\mathbf{R}_1(\mathbf{r}_1, t) - \mathbf{R}_2(\mathbf{r}_2, t)),$$

where the function U depends on the microscopic details of interaction. We assume, that there act some local adhesive forces between the contacting manifolds, and the potential energy of these forces is simply proportional to the product of two "charge densities":

$$U = V_0 \rho_1(\mathbf{r}_1)\rho_2(\mathbf{r}_2). \qquad (3)$$

Such model assumption is well justified for the case of geometrically smooth sliders, for example, in the presense of a surface chemical disorder. One could also regard (3) as a simple way to mimic the interaction between the geometrically rough surfaces [1]. Other possible implications can be related with the friction dynamics of polycrystalline or amorphous materials, or flat tethered surfaces with quenched topological defects [9], or membranes which adhere to each other via randomly distributed stickers [10] etc. Note also that if to neglect the substrate elasticity and substitute $\rho_2(\mathbf{r}) = \rho_0 \cos 2k_F x$, then our equations of motion coinside with those describing the dynamics of sliding CDW in a random potential [11]. Among some other possibilities one should mention the mechanical model of Ref.[3], which seems rather close to a discretized version of our system if to replace the random springs there by randomly charged beads; and also the random heights model [4], which explicitly takes into account the influence of the normal load.

Let us first demostrate the existence of a zero-temperature depinning transition in our model. For both sliders we take into account the elastic displacements in all lateral directions and obtain from (2) the following equation of motion:

$$\frac{1}{\Gamma}\frac{\partial u_{1,\alpha}(\mathbf{r},t)}{\partial t} + \frac{\delta H_{elastic}}{\delta u_{1,\alpha}(\mathbf{r},t)} = F_\alpha - \frac{1}{\Gamma}v_{1,\alpha} -$$

$$-V_0\rho_1(\mathbf{r})\int d^D r' \rho_2(\mathbf{r}')\partial_\alpha \delta(\mathbf{r} + \mathbf{v}_1 t + \mathbf{u}_1(\mathbf{r},t) - \mathbf{r}' - \mathbf{v}_2 t - \mathbf{u}_2(\mathbf{r}',t)), \qquad (4)$$

and a similar equation for $\mathbf{u}_2$. Expanding the generalized forces in the r.h.s. of Eqs.(4) in powers of displacements, we obtain the system of non-linear differential equations, describing the interaction of elastic degrees of freedom of the manifolds, with the vertices being quenched random variables. In order to evaluate corrections to the average velocities, we solve these equations self-consistently by iterations. Similar procedure has been used repeatedly in different problems – see, for example, Refs.[12, 13].

After first iteration and averaging over the random variables ρ_i we obtain:

$$\delta(\Gamma^{-1}v_{1,\alpha}) = 2V_0^2\Gamma v_\beta \int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\; p_\alpha p_\beta p_\gamma p_\delta K_1(\mathbf{q}-\mathbf{p})K_2(\mathbf{p}) \times$$
$$\times \left(\frac{q_\gamma q_\delta}{q^2(\gamma_\parallel^2\Gamma^2 q^{2n} + (\mathbf{pv})^2)} + \frac{q^2\delta_{\gamma\delta} - q_\gamma q_\delta}{q^2(\gamma_\perp^2\Gamma^2 q^{2n} + (\mathbf{pv})^2)}\right),$$

where $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ is the relative velocity, and upper cut-off of the integrals at $p, q \sim \sigma^{-1}$ is understood. The correction to $\mathbf{v}_2$ has exactly the same form, but with the opposite sign. Since the most divergent contribution comes from the region of small momenta, one can replace the product of the correlation functions by its value at $\mathbf{p} = \mathbf{q} = 0$, and then perform integration over $\mathbf{q}$ explicitly:

$$\delta(\Gamma^{-1}v_\alpha) \sim V_0^2K_1K_2\Gamma^{(n-D)/n}\gamma^{-D/n}\int d^Dp\; p_\alpha p_\beta|\mathbf{pv}|^{(D-2n)/n}v_\beta \sim$$
$$\sim \Gamma^{-1}v_\alpha\left(\Gamma^{-1}v\right)^{-\epsilon/n}\left(V_0^2K_1K_2\gamma^{-D/n}\sigma^{-(n+1)D/n}\right), \quad (5)$$

where $\epsilon = 2n - D$, $\gamma^{-D/n} = \gamma_\parallel^{-D/n} + (D-1)\gamma_\perp^{-D/n}$, and $K_i = K_i(\mathbf{p}=0) = \int d^Dr\; K_i(\mathbf{r})$. Therefore, the corrections to the average velocity (or, in other words, to the inverse mobility coefficient) diverge at small velocities below the critical dimensionality $D_c = 2n$, depending on a kind of the elastic response of the manifolds.

Although the analysis, given above, is applicable quantitatively only in the limit of high velocities, we believe, that the vanishing of mobility coefficient results in some non-zero threshold value of the external force F_c, below which steady relative motion of the sliders becomes impossible. Such picture resembles the one, which is encountered in consideration of the transverse motion of an elastic D-dimensional interface through a quenched random medium (see, e.g., Ref.[14]). The threshold F_c can be estimated in assumption that the manifolds get stuck to each other when the corrections to $v \sim \Gamma F$ become equal to or larger than the average velocity itself [13]. Then it follows from (5), that

$$F_c \sim \left((V_0K_1K_2)^{2n}\gamma^{-D}\sigma^{-(n+1)D}\right)^{1/\epsilon}.$$

By analogy with the interface depinning problem one can expect, that just above F_c the average velocity obeys a scaling law: $v \sim (F - F_c)^\theta$, however, to calculate θ one has to use a more elaborate technique. In the rest of the article we shall establish explicit equivalence between the driven dynamics of our system and the (extensively studied) critical behaviour of a certain class of moving elastic interfaces. Note, that such equivalence holds for the equilibrium static properties as well [15].

For the sake of simplicity we neglect the elasticity of substrate (i.e. $\mathbf{u}_2 = \mathbf{v}_2 = 0$ in (4)) and write down the overdamped equations of motion for $\mathbf{u} = \mathbf{u}_1$:

$$\frac{1}{\Gamma}\frac{\partial u_\alpha}{\partial t} + \frac{\delta H_{elastic}}{\delta u_\alpha} = f_\alpha - V_0 \rho_1(\mathbf{r}) \partial_\alpha \rho_2(\mathbf{r} + \mathbf{v}t + \mathbf{u}(\mathbf{r}, t)), \tag{6}$$

where $\mathbf{f} = \mathbf{F} - \Gamma^{-1}\mathbf{v}$. Next, we introduce an auxiliary vector field $\hat{\mathbf{u}}$ and construct the Martin–Siggia–Rose generating functional [16]:

$$Z[\eta, \hat{\eta}] = \int \mathcal{D}\mathbf{u}\mathcal{D}\hat{\mathbf{u}} \exp\Big\{ i \int d^D r dt \Big[\hat{u}_\alpha \Big(\frac{1}{\Gamma}\frac{\partial u_\alpha}{\partial t} + \frac{\delta H_{elastic}}{\delta u_\alpha} \Big) + $$
$$+ (V_0 \hat{u}_\alpha(\mathbf{r}, t) \rho_2(\mathbf{r}) \partial_\alpha \rho_1(\mathbf{r} + \mathbf{v}t + \mathbf{u}(\mathbf{r}, t)) + \eta_\alpha u_\alpha + (\hat{\eta}_\alpha - f_\alpha)\hat{u}_\alpha) \Big] \Big\}.$$

Since $Z[0,0] \equiv 1$, we are able to average the generating functional over the random variables without any complications. Due to the specific form (3) of the interaction energy, integration over ρ_1 and ρ_2 can be performed explicitly, resulting in

$$Z[\eta, \hat{\eta}] = \int \mathcal{D}\mathbf{u}\mathcal{D}\hat{\mathbf{u}} \exp\Big[iI_0 + iI_{\rm int} + i \int d^D r dt \ \ (\eta_\alpha u_\alpha + (\hat{\eta}_\alpha - f_\alpha)\hat{u}_\alpha) \Big],$$

I_0 being quadratic in $\mathbf{u}$ and $\hat{\mathbf{u}}$, and $I_{\rm int}$ containing the interaction terms of different orders, for example:

$$I_{\rm int}^{(2)} = i\frac{K_1 V_0^2}{2} \int d^D r dt_1 dt_2 \hat{u}_\alpha(\mathbf{r}, t_1) \hat{u}_\beta(\mathbf{r}, t_2) \times$$
$$\times \partial_\alpha \partial_\beta K(\mathbf{v}(t_1 - t_2) + \mathbf{u}(\mathbf{r}, t_1) - \mathbf{u}(\mathbf{r}, t_2)) \tag{7}$$

(here we put $K_1(\mathbf{r}) = K_1 \delta(\mathbf{r})$ and $K_2(\mathbf{r}) = K(\mathbf{r})$).

In order to assess relevance of different terms in the effective action, we assume, that the two-point correlation function of the lateral displacements obeys a scaling law near the depinning transition:

$$\langle (\mathbf{u}(\mathbf{r}_1, t_1) - \mathbf{u}(\mathbf{r}_2, t_2))^2 \rangle \sim |\mathbf{r}_1 - \mathbf{r}_2|^{2\zeta} g\left(\frac{t_1 - t_2}{|\mathbf{r}_1 - \mathbf{r}_2|^z} \right),$$

where $g(x) \to$ const at $x \to 0$ and $g(x) \sim x^{2\zeta/z}$ at $x \to \infty$, $\zeta(D,n)$ and $z(D,n)$ being the roughness and dynamic exponents respectively. A change of scale $r \to \lambda r$, $t \to \lambda^z t$, $\mathbf{u} \to \lambda^\zeta \mathbf{u}$ in assumption, that the action I is dimensionless, yields $[\hat{u}] = n - D - \zeta - z$, $[\Gamma] = n - z$ and $[F] = \zeta - n$.

Comparing two terms in the argument of K in (7), we see that there exists the time scale $\Delta t_c \sim v^{-z/(z-\zeta)}$, beyond which one can neglect non-linearity of Eq.(6) and regard the quenched random forces as an effective thermal noise. Since $v \sim (F - F_c)^\theta$, there also exists a corresponding space scale – the correlation lenght $\xi \sim (F - F_c)^{-\nu}$ with $\nu = \theta/(z - \zeta)$, therefore, $[F] = [L^{-1/\nu}]$. As a consequence, all the exponents of interest can be expressed in terms of ζ and z [14]:

$$\nu = \frac{1}{n - \zeta}, \quad \theta = \frac{z - \zeta}{n - \zeta}.$$

At $L > \xi$ we neglect all interactions between $\mathbf{u}$ and $\hat{\mathbf{u}}$, and the trivial "thermal-noise" exponents $\zeta = 0$, $z = n$ [17] are recovered near D_c. However, at smaller scales we can instead neglect the $v(t_1 - t_2)$-terms:

$$I_{\rm int}^{(2)} = i\frac{K_1 V_0^2}{2} \int d^D r dt_1 dt_2 \hat{u}_\alpha(\mathbf{r}, t_1) \hat{u}_\beta(\mathbf{r}, t_2) \partial_\alpha \partial_\beta K(\mathbf{u}(\mathbf{r}, t_1) - \mathbf{u}(\mathbf{r}, t_2)). \tag{8}$$

Therefore, in the critical region the interactions become important, and one can expect that $\zeta \sim O(\epsilon)$, $z \sim n + O(\epsilon)$. If to substitute in the r.h.s. of (8) the expansion

$$K(\mathbf{r}) = \sum_{m=0}^{\infty} \frac{K_{2m}}{(2m)!} r^{2m},$$

then straightforward power counting yields, that the dimensionalities of all coefficients K_{2m} are positive near D_c, i.e. we have to renormalize the whole function $K(\mathbf{r})$ [18]. The mobility coefficient Γ and the force F are also subject to renormalization.

The crucial point is that the expression (8) looks exactly the same as the effective interaction of elastic degrees of freedom of a D-dimensional interface, driven through a $2D$-dimensional disordered medium [19, 20], the transverse coordinate (height) of interface being substituted by the lateral deformations $\mathbf{u}$. This observation allows us to apply all results of the interface depinning theory directly to our system. But before one should justify the possibility of neglecting the higher-order interaction terms. In the critical region, using the condition $\zeta < 1$, we obtain, that the only term with coinciding space arguments gives a contribution in the long-wavelenght limit:

$$I_{\text{int}}^{(4)} \sim \int d^D r dt_1 dt_2 dt_3 dt_4 \hat{u}_\alpha(\mathbf{r}, t_1) \hat{u}_\beta(\mathbf{r}, t_2) \hat{u}_\gamma(\mathbf{r}, t_3) \hat{u}_\delta(\mathbf{r}, t_4) \times$$
$$\times \partial_\alpha \partial_\delta K(\mathbf{u}(\mathbf{r}, t_1) - \mathbf{u}(\mathbf{r}, t_4)) \partial_\beta \partial_\gamma K(\mathbf{u}(\mathbf{r}, t_2) - \mathbf{u}(\mathbf{r}, t_3)). \tag{9}$$

One can easily convince oneself that such interaction is irrelevant. Indeed, if to count powers in the r.h.s. of (9), then for a short-range correlator we obtain the condition $3D - 4n + 2(D+4)\zeta > 0$, which is fulfilled near D_c.

Unfortunately, our understanding of the behaviour of interfaces near the depinning transition is far from being complete. In particular, different analytical approaches diverge in predictions about the values of ζ and z. Nattermann et al. [14] observed, that the functional renormalization group equations for $K(\mathbf{r})$ (at $n = 2$) have exactly the same form as in the static case [18], and concluded that the corresponding roughness exponents should also coincide. Substituting the fixed-point solution for $K(\mathbf{r})$ into the flow equations for Γ, they evaluated the dynamic exponent z, and obtained: $z = 2 - 5/18\epsilon$, $\theta = 1 - 5/36\epsilon$.

On another hand, Narayan and Fisher [20], using expansion around the mean-field solution of the equations of motion, came to essentially the same functional flow equations, but with somewhat different interpretation. In the simplest case of $n = 2$ and codimension $d = 1$ (in terms of our original system it means, that the only deformations along $\mathbf{v}$ are taken into account) they found $\zeta = \epsilon/3$, $z = 2 - 2\epsilon/9$ and $\theta = 1 - \epsilon/9$.

All these results, however, should be taken with caution, since it is not clear how they could be affected by allowing for the displacements in all lateral directions. Besides that, from the analysis of static behaviour of the system under consideration we know [15], that the functional flow equations change their form if $\gamma_\parallel \neq \gamma_\perp$ in (2). These problems obviously need a careful analytical treatment.

In summary, it is shown, that the driven dynamics of two D-dimensional elastic manifolds with random interaction near the depinning transition belongs to the universality class of transverse motion of a single manifold through a $2D$-dimensional disordered medium. This conclusion is in agreement with the numerical result of Cule and Hwa [3], who studied the dynamics of a one-dimensional random mechanical chain on a rough substrate and found, that it falls into the universality class of driven directed polymers in a two-dimensional random environment.

I would like to thank T.Nattermann and A.Volmer for numerous stimulating discussions. This work was financially supported by the Deutscheforschungsgemeinschaft through the Project SFB 341.

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