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RARE PROCESSES AND COHERENT PHENOMENA IN CRYSTALS

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We study coherent enhancement of Coulomb excitation of high energy particles in crystals. We develop multiple scattering theory description of coherent excitation which consistently incorporates both the specific resonant properties of particle-crystal interactions and the final/initial state interaction effects typical of the diffractive scattering. Possible applications to observation of induced radiative neutrino transitions are discussed.

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1. In a two-level quantum system (the ground state $|0\rangle$ and the excited state $|1\rangle$) under periodical perturbation $V \sin \nu t$ with the frequency ν equal to the level splitting $\nu_{10} = E_1 - E_0$ there develop quantum beats with the o scillation frequency $\omega = \langle 1|V|0\rangle$. If the perturbation V is weak, then for $\omega t \ll 1$ the $|0\rangle \to |1\rangle$ the transition probability $P_{10}(t)$ increases rapidly with the time

$$P_{10}(t) \propto \omega^2 t^2 . \tag{1}$$

In particle physics, examples of such rare processes are the weak radiative transitions of hyperons, $N\gamma \to Y$, beyond-the-standard-model-decays like $\mu \to e\gamma$ and radiative (magnetic) neutrino conversion $\nu_1\gamma \to \nu_2$. For instance, if one could subject hyperons to a high frequency field, $\nu \sim m_Y - m_N$, the rates of rare decays can be enhanced substantially. The high monochromaticity is an evident condition to sustain the growth (1) over large time scale.

Okorokov [1-3] was the first to suggest that all of the above requirements are met best in Coulomb interaction of a high-energy particle propagating in a crystal along the crystallographic axis. Here the rôle of "time" is played by the crystal thickness L. In [1] the resonant excitation of atoms and nuclei by a periodical Coulomb field of a crystal was predicted. In [2] the first observation of the resonant transition of $He^+(n=1)$ to $He^+(n=4)$ in Ag crystal was reported. For a propagation through a crystal with the lattice spacing d, the frequency $\nu = 2\pi v/d$, where v is the velocity of the atom. For ultrarelativistic particles ν is enhanced due to the Doppler shift, $\nu = 2\pi\gamma v/d$, where γ is the Lorentz factor. It is the Lorentz factor which can boost ν to the hundreds MeV's range. Since [1] the Okorokov effect has been studied extensively both experimentally [4] and theoretically [5].

In early works on the subject the Coulomb field of a crystal was evaluated in the Weizsäker-Williams approximation [6, 7] and then applied to the calculation

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of the transition amplitude in the plane wave Born approximation. The $N\gamma \to Y$ transitions in the Coulomb field of the nucleus were discussed by Pomeranchuck and Shmushkevich[8] . The $p\gamma \to \Sigma^+$ excitation in crystals based on the approach [6, 7] and [8] was considered recently in [9] and it was claimed that the law $P_{\Sigma p}(N) \propto N^2$, holds up to the crystal thicknesses $N = L/d \simeq 10^7$ [9].

However, the plane wave Born approximation is not self-contained, it does not allow to assess what is the upper limit on N and, as a matter of fact, it grossly overestimates the enhancement factor. Our point is that the coherency of $p\gamma \to \Sigma^+$ transitions depends on the initial state interactions (ISI) of the proton and final state interactions (FSI) of the hyperon. It also depends on thermal vibrations of the atoms in a crystal.

The purpose of this communication is to derive the Okorokov effect for the Pomeranchuck–Shmushkevich processes directly from the multiple scattering (MS) theory. We find a dramatic impact of the ISI and FSI effects on $p \to \Sigma^+$-transition resulting in the very slow, $\sim \log N$, rise of the amplitude rather than $\sim N$. This result is quite general and is equally applicable to the excitation of ultrarelativistic nuclei and ions passing through the crystals.

On the contrary, the magnetic conversion of the neutrino in crystals as shown to have a pronounced resonant structure and we comment on possible implications for the future laboratory investigations of the neutrino electrodynamics.

2. Consider Coulomb interaction of high-energy particle a moving along the $< 001 >$ axis in monatomic crystal. The interatomic distances are large, $d \sim 5\,\text{Å}$, compared to the Thomas–Fermi screening radius a_{TF}, $a_{TF} = \mu^{-1} = (m_e \alpha Z^{1/3})^{-1} \simeq 0.5 Z^{-1/3}\,\text{Å}$, where Z is the atomic number and $\alpha = 1/137$. The relevant impact parameters, b_{eff}, satis

fy $b_{eff} \sim \mu^{-1} \ll d$ and the widely used one-chain approximation is applicable. At the same time, b_{eff} is much larger than the nuclear radius $R_A \sim 10^{-5} Z^{1/3} \text{Å}$ and the effects of strong interactions can be safely neglected.

The amplitude $\mathcal{F}_{ba}$ of coherent transition $a \to b$ on a chain of N identical atoms is

$$\mathcal{F}_{ba} = \langle \Psi(\mathbf{r}_1, ..., \mathbf{r}_N) | S_{ba} | \Psi(\mathbf{r}_1, ..., \mathbf{r}_N) \rangle \,. \tag{2}$$

Here $\Psi(\mathbf{r}_1, ..., \mathbf{r}_N)$ is the full ground-state wave function of the N-atomic chain and $\mathbf{r}_i = (\mathbf{b}_i, z_i)$ are positions of atoms. The S-matrix, S_{ba}, admits the expansion into multiple scattering series, $\mathcal{F}_{ba} = \sum_n F_{ba}^{(n)}$, and describes the a-to-b transition as well as both the initial and final state interactions of both particles a and b with crystal. The high momentum projectile propagates along straight-line trajectories at fixed impact parameter b. Then one can use Gribov's dispersion integral representation for the coupled channel n-fold scattering amplitude $F_{ba}^{(n)}(\mathrm{b})$ in the basis of physical states $|a\rangle, |i\rangle, ..., |b\rangle$ [10]. This allows one to consider the $a \to b$ transition as a sequence of n both diagonal and off-diagonal transitions $|a\rangle \to |i\rangle \to ... |k\rangle \to |j\rangle \to |b\rangle$ ordered along the beam direction. Each off-diagonal transition, involves a longitudinal momentum transfer [10]

$$\kappa_{ji} = \frac{\Delta m_{ji}^2}{2E} \,, \tag{3}$$

where $\Delta m_{ji}^2 = m_j^2 - m_i^2$, E is the projectile energy and m_j is the particle j mass. Associated with the κ_{ji} is the phase factor $\exp[i\kappa_{bj} z_\nu + i\kappa_{jk} z_{\nu-1} + ... + i\kappa_{ia} z_1]$, in the amplitude $F_{ba}^{(n)}(\mathbf{b})$:

$$F_{ba}^{(n)}(\mathbf{b}) = (-i)^{n-1}\frac{N!}{(N-n)!}\sum_{i,\dots,k,j}\int_0^L dz_n\int d^2\mathbf{b}_n f_{bj}(\mathbf{b}-\mathbf{b}_n)\exp[i\kappa_{bj}z_n]\times \tag{4}$$
$$\times\int_0^{z_n} dz_{n-1}\int d^2\mathbf{b}_{n-1}f_{jk}(\mathbf{b}-\mathbf{b}_{n-1})\exp[i\kappa_{jk}z_{n-1}]\dots$$
$$\dots\int_0^{z_2} dz_1\int d^2\mathbf{b}_1 f_{ia}(\mathbf{b}-\mathbf{b}_1)\exp[i\kappa_{ia}z_1]\chi(\mathbf{r}_1,\dots,\mathbf{r}_n),$$

where

$$\chi(\mathbf{r}_1,\dots,\mathbf{r}_n) = \int d^3\mathbf{r}_{n+1}\dots d^3\mathbf{r}_N|\Psi(\mathbf{r}_1,\dots,\mathbf{r}_N)|^2. \tag{5}$$

For our purposes it is sufficient to use the uncorrelated wave function

$$|\Psi(\mathbf{r}_1,\dots,\mathbf{r}_N)|^2 = \prod_{i=1}^N \phi(\mathbf{u}_i), \tag{6}$$

where the normal coordinates $\mathbf{u}_i$ are defined by $\mathbf{r}_i = (i-1)\mathbf{d}+\mathbf{u}_i$, $i=1,\dots,N$, $\mathbf{d}=(0,\,0,\,d)$ and $\mathbf{u}_i=(\mathbf{b}_i,u_{zi})$.

For the rare processes of interest

$$f_{ij} \ll f_{ii}, f_{jj}\,, \quad i\neq j \tag{7}$$

and we keep only the lowest order terms in powers of the off-diagonal transitions. Then,

$$\mathcal{F}_{ba}(\mathbf{b}) = \sum_{n=1}^N F_{ba}^{(n)}(\mathbf{b}) = \sum_{m=1}^N\sum_{n_1=0}^{m-1}\sum_{n_2=0}^{N-m} h_{aa}(\mathbf{b})^{n_1}h_{bb}(\mathbf{b})^{n_2}h_{ba}(\mathbf{b})S_L^{(m)}(\kappa_{ba}), \tag{8}$$

where

$$h_{ba}(\mathbf{b}) = \int d^2\mathbf{b}_i f_{ba}(\mathbf{b}-\mathbf{b}_i)\rho(\mathbf{b}_i), \tag{9}$$

$$\rho(\mathbf{b}_i) = \int du_{zi}\phi(\mathbf{u}_i) = \frac{3}{2\pi\langle\mathbf{u}^2\rangle}\exp\left[-\frac{3\mathbf{b}_i^2}{2\langle\mathbf{u}^2\rangle}\right], \tag{10}$$

$$S_L^{(m)}(\kappa_{ba}) = \int_0^L du_{zm}\phi(\mathbf{u}_m)\exp(i\kappa_{ba}u_{zm}) = \exp\left[-\frac{1}{6}\kappa_{ba}^2\langle\mathbf{u}^2\rangle\right]\exp\left[i\kappa_{ba}(m-1)d\right]. \tag{11}$$

For the sake of simplicity, we have assumed the Gaussian form of $\phi(\mathbf{u}_i)$. In (10) $\langle\mathbf{u}^2\rangle$ is the mean squared amplitude of thermal vibrations of the lattice . Note that the relevant longitudinal momenta are as follows $\kappa_{ji}\sim d^{-1}\ll\langle\mathbf{u}^2\rangle^{-1/2}$. The summation over m in eq.(8) results in

$$\mathcal{F}_{ba}(\mathbf{b}) = h_{ba}(\mathbf{b})\exp\left[-\frac{1}{6}\kappa_{ba}^2\langle\mathbf{u}^2\rangle\right]\times \tag{12}$$
$$\times\frac{[1+ih_{bb}(\mathbf{b})]^N-[1+ih_{aa}(\mathbf{b})]^N\exp\left[i\kappa_{ba}Nd\right]}{[1+ih_{bb}(\mathbf{b})]-[1+ih_{aa}(\mathbf{b})]\exp\left[i\kappa_{ba}d\right]}.$$

In the practically interesting case like $p\gamma \to \Sigma^+$ and/or $e\gamma \to \mu$ we have $f_{aa} = f_{bb}$ and the amplitube $\mathcal{F}_{ba}$ reads

$$\mathcal{F}_{ba}(\mathbf{b}) = h_{ba}(\mathbf{b})\,[1 + ih_{aa}(\mathbf{b})]^{N-1}\,S_L(\kappa_{ba})\,, \tag{13}$$

where $S_L(\kappa)$ is the longitudinal form factor

$$S_L(\kappa_{ba}) = \exp\left[-\frac{1}{6}\kappa_{ba}^2\langle \mathbf{u}^2\rangle\right]\frac{\sin(\kappa_{ba}Nd/2)}{\sin(\kappa_{ba}d/2)}\,. \tag{14}$$

Evidently, for the resonant values of κ_{ba},

$$\kappa_{ba} = \frac{2\pi n}{d}\,,\; n = 0, \pm 1, \pm 2\ldots\,, \tag{15}$$

we have $S_L \propto N$. The resonance energy E_n,

$$E_n = \frac{\Delta m_{ba}^2 d}{4\pi n} \tag{16}$$

can be lowered going to $n \gg 1$. However, there is an upper limit on n.

$$n_{max} \simeq \frac{1}{2\pi}\left(\frac{3d^2}{\langle \mathbf{u}^2\rangle}\right)^{1/2} \simeq 5 - 10\,. \tag{17}$$

which comes from the Debye-Waller factor in $S_L(\kappa)$, eq.(14).

Above we focused on the S-matrix at a fixed impact parameter $\mathbf{b}$. The experimentally observable quantity is the differential cross section $d\sigma_{ba}/dq_\perp^2 = |\mathcal{T}_{ba}(\mathbf{q}_\perp)|^2$, where

$$\mathcal{T}_{ba}(\mathbf{q}_\perp) = S_L(\kappa_{ba})\int \frac{d^2\mathbf{b}}{2\pi} h_{ba}(\mathbf{b})\,[1 + ih_{aa}(\mathbf{b})]^{N-1}\exp[i\mathbf{q}_\perp \mathbf{b}] \tag{18}$$

and $\mathbf{q}_\perp$ - is the two-dimensional vector of the transverse momentum ($|\mathbf{q}_\perp| = q_\perp$).

As we shall see below, the dominant contribution to (18) comes from $b_{eff} \sim \mu^{-1}N_{eff}$, where

$$N_{eff} = \log(Z\alpha N) \gg 1\,.$$

Then, because $\langle \mathbf{u}^2\rangle \sim \mu^{-2}$

$$h_{aa}(\mathbf{b}) = \int d^2\mathbf{b}_i f_{aa}(\mathbf{b} - \mathbf{b}_i)\rho(\mathbf{b}_i) \simeq f_{aa}(\mathbf{b})\,. \tag{19}$$

The amplitude f_{aa} of elastic scattering of a charged particle a in a screened Coulomb field of an isolated atom is purely real if $q_\perp^2 \ll \mu p$, where p is the projectile momentum:

$$f_{aa}(b) = Z\alpha K_0(\mu b)\,. \tag{20}$$

Here we neglect the contributions of anomalous magnetic moments to the small-angle elastic scattering.

Let the resonance condition (15) be satisfyed so that $S_L(\kappa) \propto N$. Our point is that because of the distortion factor $[1 + ih_{aa}(\mathbf{b})]^N$ in the integrand of eq.(18) the law $\mathcal{T}_{ba} \propto N$ does not hold. The evaluation of the distortion effects for the $p\gamma \to \Sigma^+$ transition proceeds as follows.

For the phenomenological transition matrix element [8, 11]

$$\mathcal{M}(p\gamma \to \Sigma^+) = i\bar{u}_\Sigma \left(\mu_{\Sigma p} + \gamma_5 d_{\Sigma p}\right) \sigma_{\mu\nu} q^\nu \varepsilon^\mu u_p \tag{21}$$

one readily finds

$$f_{\Sigma p}(\mathbf{b}) = \left[\mu_{\Sigma p} \boldsymbol{\sigma} \left[\mathbf{b} \times \mathbf{n}\right] + i d_{\Sigma p} \boldsymbol{\sigma} \mathbf{b}\right] b^{-1} Z \sqrt{\alpha} \mu K_1(\mu b) . \tag{22}$$

Here $\mathbf{n}$ - is a unit vector along the projectile direction, $\boldsymbol{\sigma}$ is the Pauli spin vector and $K_1(x)$ is a modified Bessel function.

Then, the evaluation of the full transition amplitude reads

$$\begin{aligned} \mathcal{T}_{\Sigma p}(\mathbf{q}_\perp) = {} & \left[i\mu_{\Sigma p} \boldsymbol{\sigma} \left[\mathbf{q}_\perp \times \mathbf{n}\right] + d_{\Sigma p} \boldsymbol{\sigma} \mathbf{q}_\perp\right] q_\perp^{-1} \mu S_L(\kappa_{ba}) \times \\ & \times Z \sqrt{\alpha} \int b db \, J_1(q_\perp b) K_1(\mu b)) \left[1 + iZ\alpha K_0(\mu b)\right]^{N-1} \end{aligned} \tag{23}$$

and the steepest descent from the saddle-point at $\mu b = N_{eff} - i\pi/2$ yields for $q_\perp \lesssim \mu N_{eff}^{-1}$

$$\mathcal{T}_{\Sigma p}(q_\perp) \propto i \left(N_{eff} - i\pi/2\right) J_1 \left(q_\perp \mu^{-1} (N_{eff} - i\pi/2)\right) . \tag{24}$$

where $J_1(x)$ is the Bessel function. Consequently, as soon as $N\alpha Z \gg 1$, which holds for all practical purposes, even under the resonance condition (15) one only has the logarithmic growth

$$\mathcal{T}_{\Sigma p} \propto \log(Z\alpha N) . \tag{25}$$

Although the relevant impact parameters rise, $b_{eff} \sim \mu^{-1} N_{eff}$, for all the practical purposes $b_{eff} \ll d$ so that the one-chain approximation holds. The total cross-section, $\sigma = \int dq_\perp^2 |\mathcal{T}_{\Sigma p}(q_\perp)|^2$ appeares to be independent of N.

We conclude that ISI and FSI effects completely destroy the resonant enhancement effect. Similar phenomena are well known in the high-energy diffractive scattering of hadrons, though for purely imaginary amplitudes. For the screening of real elastic amplitudes in the Coulomb potential scattering see e.g. [12].

3. Several proposals [13] of experiments on the neutrino conversion induced by the neutrino magnetic moment in external electromagnetic field

$$\nu_1 \gamma \to \nu_2 \tag{26}$$

are under discussion [14]. Hereafter we are dealing with the non-diagonal magnetic (μ_{21}) and electric (d_{21}) dipole moments which exist in general for both Dirac and Majorana neutrinos [15].

The experimental bounds on μ_{21} are such [14] that the FSI/ISI effects are negligible for all practical purposes. Then, in terms of the matrix element (21) the differential cross section of the process (26) reads

$$\frac{d\sigma_{21}}{dq_\perp^2} = \alpha Z^2 b_{21}^2 \frac{q_\perp^2}{(q_\perp^2 + \mu^2)^2} S_T^2(q_\perp^2) S_L^2(\kappa_{21}) , \tag{27}$$

where $b_{21}^2 = |\mu_{21}|^2 + |d_{21}|^2$ for Dirac neutrinos and $b_{21}^2 = 4\left[(\mathrm{Im}\mu_{21})^2 + (\mathrm{Re} d_{21})^2\right]$ for Majorana neutrinos. The transverse form factor $S_T(q_\perp^2)$ equals

$$S_T(q_\perp^2) = \exp\left[-\frac{1}{3} q_\perp^2 \langle \mathbf{u}^2 \rangle\right] . \tag{28}$$

The observation of the $\nu_1\gamma \to \nu_2$-transition which would look like a resonance at some neutrino energy $E \simeq E_n$ (see eq.(16)) would enable one to evaluate the mass difference of the neutrinos of different species and estimate the neutrino transition magnetic moment.

For these purposes it is useful to represent the resonant part of the differential cross section (27) as a sum of Lorentz curves. Then, the neutrino conversion rate, $R_c = \sigma_{21}/d^2$, equals

$$R_c \simeq N^2 \frac{b_{21}^2 \alpha Z^2}{d^2 \left[1 + \frac{2}{3}\mu^2 \langle \mathbf{u}^2 \rangle\right]^2} \sum_n \frac{(\Gamma_n/2)^2}{(\Gamma_n/2)^2 + (E - E_n)^2} \exp\left[-\frac{4\pi^2 n^2}{3} \frac{\langle \mathbf{u}^2 \rangle}{d^2}\right]. \qquad (29)$$

The width of n-th resonance is

$$\Gamma_n = \frac{\sqrt{3}}{2} \frac{\Delta m_{21}^2 d}{\pi^2 n^2 N} \qquad (30)$$

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1. V.V.Okorokov, Sov. J. Nucl. Phys. **2**, 719 (1965); JETP Lett. **2**, 111 (1965).
2. V.V.Okorokov, D.L.Tolchenkov, Yu.P.Cheblukov et al., JETP Lett **16**, (1972); Phys. Lett. **A43**, 485 (1973).
3. V.V.Okorokov, JETP Lett. **62**, 911 (1995).
4. M.J.Gaillard, J.C.Poizat, J.Remillieux, and M.L.Gaillard, Phys. Lett. **A45**, 306 (1973); H.G.Berry, D.S.Gemmel, R.E.Holland et al., Phys. Lett. **A49**, 123 (1975); M.Mannami, H.Kudo, M.Matsushita, and K.Ishii, Phys. Lett. **A64**, 136 (1977); S.Datz, C.M.Moak, O.H.Crawford et al., Phys. Rev. Lett. **40**, 843 (1987); C.M.Moak, S.Datz, O.H.Crawford et al., Phys. Rev. **A19**, 977 (1979); F.Fujimoto, Nucl. Instr. Methods **B40/41**, 165 (1989); Y.Iwata, K.Komaki, Y.Yamazaki et al., Nucl. Instr. Methods **48**, 163 (1990).
5. J.Kondo, J.Phys. Soc. Jpn. **36**, 1406 (1974); S.Shindo and Y.H.Ohtsuki, Phys. Rev. **B14**, 3929 (1976); Y.Yamashita and Y.H.Ohtsuki, Phys. Rev. **B22**, 1183 (1980); Yu.L.Pivovarov and A.A.Shirokov, Sov. J. Nucl. Phys. **37**, 653 (1983).
6. M.L.Ter-Mikaelyan, Sov. Phys. JETP **25**, 289 (1953).
7. B.Ferretti, Nuovo Cimento **7**, 1 (1950); **B7**, 225 (1972); **B9**, 399 (1972).
8. I.Ya.Pomeranchuck and I.M.Shmushkevich, Nucl. Phys. **23**, 452 (1961).
9. A.Yu.Dubin, Sov. J. Nucl. Phys. **52**, 790 (1990).
10. V.N.Gribov, Sov. Phys. JETP **29**, 483 (1969); **30** , 709 (1970).
11. R.F.Behrends, Phys. Rev. **111**, 1691 (1958); V.I.Zakharov and A.B.Kaidalov, Sov. J. Nucl. Phys. **5**, 259 (1967).
12. L.I.Schiff, Phys. Rev. **103**, 443 (1956); N.P.Kalashnikov and V.D.Mur, Sov. J. Nucl. Phys. **16**, 613 (1973).
13. M.C.Gonzalez-Garcia, F.Vannucci, and J.Castromonte, Phys. Lett. **B373**, 153 (1996); S.Matsuki and K.Yamamoto, Phys. Lett. **B289**,194 (1992); M.Sukuda, Phys. Rev. Lett. **72**, 804 (1994); J.Bernabeu, S.M.Bilenkii, F.G.Botella, and J.Segura, Nucl. Phys. **B426**, 434 (1994).
14. J.-M.Frere, R.B.Nevzorov, and M.I.Vysotsky, Preprint ULB-TH/96/14.
15. J.Schechter and J.W.F.Valle, Phys. Rev. **D24**, 1883 (1981); P.B.Pal and L.Wolfenstein, Phys. Rev. **D25** , 766 (1982); J.Nieves, Phys. Rev. **D26**, (1982); M.B.Voloshin and M.I.Vysotsky, Sov. J. Nucl. Phys. **44**, 544 (1986); M.B.Voloshin, M.I.Vysotsky, and L.B.Okun', Sov. Phys. JETP **64**, 446 (1986).