Pis'ma v ZhETF, vol.64, iss.11, pp.794 - 800 © 1996 December 10

POISSON BRACKETS SCHEME FOR VORTEX DYNAMICS IN SUPERFLUIDS AND SUPERCONDUCTORS AND EFFECT OF BAND STRUCTURE OF CRUSTAL.

G.E.Volovik

Low Temperature Laboratory, Helsinki University of Technology Otakaari 3A 02150 Espoo, Finland

L.D.Landau Institute for Theoretical Physics RAS 117940 Moscow, Russia

Submitted 11 November 1996

Poisson brackets for the Hamiltonian dynamics of vortices are discussed for 3 regimes, in which the dissipation can be neglected and the vortex dynamics is reversible: (i) The superclean regime when the spectral flow is suppressed. (ii) The regime when the fermions are pinned by crystal lattice. This includes also the regime of the extreme spectral flow of fermions in the vortex core: these fermions are effectively pinned by the normal component. (iii) The case when the vortices are strongly pinned by the normal component. All these limits are described by the single parameter C_0, which physical meaning is discussed for superconductors containing several bands of electrons and holes. The effect of the Fermi-surface topology on the vortex dynamics is also discussed.

PACS: 03.40.Ge, 47.37.+q, 67.40.Vs, 74.60.Ge

1. Introduction. The problem of the vortex motion is well understood for the translational invariant Fermi superfluids [1-4]. When the theory is applied to superconductors, one should take into account the effect of the band structure of the crystal on the vortex dynamics. In this case the topology of the Fermi-surface is to be important and we make an attempt to consider this effect in the regimes, when the dissipation is small and can be neglected.

2. Poisson Brackets Formalism for the Vortex Motion. We start with the phenomenological hydrodynamic equations for the system of distributed vortices in superfluids or superconductors. In the limit of vanishing dissipation the dynamics of the collective variables becomes conservative and in principle can be described by the effective action. However, as usually occurs in the hydrodynamic systems, such action is not well defined and the Hamiltonian formalism in terms of the Poisson brackets (PB) becomes preferrable. In this formalism the Hamiltonian is the function of the relevant hydrodynamic variables characterized by the algebra of the Poisson brackets.

We assume that the normal component is clamped, ie its velocity $\mathbf{v}_n = 0$. This is typical for superfluid ^{3}He due to its high viscosity and for superconductors where $\mathbf{v}_n$ is fixed by the impurities in the crystal lattice. The remaining hydrodynamic variables at low temperature are the mass density ρ and the superfluid velocity $\mathbf{v}_s$, which is non-potential in the presence of the distributed vorticity. The Hamiltonian

$$H = \int d^3r \ \left[\frac{1}{2}\rho_s^{ij} v_s^i v_s^j + \epsilon(\rho)\right] \quad , \tag{2.1}$$

contains the internal energy density $\epsilon(\rho)$ and the kinetic energy. In crystals, due to absence of the Galilean invariance, the superfluid component does not coincide with the density ρ of the electrons even at $T=0$: $\rho_s(T=0) \neq \rho$. Actually ρ_s can be much smaller than ρ because most of the electrons are concentrated in the completely filled bands. Nevertheless the hydrodynamic equations are valid for the descrition of the long-wave-length dynamics of ρ and $\mathbf{v}_s$.

The motion equations are obtained as the Liouville equations

$$\frac{\partial \rho}{\partial t} = \{H, \rho\} \quad , \quad \frac{\partial \mathbf{v}_s}{\partial t} = \{H, \mathbf{v}_s\} \quad . \tag{2.2}$$

if one uses the PB between the variables. These PB are universal, i.e. they do not depend on the Hamiltonian [5], and we propose:

$$\{\rho(\mathbf{r}), \mathbf{v}_s(\mathbf{r}')\} = \vec{\nabla}\delta(\mathbf{r}-\mathbf{r}') \quad , \tag{2.3}$$

$$\{\rho(\mathbf{r}), \rho(\mathbf{r}')\} = 0 \quad , \tag{2.4}$$

$$\{v_{si}(\mathbf{r}), v_{sj}(\mathbf{r}')\} = -e_{ijk}\frac{(\vec{\nabla}\times\mathbf{v}_s)_k}{\rho - C_0}\delta(\mathbf{r}-\mathbf{r}') \quad . \tag{2.5}$$

The first two PB are conventional (see [5]), eg the Eq.(2.3) follows from the fact that the particle number and the phase of the condensate are canonically conjugated variables. The Eq.(2.5) contains a new variable C_0, which is to be the dynamical invariant of the system, $\partial C_0/\partial t = 0$. With this constraint the Poisson brackets satisfy the Jacobi identity $\{a\{bc\}\} + \{b\{ca\}\} + \{c\{ab\}\} = 0$. The Eq.(2.5) with $C_0 = 0$ was derived in superfluids in the $T=0$ limit [6]. In derivation it was assumed that each element of the vortex moves with the local superfluid velocity $\mathbf{v}_s(\mathbf{r})$, which corresponds to the Helmholtz theorem for the perfect liquid. The PB Eq.(2.5) with $C_0 = \infty$ was written in [5]. This corresponds to the motion of vortices with the local normal velocity $\mathbf{v}_n(\mathbf{r})$. This is actually a true hydrodynamic regime, which requires the complete pinning of the vortex lines by the normal component or by the heat bath of the crystal lattice [7].

The range $0 < C_0 < \infty$ corresponds to the intermediate regimes when the vortex is unpinned, but some groups of particles (electrons) are pinned by the heat bath during the vortex motion. In addition to the trivial localization of the electrons by the crystal lattice and impurities, the quasiparticles are also pinned by the anomalous process of the spectral flow in the core of the vortex, discussed in [8, 9, 3, 4]. When the vortex moves in the regime of the extreme spectral flow, the momentum is effectively transferred from the vortex to the heat bath, which corresponds to the pinning of fermions by the heat bath.

With the PB in Eqs.(2.3-5), the Liouville equations (2.2) of the nondissipative dynamics become

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\hat{\rho}_s \mathbf{v}_s) = 0 \quad , \tag{2.6}$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + \frac{1}{\rho - C_0}(\hat{\rho}_s\mathbf{v}_s)\times(\vec{\nabla}\times\mathbf{v}_s) + \vec{\nabla}\mu = 0 \quad , \quad \mu = \frac{\delta H}{\delta \rho} \quad . \tag{2.7}$$

Using the kinematic definition of local velocity $\mathbf{v}_L$ of vortex lines[10, 5]:

$$\partial_t \mathbf{v}_s + \vec{\nabla}\mu = -\mathbf{v}_L \times (\vec{\nabla}\times\mathbf{v}_s) \quad , \tag{2.8}$$

one obtains the following relation between $\mathbf{v}_L$ and $\mathbf{v}_s$:

$$(\rho - C_0)\mathbf{v}_L = \hat{\rho}_s \mathbf{v}_s \quad . \tag{2.9}$$

For superconductors the lhs of Eq.(2.9) determines the Hall conductivity in the limit of vanishing dissipation:

$$\sigma_{\rm Hall} = \frac{ec}{mB}(\rho - C_0) \quad ,$$

where B is magnetic field.

The equation (2.9) is not Galilean invariant. The Galilean invariance is restored by introducing the velocity of the normal component $\mathbf{v}_n$, which coincides with the velocity of crystal lattice in the case of superconductors. Then multiplying by the circulation $\vec{\kappa}$ (with $|\vec{\kappa}| = N\pi\hbar/m$, and N being the winding number of the vortex) one obtains the equation for the balance of Magnus, spectral flow and Iordanskii forces acting on the vortex [3]:

$$\vec{\kappa} \times [\rho(\mathbf{v}_s - \mathbf{v}_L) + C_0(\mathbf{v}_L - \mathbf{v}_n) + \hat{\rho}_n(\mathbf{v}_n - \mathbf{v}_s)] = 0 \quad , \tag{2.10}$$

where $\hat{\rho}_n = \rho - \hat{\rho}_s$ is the tensor of the normal density.

The density ρ and the superfluid density tensor $\hat{\rho}_s$ are well determined quantity, the latter being determined by the current-current correlation function. Let us now discuss the parameter C_0. For the systems with translational invariance [3, 8] the parameter C_0 was determined only in the special limit case, the so called hydrodynamic regime. As we see below, in both hydrodynamic and collisionless regimes the dissipation can be neglected and thus the Hamiltonian approach is valid. As a result the Eqs.(2.9−10) are valid in both regimes, but with different values of the parameter C_0. For example, as follows from Refs. [3, 8], in the systems with the translational invariance the collisionless value of the parameter C_0 in Eqs.(2.9-10) is zero. But for superconductors with several electronic bands the situation is more complicated and the parameter C_0 can be nonzero even in the collisionless regime. Moreover one can have the collisionless regime for one band and the hydrodynamic regime for the other band.

3. Particles and holes contributions. The low-frequency dynamics and thermodynamics of the fermi-liquid or superconductors are determined by the low-energy quasiparticles. In the same way the low-frequency dynamics of vortices is determined by the low-energy excitations in the vortex core. The latter are concentrated on the anomalous branch of the spectrum [11]. Here we follow the simplified version of [1, 2] (see Refs.[8, 4]). Let us start with the axisymmetric vortex in the translational invariant surrounding. The spectrum of the low-energy excitations in the core is defined in the frame of the moving vortex, where the Hamiltonian does not depend on time, if there are impurities. The energy of the low-energy branch is expressed in terms of the canonically conjugated variables: the angular momentum Q and the angle α of the linear momentum, $\mathbf{k} = (k_F \cos\theta, k_F \sin\theta \cos\alpha, k_F \sin\theta \sin\alpha)$, in the transverse plane:

$$\mathcal{H} = C\omega_0 Q + \mathbf{v}_s \cdot \mathbf{k} \quad . \tag{3.1}$$

The first term describes the orbital motion of fermions bound in the core around the vortex axis [11]. The effect of the vortex on the motion of the quasiparticle is similar to the magnetic field. The quantization of this orbital motion leads to the descrete levels (see also Sec.4) with either integer or half-odd integer generalized angular momentum Q [12]. The frequency of rotation ω_0 is the function of θ. The direction of rotation is determined by the "chirality" factor $C = \pm 1$. For the conventional case of the particle-like excitations in the vortex with winding number $N = 1$ one has $C = -1$ [11], while $C = 1$ for the hole-like excitations in the vortex with the same winding number. This follows from the index theorem, which relates the number of fermion zero modes in the vortex core to the vortex winding number N [8]: in the case of holes the topological invariant, which determines the number of zero modes, changes sign.

The second term is the energy due to the superflow in the vortex frame.

When the vortex moves with respect to the heat bath, its dynamics is nonequilibrium. The kinetics of the fermions on the branch in Eq.(3.1) is governed by the Boltzmann equation for the distribution function, $n(Q,\alpha)$, in the $Q-\alpha$ phase space[4]:

$$\frac{\partial n}{\partial t} + C\omega_0 \frac{\partial n}{\partial \alpha} - \frac{\partial((\mathbf{v}_s - \mathbf{v}_L)\cdot \mathbf{k})}{\partial \alpha}\frac{\partial n}{\partial Q} = -\frac{n(Q,\alpha) - n_{\rm eq}(Q,\alpha)}{\tau} \tag{3.2}$$

The last term describes the relaxation to the equilibrium distribution function $n_{\rm eq}$ determined by the heat bath outside the core:

$$n_{\rm eq}(Q,\alpha) = f(\mathcal{H} - \mathbf{v}_n\cdot\mathbf{k}) = f(C\omega_0 Q + (\mathbf{v}_s - \mathbf{v}_n)\cdot\mathbf{k}) \quad , \tag{3.3}$$

where $f(E) = (1+\exp(E/T))^{-1}$ is the Fermi-function.

If τ does not depend on Q one gets the equation for average momentum

$$\partial_t \bar{\mathbf{k}} + C\omega_0 \hat{\mathbf{z}}\times\bar{\mathbf{k}} + \frac{C}{4}k_F^2\sin^2\theta\hat{\mathbf{z}}\times(\mathbf{v}_n - \mathbf{v}_L)(f(\Delta(T)) - f(-\Delta(T))) = -\frac{\bar{\mathbf{k}}}{\tau} \ , \tag{3.4}$$

$$\bar{\mathbf{k}} = \frac{1}{2}\int dQ\frac{d\alpha}{2\pi}(n(l,\alpha) - n_{\rm eq}(Q,\alpha))\mathbf{k} \quad . \tag{3.5}$$

Here we take into account that $\int dQ\partial_Q n$ is limited by the bound states below the gap $\Delta(T)$, since above the gap $\Delta(T)$ the spectrum of fermions is continuous [2]. The effective interlevel distance for the unbound (delocalized) states is $\omega_0 = 0$ and they will be considered below.

In the steady state of the vortex motion one has $\partial_t\bar{\mathbf{k}} = 0$ and the Eq.(3.4) is easily solved. The rhs of Eq.(3.4) gives the momentum flow to the heat bath and thus the following force due to bound states below $\Delta(T)$

$$\mathbf{F}_{loc} = \int\frac{dk_z}{8\pi}k_F^2\sin^2\theta\frac{1}{1+\omega_0^2\tau^2}\tanh\frac{\Delta(T)}{2T}[(\mathbf{v}_L - \mathbf{v}_n)\omega_0\tau - C\hat{\mathbf{z}}\times(\mathbf{v}_L - \mathbf{v}_n)] \ . \tag{3.6}$$

The spectral flow of unbound states above $\Delta(T)$ is not suppressed, since the corresponding $\omega_0\tau = 0$. This gives

$$\mathbf{F}_{deloc} = -C\frac{k_F^3}{8\pi}\int d\cos\theta\ \sin^2\theta\left(1 - \tanh\frac{\Delta(T)}{2T}\right)\hat{\mathbf{z}}\times(\mathbf{v}_L - \mathbf{v}_n) \ . \tag{3.7}$$

Thus the total nondissipative spectral-flow force $\mathbf{F}_{ndiss}$ is

$$\mathbf{F}_{ndiss} = -C\frac{k_F^3}{8\pi}\int d\cos\theta\ \sin^2\theta\left[1 - \frac{\omega_0^2\tau^2}{1+\omega_0^2\tau^2}\tanh\frac{\Delta(T)}{2T}\right]\hat{\mathbf{z}}\times(\mathbf{v}_L - \mathbf{v}_n) \ . \tag{3.8}$$

The dissipative part of the spectral-flow force $\mathbf{F}_{diss}$ is

$$\mathbf{F}_{diss} = (\mathbf{v}_n - \mathbf{v}_L)\frac{k_F^3}{8\pi}\int d\cos\theta\ \sin^2\theta\tanh\frac{\Delta(T)}{2T}\ \frac{\omega_0\tau}{1+\omega_0^2\tau^2} \ , \tag{3.9}$$

can be neglected when $\lambda = \omega_0\tau/(1+\omega_0^2\tau^2) \ll 1$. So, the vortex motion is governed by the conservative Hamiltonian dynamics either in the so called hydrodynamic regime, when $\omega_0\tau \ll 1$, or in collisional regime, when $\omega_0\tau \gg 1$. In both cases the contribution of the spectral flow to the parameter C_0 in Eq.(2.10) from one spin direction acquires universal values at low T:

$$\frac{C_0}{m} = 0 \quad , \quad \omega_0\tau \gg 1 \quad , \tag{3.10}$$

$$\frac{C_0}{m} = V^p \quad , \quad \omega_0\tau \ll 1 \quad , \quad \text{particle states} \quad , \tag{3.11}$$

$$\frac{C_0}{m} = -V^h \quad , \quad \omega_0\tau \ll 1 \quad , \quad \text{hole states} \quad . \tag{3.12}$$

Here $V^p = p_F^3/6\pi^2$ is the volume of the Fermi-sphere of particle states (with one spin direction), while $V^h = p_F^3/6\pi^2$ is the volume of the Fermi-sphere of the hole states.

In conventional superconductors there are no zeroes of energy in the quasiparticle spectrum and thus the Fermi surface is absent. Zeroes however appear due to vortices. In the continuous description of the vortex core, the zeroes in the classical energy spectrum $E = \sqrt{\epsilon^2(\mathbf{k}) + |\Delta(\mathbf{k},\mathbf{r})|^2}$ are concentrated on the vortex axis, but can be splitted into point nodes distributed in the vortex core [13]. The function $\mathbf{k}(\mathbf{r})$, which shows the position of zero of E in the momentum space as a function of the coordinate $\mathbf{r}$ in the real space, maps the cross section of the vortex into the momentum space. Thus, if one sweeps the cross section of the N-quantum vortex, one obtains the closed surface in the momentum space, swept by zeroes. The volume within this surface is just C_0/m times N [9]. The physical meaning of C_0/m is the number of the electronic states which remain kept by the heat bath during the vortex motion. In the frame moving with the vortex this corresponds to the number of the states flowing from the vortex to the heat bath in the extreme spectral flow regime.

4. Electronic bands in crystals, open orbits. Let us consider the crystal with anisotropic Fermi-surface. In the case of the particle states the Eq.(3.11) remains to be valid for the arbitrary closed surface of zeroes with V^p being the volume within the surface [9]. If one neglects the dependence of $\omega_0\tau$ on α, θ and Q one can write an interpolating equation for the closed surface of particle states:

$$\frac{C_0(\text{particles})}{m} = \frac{V^p}{1+\omega_0^2\tau^2}. \tag{4.1}$$

The contribution of the holes to C_0 is

$$\frac{C_0(\text{holes})}{m} = V^B - V^h \frac{1}{1+\omega_0^2\tau^2} = V^B \frac{\omega_0^2\tau^2}{1+\omega_0^2\tau^2} + V^p \frac{1}{1+\omega_0^2\tau^2} \quad , \tag{4.2}$$

where V^B is the total volume of the Brillouin zone, $V^h = V^B - V^p$ is the volume of the hole states. The Eq.(4.2) follows from two arguments. (1) The spectral flow parameter C_0 changes sign for holes, as was discussed in the previous Section. (2) The completely filled band should be considered as uneffected by the motion of the vortex, the electrons on these bands are completely pinned by the crystal lattice. This corresponds to the limit of the extremely fast relaxation and thus to the extreme spectral flow.

The Eq.(4.2) agrees with the microscopic calculations in Ref.[2]. If the gap Δ is small compared to the Fermi energy, then ρ is very close to the volume of all the particle states: $\rho \approx m \sum V^p$. If there is only one band and this band contains holes, then $\rho \approx m(V^B - V^h)$. In the superclean regime, $\omega_0 \tau \gg 1$, the Eq.(4.2) gives $\rho - C_0 \approx m(V^B - V^h) - mV^B = -mV^h$. As a result $\sigma_{\rm Hall} = -(ec/B)V^h$ in agreement with Ref.[2].

The Eqs.(4.1) and (4.2) should transform into each other during the filling of the Brillouin zone, when the Fermi surface of particles transforms to the Fermi surface of holes. However Eqs.(4.1) and (4.2) differ by the value $V^B\omega_0^2\tau^2/(1+\omega_0^2\tau^2)$. The key to the problem of matching these equations in the Lifshitz transition is provided by open orbits, which appear as intermediate stage between particle and hole Fermi-surfaces. The open Fermi-surfaces with complicated topology were discussed in the relation to the Hall effect in the normal metal (see recent paper [14] and references there). Here we show that in the presence of the open surfaces of zeroes the parameter $\omega_0\tau$ is small and Eqs.(4.1) and (4.2) match each other.

In the semiclassical approach the energy of fermions in the vortex core:

$$E^2 = \epsilon^2(\mathbf{k}) + |\Delta(\mathbf{r})|^2 \quad . \tag{4.3}$$

The lowest energy levels are concentrated in the vicinity of zeroes $\mathbf{k}_0$ of the spectrum $\epsilon(\mathbf{k})$, ie close to the former Fermi-surface of the normal metal. Near the Fermi-surface one has $\epsilon(\mathbf{k}) = \mathbf{v}(\mathbf{k}_0) \cdot (\mathbf{k} - \mathbf{k}_0) = -iv_\perp \partial_s$, where s is the coordinate along $\mathbf{v}_\perp = \mathbf{v}(\mathbf{k}_0) - \hat{\mathbf{z}}(\hat{\mathbf{z}} \cdot \mathbf{v}(\mathbf{k}_0))$. Close to the vortex axis one has

$$|\Delta(\mathbf{r})|^2 \approx \gamma^2 r^2 = \gamma^2(s^2 + b^2) \quad , \tag{4.4}$$

where b is the impact parameter. We assume for simplicity that the core radius is large compared to the size of the electronic orbits: since the result is of the topological origin, it should not depend on the model. First we quantize the fast motion along s. According to supersymmetry the lowest energy level of this motion lies exactly at zero energy [8]. As a result the spectrum of the excitations in the core is determined by the slow motion along b, ie along the line of the intersection of the Fermi-surface $\epsilon(\mathbf{k}) = 0$ with the plane $k_z = const$. It is given by $E(k_\parallel, b) = \gamma b$, where $k_\parallel$ is the coordinate along the line of zeroes in the momentum space. The quantization of the slow motion, $\oint dk_\parallel b(k_\parallel) = 2\pi n$, with $b(k_\parallel) = E/\gamma$ gives the levels of bound states in the core

$$E_n = -n\omega_0 \quad , \quad \frac{1}{\omega_0} = \frac{1}{\gamma} \oint \frac{dk_\parallel}{2\pi} \quad . \tag{4.5}$$

For the closed spherical Fermi-surface this leads to the conventional result for the states in the vortex with large core radius:

$$E_n = -n\omega_0 \quad , \quad \omega_0 = \frac{\gamma}{k_F \sin\theta} \quad . \tag{4.6}$$

For the open orbits the integral in Eq.(4.5) diverges which gives $\omega_0 = 0$ as it was expected.

5. Discussion. The Eqs.(4.1) and (4.2) can be generalized to the case of several bands. If there are no open surfaces of zeroes, the total C_0/m contains the positive contribution from the particles, the negative contribution from holes and the positive contribution kV^B where k is the number of Brillouin zones which are either completely filled or contain the hole states:

$$\frac{C_0}{m} = \sum_a V_a^p \frac{1}{1+\omega_{0a}^2\tau_a^2} - \sum_b V_b^h \frac{1}{1+\omega_{0b}^2\tau_b^2} + kV^B \quad . \tag{5.1}$$

This also includes the summation over the spin indices.

Each Fermi surface has its own $\omega_0\tau$. The conservative Hamiltonian approach applies and the Eq.(5.1) holds, only if for each band the parameter $\lambda = \omega_0\tau/(1+\omega_0^2\tau^2) \ll 1$. If λ are not small, the Eq.(5.1) can be considered only as interpolation since the real ω_0 and τ are complicated functions of the impact parameter and momentum p_z. The limiting cases, when the Eq.(5.1) holds, can include the cases when $\omega_0\tau$ is small in one zone and large in the other. Thus C_0 as a function of external parameters (doping or direction of magnetic field) should have plateaus interrupted by regions where one of the parameters λ changes between 0 and 1. The latter occurs also during the change of the topology of orbits.

The Eq.(2.9) can be also applied to other inhomegeneous systems which become homogeneous on a large scale, such as Josephson junction arrays (JJA). In some cases the corresponding quantities ρ, ρ_s and C_0 can be obtained after averaging over the scale of the inhomogeneity. In the system of the SNS contacts, the parameter $\rho - C_0$ is again small in the hydrodynamic limit due to approximate particle-hole symmetry [15]. This leads to the almost ballistic motion of vortices in the absence of the supercurrent, when $\mathbf{j}_s = \hat{\rho}_s \mathbf{v}_s = 0$. It is still unclear whether the approximate cancellation of ρ and C_0 occurs in SIS contacts.

I thank N.B. Kopnin for illuminating discussions. This work was supported through the ROTA co-operation plan of the Finnish Academy and the Russian Academy of Sciences and by the Russian Foundation for Fundamental Sciences, Grant 96-02-16072.

---

1. N.B.Kopnin and V.E.Kravtsov, Pis'ma ZhETF, **23**, 631 (1976); [JETP Lett. , **23**, 578 (1976)]; ZhETF, **71**, 1644 (1976); [JETP, **44**, 861 (1976)].
2. N.B.Kopnin and A.V.Lopatin, Phys. Rev., **B 51**, 15291 (1995).
3. N.B.Kopnin, G.E.Volovik and Ü.Parts, Europhys. Lett. **32**, 651 (1995).
4. M.Stone, Phys. Rev. **B 54**, 13222 (1996).
5. I.E.Dzyaloshinskii and G.E.Volovick, Ann. Phys. **125** 67 (1980).
6. G.E.Volovik and V.S.Dotsenko (jr), Pis'ma ZhETF **29**, 630 (1979) [JETP Lett. **29**, 576 (1979)].
7. A.F.Andreev and M.Yu.Kagan, ZhETF, **86**, 546 (1984); [JETP, **59**, 318 (1984)].
8. G.E.Volovik, Pis'ma ZhETF, **57**, 233 (1993);[JETP Lett., **57**, 244 (1993)].
9. G.E.Volovik, ZhETF , **104**, 3070 (1993); [JETP, **77**, 435 (1993)].
10. E.B.Sonin, Rev. Mod. Phys. **59**, 87 (1987).
11. C.Caroli, P.G. de Gennes and J.Matricon, Phys. Lett., **9**, 307 (1964).
12. T.Sh.Missirpashaev and G.E.Volovik, Physica, **B 210**, 338 (1995).
13. G.E.Volovik and V.P.Mineev, ZhETF **83**, 1025 (1982) [JETP **56**, 579 (1982)].
14. S.P.Novikov and A.Ya.Mal'tsev, Pis'ma ZhETF **63**, 809 (1996) [JETP Lett., **63**, 855 (1996)].
15. Yu.G.Makhlin and G.E.Volovik, Pis'ma ZhETF **62**, 923 (1995) [JETP Lett., **62** , 941 (1995)].