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COMMENT ON "INTEGRABLE CHAIN OF ELECTRONS INTERACTING WITH PHONONS", BY M.E.ZHURAVLEV, V.A.IVANOV, V.V.ACHKASOV

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We note that the exactly solvable electron-phonon Hamiltonian presented in this paper can be solved by a more direct route via the Lang-Firsov transformation.

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While many spin and electron Hamiltonians are exactly solvable in one dimension by Bethe ansatz techniques, an exactly solvable Hamiltonian with coupled electron and ion degrees of freedom has only recently been found [1]. These authors constructed the following electron-ion Hamiltonian:

$$H = -t\sum_{\substack{ij\\\sigma}}\left(c^{\dagger}_{i\sigma}c_{j\sigma}\exp\{g(a_i-a^{\dagger}_i)\}\exp\{-g(a_j-a^{\dagger}_j)\}+\text{H.c.}\right)+ \\ +\ \omega_E\sum_i a^{\dagger}_i a_i + g\omega_E\sum_i n_i\left(a_i+a^{\dagger}_i\right), \tag{1}$$

where we have changed notation slightly to conform with that used in the current literature of the Holstein model. The Hamiltonian (1) is similar to the Holstein model; electrons ($c^{\dagger}_{i\sigma}$) interact with Einstein oscillators ($a^{\dagger}_i$) through a site-diagonal term with coupling $g\omega_E$. The difference resides in the hopping term. Here, electrons hop accompanied by complicated phonon excitations given by the exponential operators in the first term of Eq. (1). Zhuravlev et al. proceed to solve Eq. (1) by mapping it onto an attractive Hubbard model which can then be solved by using Bethe ansatz techniques. The purpose of this comment is to give a much more direct route to the mapping by using the well-known Lang–Firsov transformation [2]:

$$\overline{H} = e^{S}He^{-S} \tag{2}$$

where $S = -g\sum_j n_j(a_j - a^{\dagger}_j)$. The operators transform in the usual way:

$$\bar{c}_{i\sigma} \equiv e^{S}c_{i\sigma}e^{-S} = c_{i\sigma}X_i \tag{3}$$

where $X_i \equiv \exp\{g(a_i - a^{\dagger}_i)\}$, and

$$\bar{a}_i = a_i - gn_i. \tag{4}$$

The latter relation in particular implies that the displacement operator, $q_i \propto a_i + a_i^\dagger$, is shifted while the momentum operator, $p_i \propto a_i - a_i^\dagger$, is not. It thus follows that the site-diagonal terms in Eq. (1) become, as usual,

$$\overline{H}_{site-diag} = \omega_E \sum_i a_i^\dagger a_i - g^2 \omega_E \sum_i (n_i + 2n_{i\uparrow} n_{i\downarrow}). \tag{5}$$

The last two terms respectively correspond to a shift in the chemical potential and a Hubbard-like attraction with effective coupling $U_{eff} = -2g^2\omega_E$. Meanwhile the transformation of the phonon operators does not affect the combination that occurs in the exponents in the hopping term (since the combination is proportional to the momentum) while the transformation of the electron operators is (by design) such that the phonon-dependent terms cancel one another. The hopping term becomes independent of phonon operators, and one is left with an effective attractive Hubbard model which can be solved in the usual fashion by Bethe ansatz techniques [3].

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