Pis'ma v ZhETF, vol.64, iss.6, pp.397 - 402 © 1996 September 25

FLUCTUATION-INDUCED PSEUDOGAP IN HIGH-TEMPERATURE SUPERCONDUCTORS

F.Federici, A.A.Varlamov[1)]

Laboratorio "Forum" dell'Istituto Nazionale di Fisica della Materia
Dipartimento di Fisica

Università di Firenze
50125 Firenze, Italy

Submitted 9 July 1996

We study the effect of fluctuations on the ac conductivity of a layered superconductor for c-axis electromagnetic wave polarization. The fluctuation contributions of different physical nature and sign (paraconductivity, Maki–Thompson anomalous contribution, one-electron density-of-states renormalization) are found to be suppressed by the external field at different characteristic frequencies ($\omega_{\mathrm{AL}} \sim T - T_c$, $\omega_{\mathrm{MT}} \sim \max\{T - T_c, \tau_\varphi^{-1}\}$, $\omega_{\mathrm{DOS}} \sim \min\{T, \tau^{-1}\}$). As a result, the appearance of a nonmonotonic frequency dependence (pseudogap) in the infrared optical conductivity of high-temperature superconductor is predicted.

PACS: 74.25.Gr, 74.40.+k

There is no doubt that the normal properties of high-temperature superconductors (HTSs) are highly anomalous in comparison with conventional superconductors and contain the secret of the superconducting phenomenon itself. One such property is the opening of a large pseudogap in the c-axis component of the conductivity, which was observed in Ref. [1]. This feature is now attracting special attention in connection with the very recent experimental data on photoemission [2].

Much effort has been devoted to the creation of unconventional theories of superconductivity based on strongly interacting electrons, preformed Cooper pairs, etc. [2] Nevertheless, one can start from the BCS approach and find that the consistent development of perturbation theory for weakly interacting electrons in the normal phase of highly anisotropic superconductors leads to the appearance of precursor effects of superconductivity, like those mentioned above, at temperatures above T_c.

The first (and well-known) result is that in this case "preformed Cooper pairs" appear automatically: taking into account the thermal fluctuations (or, what is the same, the electron–electron interaction in the Cooper channel), one can find that some nonzero density of fluctuational Cooper pairs (with finite lifetime) arises in layers without the manifestation of long-range order in the system. It is important that their density decreases very slowly as the temperature increases: in the 2D case this decrease is described by a $\ln(1/\epsilon)$ law ($\epsilon = (T - T_c)/T_c$ is the reduced temperature). The necessity of forming these pairs leads to a decrease of the one-electron density of states (DOS) at the Fermi level.

In the present communication we show that the suppression of the DOS at the Fermi level due to electron–electron interaction in the Cooper channel gives rise to a considerable negative contribution to the optical conductivity. It manifests itself in a wide range of frequencies ($\omega_{\mathrm{DOS}} \sim \tau^{-1}$) and exceeds the

[1)]On leave of absence from Department of Theoretical Physics, Moscow Institute of Steel and Alloys positive Aslamazov–Larkin (AL) and Maki–Thompson (MT) contributions. We study the ac fluctuation-conductivity tensor for a layered superconductor, taking all contributions into account and focusing attention on the most interesting case, when the electric field is directed along the c axis. The optical conductivity can be expressed by the retarded analytical continuation of the current–current correlator (electromagnetic response operator) $Q^{(R)}(\omega)$: its diagrams are presented in Fig.1. The problem of fluctuation conductivity of layered superconductors has been extensively studied, and for a detailed description of the model and the diagrammatic representation of the electromagnetic response-operator tensor $Q_{\alpha\beta}$ we direct the reader to Ref. [3]. In what follows we will use the results and notation of that article.

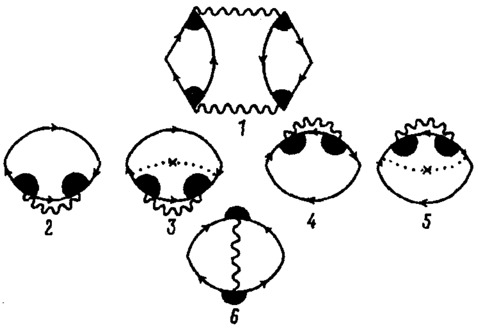

Fig.1 The leading Feynman diagrams contributing to the optical conductivity. The wavy lines indicate the fluctuation propagators; the thin solid lines with arrows stand for the impurity-averaged normal-state electron Green's functions; the shaded partial circles are the vertex corrections arising from impurities; the dashed curves with central crosses are additional impurity renormalizations. Diagram 1 is the Aslamazov–Larkin contribution, diagrams 2–5 are the corrections from the density-of-states renormalization, and diagram 6 is the Maki–Thompson diagram

We assume that the electron spectrum of the layered metal has the form of a corrugated cylinder. The electron mean free path in HTS layered single crystals or epitaxial films turns out to be of the same order as or several times larger than the coherence length $\xi_{ab}(0)$, so the parameter $T_c\tau \sim 1$ and the theory has to be constructed for an arbitrary impurity concentration.

Paraconductivity. Let us first examine the AL contribution (diagram 1 of Fig.1) to the ac fluctuation conductivity. We will concentrate on the most interesting (for analysis of HTSs) case of 2D fluctuations for which $\xi_c(T) \ll s$ (ξ_c is the Ginzburg–Landau (GL) coherence length along the c axis, and s is the interplane spacing). In this case $\sigma_{\perp}^{\mathrm{AL}}$ turns out to be suppressed by the necessity for independent tunneling of each electron participating in the fluctuational pairing from one CuO_2 layer to the neighboring one [4]. Following the outlines of Refs. [3, 5, 6], one can extend the treatment of the AL contribution up to frequencies comparable to T_c and find:

$$\sigma_{\perp}^{\mathrm{AL(2D)}}(\epsilon,\omega) = \frac{e^2 s}{64\eta}\left(\frac{r}{2\epsilon}\right)^2 \frac{1}{\tilde{\omega}^2}\ln\left(1+\tilde{\omega}^2\right) \tag{1}$$

where $\tilde{\omega} = \dfrac{\pi\omega}{16(T-T_c)}$, r is the reduced Lawrence–Doniach crossover temperature (at which $\xi_c(T) \sim s$) and η is the positive constant which appears in the gradient term of the phenomenological GL theory in the 2D case [3].

Maki–Thompson Contribution. It has been demonstrated [5] that in the case of quasi-two-dimensional electron motion there is no formal necessity of introducing the pair-breaking time τ_{φ}, since the Maki–Thompson logarithmic divergence is automatically cut off because of the possible interlayer hopping. Nevertheless, all evidence shows that the intrinsic pair-breaking in HTSs is strong (at least one of its sources may be identified as thermal phonons), and an estimate of the corresponding time $\tau_\varphi \sim 2\text{–}5 \cdot 10^{-13}$ s is only several times larger than T_c^{-1}. Thus we are actually dealing with the overdamped regime for the Maki–Thompson contribution [3]. In the most interesting case, that of the two-dimensional overdamped regime ($r \ll \epsilon < \gamma$), one can find:

$$\sigma_\perp^{\mathrm{MT(an)(2D)}}(\omega) = \frac{e^2 s}{2^7 \eta}\frac{r^2}{\gamma\epsilon}\begin{cases} 1 & \text{for } \omega \ll \tau_\varphi^{-1} \\ \left(\dfrac{8T_c\gamma}{\pi\omega}\right)^2 & \text{for } \omega \gg \tau_\varphi^{-1} \end{cases} \tag{2}$$

where $\gamma = \dfrac{\pi}{8T_c\tau_\varphi}$.

Density-of-States Contribution. As far as the DOS contribution to the electromagnetic response-operator tensor is concerned, the four main diagrams (the other two of this kind are negligible in the case under consideration [3]) are presented in Fig. 1, (2–5). Summing up all these contributions and carrying out the analytical continuation, one can obtain for the total DOS contribution to the conductivity

$$\mathrm{Re}\left(\sigma_\perp^{\mathrm{DOS}}(\omega)\right) = -\frac{e^2}{2\pi s}\left(\frac{Ts^2J^2}{\eta}\right)\ln\left[\frac{2}{\sqrt{\epsilon + r} + \sqrt{\epsilon}}\right]\frac{1}{(\tau^{-2}+\omega^2)^2}\times \tag{3}$$

$$\times\left\{\frac{4}{\tau}\left[\psi\left(\frac{1}{2}\right) - \mathrm{Re}\psi\left(\frac{1}{2} - \frac{i\omega}{2\pi T}\right)\right] + \frac{\tau^{-2}+\omega^2}{4\pi T\tau}\frac{1}{\omega}\mathrm{Im}\psi'\left(\frac{1}{2} - \frac{i\omega}{2\pi T}\right) + \right.$$

$$\left. + \left(\tau^{-2} - \omega^2\right)\frac{1}{\omega}\left[\mathrm{Im}\psi\left(\frac{1}{2} - \frac{i\omega}{2\pi T}\right) - 2\,\mathrm{Im}\psi\left(\frac{1}{2} - \frac{i\omega}{4\pi T} + \frac{1}{4\pi T\tau}\right)\right]\right\}.$$

Let us stress that, in contrast to Eq. (1), this result has been obtained under the sole assumption that $\epsilon \ll 1$, so it is valid for any frequency, any impurity concentration, and any dimensionality of the fluctuation behavior.

Discussion. Let us start with an analysis of each fluctuation contribution separately, and then we will discuss their interplay in $\mathrm{Re}[\sigma_\perp(\omega)]$.

The AL contribution describes the response of the fluctuation condensate to the applied electromagnetic field. The component of the current associated with it can be regarded as a precursor phenomenon for the screening currents in the superconducting phase. Above T_c the binding energy of the virtual Cooper pairs is of the order of $T - T_c$, so it is not surprising that at higher frequencies the AL contribution decreases rapidly with further increase of ω. Actually $\omega_{\mathrm{AL}} \sim T - T_c$ is the only relevant scale for σ^{AL}: its frequency dependence doesn't contain $\mathcal{T}$, τ_ϕ, or τ. Its independence from the last is a mathematical manifestation of the fact that elastic impurities do not present an obstacle for the motion of Cooper pairs. The interaction of the electromagnetic wave with the fluctuational Cooper pairs resembles, in some way, the anomalous skin effect, where the wave reflection is determined by the interaction with the free electron system.

Another effect related to the formation of fluctuational Cooper pairs, but on self-intersecting trajectories (like the weak localization correction), is described by the MT *anomalous* contribution. Being the contribution due to the electric charge transfer of Cooper pairs it does not depend on the elastic scattering time, but it does turn out to be extremely sensitive to the phase-breaking mechanisms. So two characteristic scales turn out to be relevant in the frequency dependence of the MT contribution: $T-T_c$ and τ_ϕ^{-1}. In the case of HTSs, where τ_ϕ^{-1} has to be estimated as at least $0.1T_c$, the MT contribution is overdamped at temperatures up to 5–10 K above T_c, but in the 2D regime it is still dependent on temperature.

The contribution to $\mathrm{Re}\,[\sigma(\omega)]$ from the density-of-states fluctuation renormalization is quite different from the others. Physically it is related with the decrease of the one-electron density due to the involvement of some number of electrons in the fluctuational Cooper pairing. At low frequencies ($\omega \ll \tau^{-1}$) the lack of electron states at the Fermi level leads to the opposite effect as compared to the AL and MT contributions: $\mathrm{Re}\left[\sigma^{\mathrm{DOS}}(\omega)\right]$ turns out to be negative, implying an increase in the surface impedance, or, in other words, a decrease in the reflectance. Nevertheless, the applied electromagnetic field does affect the electron distribution, and at high frequencies $\omega \sim \tau^{-1}$ the DOS contribution changes sign. It is interesting that the DOS contribution, a one-electron effect, depends on the impurity scattering similiarly to the normal Drude conductivity. The decrease of $\mathrm{Re}\left[\sigma^{\mathrm{DOS}}(\omega)\right]$ starts at frequencies $\omega \sim \min\{T, \tau^{-1}\}$, which for HTSs are much higher than $T-T_c$ and τ_φ^{-1}.

The scenario of the ω dependence of $\mathrm{Re}[\sigma_\perp^{\mathrm{tot}}]$ with the most natural choice of parameters ($r \ll \epsilon < \tau_\varphi^{-1} \ll \min\{T, \tau^{-1}\}$) is presented in Fig.2. The positive AL and MT effects, in their ω dependence, are distinctly visible at low frequencies against the background of the DOS contribution, which in this region remains a negative constant. Then at $\omega \sim T-T_c$ the former decays, and $\mathrm{Re}\sigma_\perp$ remains negative up to $\omega \sim \min\{T, \tau^{-1}\}$. The DOS correction changes sign at $\omega \sim \tau^{-1}$, and the subsequent high-frequency behavior is governed by the Drude law. So one can see that the characteristic pseudogap-like behavior in the frequency dependence of the optical conductivity, mentioned in the title of this paper, takes place in the range $\omega \in [T-T_c, \tau^{-1}]$. The depth of the window increases logarithmically with ϵ as T tends to T_c.

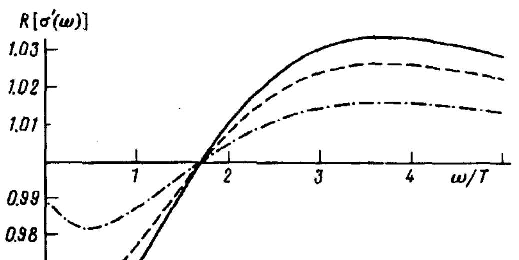

Fig.2.A plot of $\mathrm{Re}[\sigma'(\omega)] = \mathrm{Re}[\sigma_\perp(\omega)]/\sigma_\perp^{n}$, the real part of the c-axis component of the conductivity tensor normalized by the Drude normal conductivity $\sigma_\perp^n = \sigma_\parallel^n J^2 s^2/v_F^2$, as a function of ω/T for different temperatures. In this plot we have put $T\tau = 0.2$, $E_F/T = 100$, $r = 0.01$, $\epsilon = 0.04$, $T\tau_\varphi = 4$. The solid line refers to $\epsilon = 0.04$, the dashed line to $\epsilon = 0.06$, and the dot-and-dash line to $\epsilon = 0.08$

Let us now compare the results of our calculations with the experiments available. Recent measurements [1] of the c-axis reflectivity spectra in the FIR region on $YBa_2Cu_4O_8$ single crystals show the response of a poor metal, with additional contributions from IR-active phonon modes (which we do not discuss here). With decreasing temperature the c-axis optical conductivity decreases, exhibiting a transition from a Drude-like to a pseudogap-like behavior at $\omega \sim 180$ cm^{-1}. This gap becomes deeper below 180 K without any abrupt change at the superconducting transition temperature $T_c = 80$ K.

Such experimentally observed behavior of the optical conductivity is in qualitative agreement with our results. The suppression of the density of states due to the superconducting fluctuations in the vicinity of T_c actually does lead to a decrease of the reflectivity in the range of frequencies up to $\omega \sim \tau^{-1}$. The magnitude of this depression slowly (logarithmically) increases with decreasing temperature, but evidently at the edge of the transition it reaches some saturation value because of the crossover to the 3D regime in the fluctuations (where instead of $\ln(1/\epsilon)$ one has $\ln(1/\tau) - \sqrt{\epsilon}$; see Eq. (3)). Thus no singularity is expected in the value of the minimum even in the first order of perturbation theory. The fluctuation behavior of $\langle\Delta_{\rm fl}^2\rangle$ below T_c is largely symmetrical to that above T_c, and with further decrease in temperature, the fluctuation-pseudogap minimum in the optical conductivity smoothly transforms into a real superconducting gap, which in HTSs opens very sharply. Let us stress that the slow dependence of the pseudogap threshold on temperature appears naturally in our theory: it is determined by $\omega_0 \sim \tau^{-1}$ (see Eq. (3)). As to the numerical value of ω_0, if we suppose that $T_c\tau = 0.35$ (which is the value of the scattering rate for the sample used in the experiment under consideration [1] and is also in the experimental range of the inverse of the scattering rate, $T\tau \approx 0.3$–0.7), one can see that the pseudogap threshold is of the order of 200 cm^{-1}, in agreement with the experimental data, even from a quantitative point of view [1].

Strictly speaking, our theory is valid only in the vicinity of the critical temperature, where $\epsilon \ll 1$. Nevertheless the logarithmic dependence on ϵ of the result obtained gives reason to believe that qualititatively the theory can be valid up to $\epsilon = \ln(T/T_c) \sim 1$, or for temperatures up to 200 K in the experiment discussed. Thus the theory is again in agreement with the experimental value (180 K) of the temperature up to which the pseudogap is observable.

We would like to thank P. Calvani, M. Capizzi, C. Di Castro, D. Livanov, M. Randeria and V. Tognetti for valuable discussions. This work was partially supported by NATO Collaborative Research Grant #CRG 941187.

---

1. D.N.Basov, T.Timusk, B.Dabrowski, and H.A.Mook, Phys. Rev B **52**, 13141 (1995), Phys. Rev. B **50**, 3511 (1994).
2. Physics Today **49**, No. 6, pp. 17–19 (June 1996).
3. V.V.Dorin, R.A.Klemm, A.A.Varlamov, A.I.Buzdin, and D.V.Livanov, Phys. Rev. B **48**, 12951 (1993).
4. L.B.Ioffe, A.I.Larkin, A.A.Varlamov, and L.Yu, Phys. Rev. B **47**, 8936 (1993).
5. L.G.Aslamazov and A.A.Varlamov, J. Low Temp. Phys. **38**, 223 (1980).
6. H.Schmidt, Ann. Phys. (Leipzig) **216**, 336 (1968).