

ON PHASE ORDERING BEHIND THE PROPAGATING FRONT OF A SECOND-ORDER TRANSITION

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In a real system the heating is nonuniform and a second-order phase transition into a broken symmetry phase occurs by propagation of the temperature front. Two parameters, the cooling rate τ_Q and the velocity v_T of the transition front, determine the nucleation of topological defects. Depending on the relation of these parameters two regimes are found: in the regime of fast propagation defects are created according to the Zurek scenario for the homogeneous case, while in the slow propagation regime vortex formation is suppressed.

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1. Introduction. A common issue of particular interest both in cosmology and in condensed matter physics is an estimation of the initial defect density produced during a phase transition into a broken symmetry state [1, 2]. In cosmology this is an important issue so far as the later transitions are concerned – for example in extended models where there are stable defects produced at the electroweak transition. It is probably not of much practical importance for the GUT transition (if our ideas about scaling are correct) because in that case so much time has elapsed that no trace of the initial conditions remains [3, 4]. The same thing is true no doubt for some condensed matter systems, but it would be good to identify some situations in which predictions about the initial density (not just the later scaling value) could be tested. We really need to find cases (i) in which the transition is second-order, (ii) in which it is of first order and goes by bubble nucleation, and (iii) in which it goes by the ‘false vacuum’ becoming totally unstable, which has been called spinodal decomposition. All these cases correspond to different scenarios of the phase ordering below the phase transition, and the creation and evolution of the defects prior to the final establishment of long-range order (phase coherence, or generally the coherence of the Goldstone variables).

The great thing about superfluid helium-3 is that it allows such a wide range of possibilities. Most of these cases, or possibly all of them, may be represented in different regions of the parameter space: the normal ^3He to $^3\text{He-B}$ or normal ^3He to $^3\text{He-A}$ transition is second order, while the A-to-B transition is first order and proceeds in different ways depending on the pressure (and magnetic field) and the extent of supercooling [5]. Also of course, it adds another example to the list of possible systems, including nematics [6], helium-4 [7] and high- and low-temperature superconductors, but an example which is in important ways more

relevant since the fermion-boson interaction in ${}^3\text{He}$ shares many properties of the quantum field theory in high energy physics [8]. This is why it is so valuable as an analogue of the early universe.

One of the most interesting experiments, which can shed light on the phase ordering is the 'mini big bang' produced by thermal neutrons [9, 10]. There the exothermic nuclear reaction $n + {}^3_2\text{He} \rightarrow p + {}^3_1\text{H} + 0.76 \text{ MeV}$ produces a local Big Bang in ${}^3\text{He-B}$ — a region of high temperature, $T > T_c$, where the symmetry is restored. The subsequent cooling of this region back through the second-order superfluid transition results in the creation of a network of vortex lines. These seeds of vortex lines are grown by the applied rotation and measured with the NMR technique [9].

Theoretically the formation of defects in a second-order transition into a broken symmetry state, discussed in [1, 2], corresponds to a homogeneous phase transition. In real experiments the heating is never uniform. For example, in the neutron experiments the heating above T_c by the neutron and the subsequent cooling occur locally in a region of about $10 \mu\text{m}$ size. This results in a temperature gradient and thus in a propagating boundary of the second-order transition. The temperature gradient is present also in other experiments where defect formation is influenced by the details of the cooling through T_c ; see e.g. [11] where the type of ${}^3\text{He-A}$ vortices created depends on the cooling rate. One might think that the phase of the order parameter would be determined by the ordered state behind the propagating boundary and that vortex formation would be suppressed in this geometry. On the other hand, in the limit of very rapid motion of the boundary one comes to the situation of an instantaneous transition and the Kibble-Zurek mechanism should be restored. Here we discuss the criterion on the propagation velocity of a second-order phase transition which separates the two regimes. We shall illustrate it with the example of the experimentally studied normal ${}^3\text{He}$ to ${}^3\text{He-A}$ and normal ${}^3\text{He}$ to ${}^3\text{He-B}$ transitions.

2. Phase ordering in a spatially homogeneous transition. A nonequilibrium phase transition into the state in which the symmetry $U(1)$ is broken leads to the formation of an infinite cluster of the topological defects — vortices or strings [1]. As a result, when the temperature crosses T_c two length scales appear. One of them is the conventional coherence length, which diverges at T_c

$$\xi \approx \xi_0 \left(1 - \frac{T}{T_c}\right)^{-\nu} \quad (2.1)$$

(For superfluid ${}^3\text{He}$, where the Ginzburg-Landau theory is valid, one has $\nu = 1/2$). Another one is the mean distance $\tilde{\xi}$ between the vortices in the infinite cluster; it defines the scale within which the phase of the order parameter is correlated. This scale diverges with time when the vortex cluster decays. We are interested in the estimation of the initial density of the defects, $\tilde{\xi}_{\text{initial}}$, i.e., at the moment when these two scales become well defined.

According to Zurek [2], $\tilde{\xi}_{\text{initial}}$ is determined by the cooling rate τ_Q , which characterizes the time dependence of the temperature in the vicinity of the phase transition:

$$\epsilon(t) \equiv 1 - \frac{T(t)}{T_c} \approx \frac{t}{\tau_Q} \quad (2.2)$$

Well defined vortices are formed at the time when the regions within the Ginzburg-Landau coherence length become causally connected. Causal connection is estab-

lished by the propagation of the order parameter. In the case of superfluid ^3He the propagation velocity of the order parameter can be estimated as the velocity of spin waves which just represent the propagating oscillations of some components of the order parameter. Thus one has that the corresponding velocity also depends on time and is given by

$$c(t) \approx c_0 \epsilon^{1/2}(t) \quad , \quad (2.3)$$

where c_0 is of order of the Fermi velocity $v_F \sim 10^4$ cm.

At the moment $t \approx t_{coh}$ when

$$\xi(t_{coh}) \approx \int_0^{t_{coh}} c(t') dt' \quad , \quad (2.4)$$

the regions within the coherence length $\xi(t_{coh})$ are already connected, while outside they are causally disconnected. So the phase of the order parameter is well defined within the coherence length, but the phases in regions outside ξ do not match each other. This corresponds to well defined vortices with the separation $\tilde{\xi}_{initial} = \xi(t_{coh})$. The time t_{coh} after the transition when this happens is

$$t_{coh} \approx \sqrt{\tau_0 \tau_Q} \quad , \quad \tau_0 \approx \frac{\xi_0}{v_F} \approx \frac{\hbar}{\Delta_0} \sim 10^{-9} \text{ s} \quad , \quad (2.5)$$

and thus the initial separation between the vortices in the infinite cluster is

$$\tilde{\xi}_{initial} = \xi(t_{coh}) \approx \xi_0 \left(\frac{\tau_Q}{\tau_0} \right)^{1/4} \quad . \quad (2.6)$$

For the more general case, when the phase transition is not necessarily described by the Ginzburg–Landau theory, one has $\tilde{\xi}_{initial} \sim \xi_0 (\tau_Q / \tau_0)^\alpha$.

The further development of the infinite cluster, which leads to its final elimination, has been the subject of the intensive investigations in phase ordering kinetics (see e.g. the review [12]).

In the neutron experiments in $^3\text{He-B}$ the estimation for $\tau_Q \sim R_b^2 / D$, where R_b is the size of the bubble and D is the diffusion constant near T_c . This gives $\tau_Q \sim 10 \mu\text{s}$, and $\tilde{\xi}_{initial} \sim 10^{-4}$ cm. In the typical A-phase experiments the cooling of the A-phase through the transition from the normal liquid N occurs during $\tau_Q \sim 10^3 - 10^4$ s [11], which gives $\tilde{\xi}_{initial} \sim 10^{-2}$ cm.

3. The two regimes for a moving front. In the presence of a temperature gradient two parameters characterize the nonequilibrium phase transition. In addition to τ_Q , which determines the time scale of the temperature change, one has now the characteristic length scale of the temperature

$$\frac{1}{\lambda} \approx \frac{|\nabla T|}{T_c} \quad . \quad (3.1)$$

Combining with τ_Q this gives the velocity v_T of the propagating temperature front:

$$v_T \approx \frac{\lambda}{\tau_Q} \quad , \quad (3.2)$$

which is thus the velocity of the propagating second-order transition.

The homogeneous result Eq. (2.6) is obtained when the velocity of the front is large compared to $c(t_{coh})$, so the causality argument does work. This gives the estimation for the critical value of the velocity

$$v_{Tc} \approx \frac{\xi(t_{coh})}{t_{coh}} \approx c_0 \left(\frac{\tau_0}{\tau_Q} \right)^{1/4} . \quad (3.3)$$

If $v_T < v_{Tc}$, the slowly moving front dictates the phase of the order parameter and formation of vortices should be significantly suppressed compared to the case of a rapidly moving front. In $^3\text{He-B}$ neutron experiments one has $\lambda \sim R_b \sim 10 \mu\text{m}$ (where R_b is the maximum radius of the bubble of normal fluid above T_c) and $v_T \sim 10^3 \text{ cm/s}$. This is comparable with $c_0(\tau_0/\tau_Q)^{1/4} \sim 10^3 \text{ cm/s}$ so vortex nucleation is not suppressed by the moving interface in this experiment.

In the A-phase experiments the typical $\lambda \sim 10^2 \text{ cm}$ and $v_T \approx \lambda\tau_Q \sim 10^{-2} - 10^{-1} \text{ cm/s}$, which is much less than $c(t_{coh}) \sim 10 \text{ cm/s}$. Thus at this low v_T Eq. (2.6) does not hold since the phase correlation across the front occurs faster than due to the temporal change of T . So the creation of vortices is markedly suppressed. However even in this regime formation of defects has been experimentally observed, the formation of planar solitons in $^3\text{He-A}$ [13].

4. Phase ordering behind a slowly moving front. One may expect that in the whole range of v_T the initial density of vortices immediately behind the front is determined by the general scaling law:

$$\frac{\tilde{\xi}_{initial}(v_T, \tau_Q)}{\xi_0} = x^\alpha F(x^\beta y) \quad , \quad x = \frac{\tau_Q}{\tau_0} \quad , \quad y = \frac{v_T}{c_0} \quad , \quad (4.1)$$

where $F(u)$ is some function, which has different asymptotes in the two regimes discussed above. From the previous sections it follows that for ^3He one has $\alpha = \beta = 1/4$. The regime of fast propagation of the temperature front corresponds to the asymptote $F(u) \rightarrow 1$ when its argument $u \gg 1$. Let us find the asymptote in the regime of slow propagation, i.e. $F(u \ll 1)$.

Since the propagation velocity of the order parameter $c(t)$ slows down very near the transition, in some neighbourhood of the front, in a layer of thickness $\Delta z \approx v_T \Delta t$, the situation becomes 'homogeneous'. Here Δt is obtained from

$$v_T \Delta t \approx \int_0^{\Delta t} c(t) dt \approx c_0 \frac{(\Delta t)^{3/2}}{\tau_Q^{1/2}} \quad , \quad (4.2)$$

which gives

$$\Delta t \approx \tau_Q \frac{v_T^2}{c_0^2} . \quad (4.3)$$

Outside this layer the condensate phase is already fixed due to the phase correlations with the low temperature regions, which are transferred by order parameter waves propagating along the vertical axis. So the only source of the mismatch of the phase originates within this thin layer, which means that the magnitude of the coherence length $\xi(T)$ within the layer determines the initial distance between the vortices as a function of v_T at slow transition:

$$\tilde{\xi}_{initial}(v_T) \approx \xi(t \approx \Delta t) \approx \xi_0 \frac{c_0}{v_T} \quad , \quad \frac{v_T}{c_0} < \left(\frac{\tau_0}{\tau_Q} \right)^{1/4} . \quad (4.4)$$

Thus in the limit $u \ll 1$, we find $F(u) \sim 1/u$, so the initial length scale is essentially larger than in the case of a rapidly propagating front, where:

$$\tilde{\xi}_{initial}(v_T) \approx \xi_0 \left(\frac{\tau_Q}{\tau_0} \right)^{1/4}, \quad \frac{v_T}{c_0} > \left(\frac{\tau_0}{\tau_Q} \right)^{1/4}. \quad (4.5)$$

5. Discussion. Estimation of $\tilde{\xi}_{initial}$ for a slowly propagating front in the A-phase gives $\tilde{\xi}_{initial} \sim 0.01 - 0.1$ cm. This is significantly smaller than the size of the cell, ~ 1 cm, and thus well explains the appearance of solitons during the cooling into the A-phase in [13].

On the other hand this is of order of the intervortex distance in the rotating cryostat and thus can influence the vortex texture which appears in the rotating cryostat when the superfluid transition occurs under rotation [11]. In a field of 10 mT two types of vortices are competing in the equilibrium rotating state: singular one-quantum vortices with the core radius $r_{core} \sim \xi(T)$ and continuous two-quantum vortices (textures) with $r_{core} \sim \xi_D$, the dipole length $\xi_D \sim 10^{-3}$ cm. At low rotation velocities, $\Omega < 2$ rad/s (or in general at low vortex density), the array of singular one-quantum vortices has less energy. The experimental evidence is that well below 1 rad/s singular vortices are created after the superfluid transition under rotation, and no dependence on the cooling rate was observed. However, the experiment at higher velocity, $\Omega \approx 1.4$ rad/s, showed such dependence: at slow transition the equilibrium (singular) vortices dominate in the cell — their fraction is $n_s/(n_s + 2n_c) \sim 0.8$ — while at fast transition the fraction of continuous vortices $2n_c/(n_s + 2n_c)$ sharply increases from 0.2 up to 0.8. The change occurs in a jump-like manner at $\partial_t T \sim 6 \mu\text{K}/\text{min}$ which corresponds to $\tau_Q \sim 3 \cdot 10^4$ s. One may expect that this is related to the phase ordering process, which leads to an initial vortex density larger than the equilibrium one. The further relaxation of the initial network towards equilibrium may discriminate between textures and singular vortices, which have significantly different scales.

6. Conclusion. In a spatially inhomogeneous phase transition vortex formation depends on the velocity v_T of the propagating front of the second-order phase transition. There is a critical value of the front velocity, $v_{Tc} \approx c_0(\tau/\tau_Q)^{1/4}$, which separates two regimes. For a rapidly propagating front, with $v_T > v_{Tc}$, vortex formation is the same as in a spatially homogeneous phase transition. For a slowly propagating front, with $v_T < v_{Tc}$, vortex formation becomes less favorable with decreasing v_T .

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