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COMMENT ON VORTEX MASS AND QUANTUM TUNNELING OF VORTICES.

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Vortex mass in Fermi superfluids and superconductors and its influence on quantum tunneling of vortices are discussed. The vortex mass is essentially enhanced due to the fermion zero modes in the core of the vortex: the bound states of the Bogoliubov qiasiparticles localized in the core. These bound states form the normal component which is nonzero even in the low temperature limit. In the collisionless regime $\omega_0\tau \gg 1$, the normal component trapped by the vortex is unbound from the normal component in the bulk superfluid/superconductors and adds to the inertial mass of the moving vortex. In the d-wave superconductors, the vortex mass has an additional factor $(B_{c2}/B)^{1/2}$ due to the gap nodes.

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The vortex mass is thought to be an important issue for the problem of the quantum tunneling of vortices. The latter problem is popular now and many experiments are discussed in terms of the macroscopic quantum tunneling of vortices in superfluids or superconductors. The firm experimental prove for the quantum nucleation of vortices is still absent. On the other hand the characteristic plateau in the temperature dependence of the critical velocity, which is always ascribed to the quantum nucleation, has been also observed in superfluid ^{3}He-B [1]. However the time of the quantum nucleation of the vortex in ^{3}He-B is 10^{10^6}, which is extremely big in any units. The vortices in ^{3}He-B are created in the process of the development of the classical instability of the superflow at the pair-breaking velocity. The reason of the plateau is that the characteristic physical quantities, such as the gap amplitude Δ, which determine the treshold of instability become temperature independent at low T. The intrinsic instability thus provides an alternative explanation of the plateau observed in many different systems including superfluid ^{4}He.

In the vortex tunneling problem the inertial mass becomes important only if its effect is comparable with the effect of the Magnus force. That is why the magnitude of the inertial mass is of main importance. It appears that in Fermi superfluids and superconductors the mass of the vortex is essentially enhanced compared to the vortex mass in superfluid ^{4}He, where it is determined by the compressibility. In Fermi systems the fermions bound to the vortex core give the dominating contribution, as was first found by Kopnin [2]. We discuss this effect in details and relate it to the normal component trapped by the vortex. This effect is even more enhanced in d-wave superconductors where the vortex traps an essential part of the bulk excitations due to the gap nodes.

Volume law and area law for the quantum tunneling. In the earlier estimations of the vortex tunneling rate the mass of vortex line was neglected

[3, 4]. When the mass is neglected the tunneling exponent $\exp -S_{eff}$ is determined by the volume V within the surface swept by the classical trajectory of the vortex in the process of the quantum tunneling:

$$S_{eff}/\hbar = 2\pi\mathcal{N} \quad , \quad \mathcal{N} = nV \ . \tag{1.1}$$

Here n is the particle density; $\mathcal{N}$ is the number of particles in the volume V. The volume law for the vortex action follows from the general laws of the vortex dynamics governed by the Magnus force [5].

In Ref.[3] the tunneling trajectory between the ground state of the superfluid and the state with a vortex, was generated by irregularity (pinning center) on the container wall in the presence of the superflow with the asymptotic superfluid velocity v_s. For small v_s the tunneling exponent does not depend on the pinning center and corresponds to the volume

$$V = \frac{4\pi}{3} R_0^3 \ . \tag{1.2}$$

Here R_0 is the radius of the nucleated vortex ring:

$$R_0 = (\kappa/2\pi v_s) \ln \frac{R_0}{R_{core}} \tag{1.3}$$

and R_{core} is the core size, which is of order coherence length ξ.

The volume law for the tunneling exponent S_{eff} was confirmed in Ref.[4], where S_{eff} was found as the overlapping integral of the many-body wave function. This S_{eff} was then minimized with respect to the velocity field in the vortex. The extremal trajectory corresponds to the formation of the intermediate vortex state with the deformed velocity field around the vortex loop. The resulting volume V is logarithmically reduced compared with the Eq.(1.2) for the direct formation of the equilibrium vortex:

$$S_{eff}/\hbar = 2\pi n V \quad , \quad V = \frac{27}{\pi \ln \frac{R_0}{R_{core}}} R_0^3 \ . \tag{1.4}$$

In this approach the volume law reflects the general property of the macroscopic quantum tunneling: the tunneling exponent is proportional to the number $\mathcal{N}$ of particles, effectively participating in the tunneling. This was also found in other systems [6, 7].

When the problem of the vortex tunneling was revived due to the experiments on the vortex creep in superconductors, the effect of the vortex mass was discussed [8]. If the mass term is more important for the quantum tunneling than the Magnus force, then the volume law of Eq.(1.1) should transfer to the area law. The quadratic dependence of S_{eff} on R_0 (area law) was also obtained using the field theory in [9, 10], where the vortex nucleation was considered as a process analogous to the Scwinger production of the electron-positron pairs in electric field. The result for the semi-classical tunneling exponent is

$$S_{eff} = \int_0^{R_0} dR \ \sqrt{2M(R)E_{\rm vortex}(R)} \quad , \tag{1.5}$$

where $E_{\rm vortex}(R) \sim R \ln \frac{R}{R_{core}}$ is the energy of the vortex ring of radius R and $M(R)$ is the mass of the vortex loop. Since $M(R)$ is also $\propto R$ the tunneling rate is proportional to the area R_0^2 of the nucleated vortex ring.

This area law for the action is typical for the dynamics of string loops in systems without Magnus force, such as cosmic strings (see [11]), vortex rings in charge-density-wave system [12], in antiferromagnets, etc. The breaking of the time inversion symmetry introduces the Magnus force even in these systems (see Ref.[13] on vortices in planar magnets and Ref.[14] on spinning global strings), and the volume law can be restored.

Hydrodynamic mass of the vortex. In the hydrodynamic theory the mass of the vortex is nonzero due to compressibility of the liquid which leads to the "relativistic" expression [9, 15, 10, 16]

$$M_{\rm hydro} = \frac{E_{\rm vortex}}{s^2} \quad , \tag{2.1}$$

where s is sound velocity. For Fermi superfluids s is of order the Fermi velocity $v_F \sim p_F/m$ (m is the mass of the electron or of the ^{3}He atom), and the estimation for the hydrodynamic mass of the vortex loop of length L is

$$M_{\rm hydro} \sim Lmk_F \ln\frac{L}{\xi} \quad . \tag{2.2}$$

However in this consideration the fermions in the vortex core [17] are neglected. They produce the effective mass proportional to the core area $R^2_{core} \sim \xi^2$ [2, 18, 19, 20]:

$$M_{\rm bound\ \ states} \sim Lmk_F(k_F\xi)^2 \quad . \tag{2.3}$$

Though it does not contain the logarithmic divergence, it gives the main contribution since the core radius $\sim \xi$ in superfluid ^{3}He-B and superconductors is large compared with the interatomic space: $k_F\xi \gg 1$. The mass of the vortex is essentially enhanced, so the arguments, that the effect of the vortex mass on the vortex tunneling is negligible [3, 25], become shaky. That is why it is worthwhile to consider the effect of core fermions more thoroughly.

Bound states contribution to the vortex mass: Normal component in the vortex core in collisionless regime. The core contribution to the vortex mass was obtained by Kopnin [2] in a rigorous microscopic theory for the vortex dynamics developed by Kopnin and Kravtsov [26]. Here we associate it with the normal component trapped by the core texture. At low T the core contribution to the vortex dynamics is completely determined by the low-energy excitations in the vortex core, which energy spectrum is $E = -Q\omega_0(k_z)$ in the vortex frame [17]. Here Q is the angular momentum of fermions and $\omega_0(k_z)$ is the interlevel spacing, which depends on the linear momentum $k_z = k_F\cos\theta$ along the vortex axis ($\omega_0 \sim \Delta^2/E_F \ll \Delta$). If the temperature is large enough, $\omega_0 \ll T \ll T_c$, this branch is characterized by the density of states $N(0) = 1/\omega_0(k_z)$.

If the vortex moves with velocity $\mathbf{v}_L$ with respect to the superfluid component, the fermionic spectrum in the vortex frame is Doppler shifted $E = -Q\omega_0(k_z) - \mathbf{k}\cdot\mathbf{v}_L$. In the collisionless regime, $\omega_0\tau \gg 1$, the exchange between the fermions in the vortex core and in the heat bath vanishes and the linear momentum of the bound states fermions adds to the momentum of the moving vortex. The summation of fermionic momenta in the moving vortex leads to the extra linear momentum of the vortex $\propto \mathbf{v}_L$ (see also Eq.(5.7) in Ref.[20]):

$$\mathbf{P} = \sum \mathbf{k}\theta(-E) = M_{\rm bound\ \ states}\mathbf{v}_L \quad , \tag{3.1}$$

$$M_{\text{bound states}} = L\int_{-k_F}^{k_F} \frac{dk_z}{4\pi}\frac{k_\perp^2}{\omega_0(k_z)} \quad , \tag{3.2}$$

This is an extra vortex mass which is by factor $(k_F\xi)^2$ larger than the hydrodynamical mass of the vortex.

The Eq.(3.2) represents the dynamical mass of the vortex in the low temperature limit and only in the clean (or collisionless) regime, when the exchange between the core fermions and the heat bath is suppressed. Actually it was assumed that $T_c \gg T \gg \omega_0 \gg 1/\tau$. In this regime there is no spectral flow between the bound fermions and the heat bath, as a result during the vortex motion the momentum of core fermions is not transferred to the heat bath and adds to the momentum of the vortex, producing an extra inertia. In other words, this is the contribution of the normal component associated with the vortex core, which in the collisionless regime is trapped by the vortex and is transferred together with the vortex.

For vortices, which core size $R_{core} \gg \xi$, this extra vortex mass can be represented as the integral over the local density of the normal component

$$M_{\text{bound states}} = \int d^3r \;\; \rho_n(\mathbf{r}, T=0) \quad . \tag{3.3}$$

This nonzero normal component at $T=0$ is produced by the inhomogeneous order parameter, the texture. This can be seen on the simplest example of continuous vortex in ^{3}He-A-phase, where the corresponding texture is the field of the unit vector $\hat{\mathbf{l}}$ along the orbital angular momentum of Cooper pairs. Let us choose the texture in the form

$$\hat{\mathbf{l}}(\mathbf{r}) = \hat{\mathbf{z}}\cos\eta(r) + \hat{\mathbf{r}}\sin\eta(r) \quad , \tag{3.4}$$

with $\hat{\mathbf{l}}(0) = -\hat{\mathbf{z}}$ and $\hat{\mathbf{l}}(\infty) = \hat{\mathbf{z}}$. This texture represents the doubly quantized continuous vortex in ^{3}He-A (see Eq.(5.21) in Review [21]), the latest experiments on such vortices are discussed in [22].

The $\hat{\mathbf{l}}$-texture leads to the normal component tensor even at $T=0$ [23] (see Eq.(5.24) in [24]):

$$(\rho_n)_{ij}(\mathbf{r}) = \frac{k_F^4}{2\pi^2\Delta_A}|(\hat{\mathbf{l}}\cdot\vec{\nabla})\hat{\mathbf{l}}| \;\; \hat{l}_i\hat{l}_j \quad , \tag{3.5}$$

where Δ_A is the gap amplitude in ^{3}He-A. For the texture in Eq.(3.4) one has $|(\hat{\mathbf{l}}\cdot\vec{\nabla})\hat{\mathbf{l}}| = \sin\eta \;\; \partial_r\eta$, so the normal component contribution to the vortex mass should be

$$M_{\text{bound states}}\delta_{\perp ij} = \int d^3r(\rho_n)_{ij} = L\frac{k_F^4}{2\pi\Delta_A}\int_0^\infty dr \;\; r\sin^3\eta \;\; \partial_r\eta \quad . \tag{3.6}$$

The Eq.(3.6) for the vortex mass in terms of the local normal component coincides with the general Eq.(3.2) for the vortex mass in terms of $\omega_0(k_z)$. The interelevel spacing for this continuous vortex was found by Kopnin [18]

$$\omega_0(k_z) = \frac{\Delta_A}{k_F r(k_z)} \quad , \quad \cos\eta(r(k_z)) = \frac{k_z}{k_F}. \tag{3.7}$$

Here $r(k_z)$ is the radius where the energy of the fermion $E(r,\vec{k}) = \sqrt{v_F^2(k-k_F)^2 + \Delta_A^2(\hat{\mathbf{l}}(r)\times\hat{\mathbf{k}})^2}$ is zero at given k_z. The Eq.(3.2) gives [18]

$$M_{\text{bound states}} = L\int_{-k_F}^{k_F}\frac{dk_z}{4\pi}\frac{k_\perp^2}{\omega_0(k_z)} = \frac{k_F}{4\pi\Delta_A}\int_{-k_F}^{k_F} dk_z(k_F^2 - k_z^2)r(k_z).$$

After inverting the function $r(k_z)$ in Eq.(3.7) into $k_z(r) = k_F \cos\eta(r)$ one obtains the Eq.(3.6).

Vortex mass from the kinetic equation. The above results for the vortex mass can be proved using the kinetic equation for the fermions bound to the core [2, 18, 19]. The inertial term in the force balance for the vortex is obtained by substitution $\frac{1}{\tau}$ by $\frac{1}{\tau} - i\omega$ in the equation for the longitudinal (dissipative friction) force acting on the vortex line, where ω is external frequency identified with the frequency of the oscillations of the vortex line. In the temperature region $\omega_0 \ll T \ll T_c$ one has [18]

$$\mathbf{F}_{\rm long} = -\mathbf{v}_L \frac{k_F^3}{4\pi} L \int d\cos\theta \ \sin^2\theta \ \left(\frac{1}{\tau} - i\omega\right) \frac{\omega_0}{\omega_0^2 + \left(\frac{1}{\tau} - i\omega\right)^2} \quad . \tag{3.8}$$

In the limit case $\omega_0 \gg \omega \gg 1/\tau$ one obtains $\mathbf{F}_{\rm long} = i\omega \mathbf{v}_L M_{\rm bound\ states}$ with the vortex mass

$$M_{\rm bound\ states} = \frac{3\pi}{4} L C_0 \int d\cos\theta \ \sin^2\theta \ \frac{1}{\omega_0(\theta)} \quad , \tag{3.9}$$

where $C_0 = k_F^3/3\pi^2$ is close to the particle density n. This corresponds to Eq.(3.2).

In the high frequency limit $\omega \gg \omega_0 \gg 1/\tau$ the Eq.(3.8) leads to the "dielectric" behavior with the "pinning potential"

$$U = \frac{1}{2}\alpha \mathbf{r}_L^2 \quad , \quad \alpha = \frac{k_F^3}{4\pi} \int d\cos\theta \ \sin^2\theta \ \omega_0(\theta) \quad . \tag{3.10}$$

Discussion. The vortex inertia is essentially enhanced due to the fermion zero modes in the vortex core. This fermionic contribution to the vortex mass appears when the characteristic frequency is small compared to the interlevel distance $\omega \ll \omega_0$. The characteristic frequency of the tunneling process can satisfy this condition, since $\omega \sim \sqrt{E_{\rm vortex}(R_0)/M(R_0)R_0^2} \sim \omega_0\xi/R_0 < \omega_0$. If $\omega > \omega_0$ the more general contribution of the core fermions, the Eq.(3.8), is to be applied. But even in this case the effect of the fermions is always larger then the contribution of the hydrodynamic mass in Eq.(2.1). This is because the frequency ω of the vortex motion cannot exceed the magnitude of the gap Δ, otherwise the simple approach to the vortex dynamics is not valid. This means that the hydrodynamic mass in Eq.(2.1) never enters the tunneling rate in Fermi superfluids and superconductors.

On the other hand because of the limited frequency the effect of the inertial mass on the vortex tunneling is still small compared to the effect of the Magnus force. Since $\omega \ll \omega_0$ the kinetic term $M\dot{\mathbf{v}}_L = -i\omega M\mathbf{v}_L \sim \hbar n L(\omega/\omega_0)$ is always smaller than the Magnus force $\pi\hbar n L\hat{z} \times \mathbf{v}_L$. That is why the volume law for the tunneling exponent in Eq.(1.1) is still dominating.

Situation can change in the regime $\omega_0\tau \ll 1$, where the Magnus force is suppressed by the spectral flow of fermions: $\pi\hbar n L\hat{z} \times \mathbf{v}_L \rightarrow \pi\hbar(n - C_0)L\hat{z} \times \mathbf{v}_L$ [27, 28, 19, 20].

The vortex mass can be also important in the d-wave superconductor, where the effect of the fermions on the vortex is more pronounced due to gap nodes [29]. In these superconductors with highly anisotropic gap the interlevel spacing depends on the azimuthal angle α between the momentum $\mathbf{k}$ in the $a-b$ plane and the direction of the gap nodes [29]

$$\omega_0(\alpha) \approx \alpha^2 \ \frac{\Delta_0^2}{E_F} \ln\frac{1}{|\alpha|} \quad , \tag{4.1}$$

where Δ_0 is the gap amplitude. The vortex mass in Eq.(3.2) is:

$$M\delta_{\perp ij} = L\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\int_0^{2\pi}\frac{d\alpha}{2\pi}k_{\perp i}k_{\perp j}\frac{1}{\omega_0(k_z,\alpha)} \quad . \tag{4.2}$$

With Eq.(4.1) for $\omega_0(\alpha)$ the integral over α diverges near the gap nodes. The cut-off $\alpha_{min} \sim \xi/R_v$, where $R_v \sim \xi\sqrt{B_{c2}/B}$ is the intervortex distance, gives the $\sqrt{B_{c2}/B}$ enhancement of the vortex mass:

$$M \sim mk_F^3\xi^2 L\sqrt{\frac{B_{c2}}{B}} \quad . \tag{4.3}$$

This equation holds if $1 \gg B/B_{c2} \gg T^2/T_c^2$ and $B/B_{c2} \gg E_F/\tau\Delta_0^2$.

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