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CALORIMETER EVENT SELECTION FOR SMALL ANGLE BHABHA SCATTERING AT LEP1

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An analytical method is applied for description of calorimeter event selection in small-angle electron–positron scattering at LEP1. The selections CALO1 and CALO2 are investigated specifically. The first-order correction to the Born cross section is given in the case of wide–narrow angular acceptance.

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The small-angle Bhabha scattering (SABS) process is used to measure the luminosity of electron–positron colliders. Accurate theoretical determination of the SABS cross section therefore has a direct bearing on the physical values measured in LEP1 experiments [1]. In recent years considerable attention has been devoted to Bhabha scattering process (see [2-5] and references therein).

There are two methods of theoretical investigation of the SABS cross section at LEP1: an approach based on Monte Carlo calculations and an analytical approach. The latter is used to check different Monte Carlo programs for *ideal* experimental conditions. In this letter I give for the first time an analytical result for the two calorimeter event selections (CES) labeled in [3] as CALO1 and CALO2 in the case of wide–narrow angular acceptance. Discussion is resticted to the first-order correction. The second- and third-order leading corrections can be written with the help of the electron structure function, but the second-order next-to-leading correction requires considerable additional effort.

Before studying CES it is helpful to clarify the inclusive event selection (IES), when only the final electron and positron energies are recorded by means of wide–narrow circular detectors. The result will be widely applicable for the description of CES.

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1. We introduce the dimensionless quantity

$$\Sigma = \frac{1}{4\pi\alpha^2} Q_1^2 \sigma_{\exp} \ , \tag{1}$$

where $Q_1^2 = \varepsilon^2\theta_1^2$ (ε is the beam energy and θ_1 is the minimum angle of the wide detector). The "experimetally" measurable cross section $\sigma_{\exp}$ is defined as

$$\sigma_{\exp} = \int dx_1 dx_2 \Theta d^2 q_1^\perp d^2 q_2^\perp \Theta_1^c \Theta_2^c \frac{d\sigma(e^+ + e^- \to e^+ + e^- + X)}{dx_1 dx_2 d^2 q_1^\perp d^2 q_2^\perp} \ , \tag{2}$$

where X represents undetected final particles, and x_1 (x_2) and $\mathrm{q}_1^\perp$ ($\mathrm{q}_2^\perp$) are the energy fraction and the transverse component of the momentum of the electron (positron) in the final state. The functions Θ_i^c take into account the angular cuts, while the function Θ takes into account the cutoff on the invariant mass of detected electron and positron:

$$\Theta_1^c = \theta(\theta_3 - \theta_-)\theta(\theta_- - \theta_1) \ , \ \Theta_2^c = \theta(\theta_4 - \theta_+)\theta(\theta_+ - \theta_2) \ , \ \Theta = \theta(x_1 x_2 - x_c) \ ,$$

$$\theta_- = \frac{|\mathrm{q}_1^\perp|}{x_1\varepsilon} \ , \quad \theta_+ = \frac{|\mathrm{q}_2^\perp|}{x_2\varepsilon} \ . \tag{3}$$

For the wide–narrow case

$$\theta_3 > \theta_4 > \theta_2 > \theta_1 \ , \ \rho_i = \theta_i/\theta_1 > 1 \ .$$

The first-order correction Σ_1, which includes the contributions of virtual and real soft and hard photon emission processes, is given by

$$\Sigma_1 = \frac{\alpha}{2\pi}\left\{ \int_1^{\rho_3^2} \frac{dz}{z^2}\left[-\Delta_{42}\delta(1-x) + \int_{x_c}^1 \left((L-1)P_1(x)(\Delta_{42} + \Delta_{42}^{(x)}) + \frac{1+x^2}{1-x}\overline{K} \right) dx \right] + \right.$$

$$\left. + \int_{\rho_2^2}^{\rho_4^2} \frac{dz}{z^2}\left[-\delta(1-x) + \int_{x_c}^1 \left((L-1)P_1(x)(1+\theta_3^{(x)}) + \frac{1+x^2}{1-x} K \right) dx \right] \right\} \ , \tag{4}$$

where

$$P_1(x) = \frac{1+x^2}{1-x}\theta(1-x-\Delta) + (2\ln\Delta + \frac{3}{2})\delta(1-x) \ , \quad \Delta \to 0 \ ,$$

$$\overline{K}(x,z;\rho_4,\rho_2) = \frac{(1-x)^2}{1+x^2}(\Delta_{42} + \Delta_{42}^{(x)}) + \Delta_{42}\overline{L}_1 + \Delta_{42}^{(x)}\overline{L}_2 + (\overline{\theta}_4^{(x)} - \theta_2^{(x)})\overline{L}_3 + (\overline{\theta}_4 - \theta_2)\overline{L}_4 \ ,$$

$$K(x,z;\rho_3,1) = \frac{(1-x)^2}{1+x^2}(1+\theta_3^{(x)}) + L_1 + \theta_3^{(x)} L_2 + \overline{\theta}_3^{(x)} L_3 \ ,$$

$$\overline{L}_1 = \ln\left| \frac{(z-\rho_2^2)(\rho_4^2 - z)x^2}{(x\rho_4^2 - z)(x\rho_2^2 - z)} \right| \ , \quad \overline{L}_2 = \ln\left| \frac{(z - x^2\rho_2^2)(x^2\rho_4^2 - z)}{x^2(x\rho_4^2 - z)(x\rho_2^2 - z)} \right| \ ,$$

$$\overline{L}_3 = \ln\left| \frac{(z - x^2\rho_2^2)(x\rho_4^2 - z)}{(x^2\rho_4^2 - z)(x\rho_2^2 - z)} \right| \ , \quad \overline{L}_4 = \ln\left| \frac{(z-\rho_2^2)(x\rho_4^2 - z)}{(\rho_4^2 - z)(x\rho_2^2 - z)} \right| \ , \tag{5}$$

$$\Delta_{42}^{(x)} = \theta_4^{(x)} - \theta_2^{(x)} \ , \quad \Delta_{42} + \theta_4 - \theta_2$$

$$\theta_i^{(x)} = \theta(x^2\rho_i^2 - z) \ , \ \theta_i = \theta(\rho_i^2 - z) \ , \quad \overline{\theta}_i^{(x)} = 1 - \theta_i^{(x)} \ , \quad \overline{\theta}_i = 1 - \theta_i \ .$$

$$\theta_3^{(x)} = \theta(x^2\rho_3^2 - z) \;,\quad \overline{\theta}_3^{(x)} = 1 - \theta_3^{(x)} = \theta(z - x^2\rho_3^2) \;,$$

and L_i can be obtained from $\overline{L}_i$ by the substitution $\rho_4 \to \rho_3$, $\rho_2 \to 1$. See Ref. [4] for a definition of the variables used.

The term in the first (second) set of square brackets in Eq. (4) is the contribution due to real and virtual photon emission by the positron (electron). The terms containing x-dependent θ functions under the integral sign correspond to initial-state corrections, while the rest correspond to final-state corrections.

2. The CALO1 cluster is a cone with angular radius $\delta = 0.01$ around the final electron (or positron) momentum direction. If a photon belongs to a cluster, then the whole cluster energy is measured by the detector, and the electron can have any energy. Therefore the limits of integration of σ_{exp} over x are expanded to the interval 0 to 1 in this case. If the photon escapes from the cluster, the event looks the same as in IES. The above restrictions on the limits of integration over x can be written symbolically as follows:

$$\int_{x_c}^{1} dx + \int_{0}^{x_c} (if|\mathbf{r}| < \theta_0)dx \quad \equiv \int_{0}^{1} dx - \int_{0}^{x_c} (if|\mathbf{r}| > \theta_0)dx \;, \tag{6}$$

where $\mathbf{r} = \mathbf{k}/\omega - \mathbf{q}_1/\varepsilon_1$, and $\omega(\mathbf{k})$ is the energy (transverse momentum) of the hard photon. It is convenient to separate the contributions due to electron and positron emission.

$$\Sigma_1 = \Sigma^\gamma + \Sigma_\gamma \;,\quad \Sigma^\gamma = \Sigma_i + \Sigma_f + \Sigma_i^c + \Sigma_f^c \;,$$
$$\Sigma_\gamma = \widetilde{\Sigma}_i + \widetilde{\Sigma}_f + \widetilde{\Sigma}_i^c + \widetilde{\Sigma}_f^c \;. \tag{7}$$

The contributions in Eq. (7) labeled with a superscript c depend on both the shape and size of the cluster, while the rest are universal and are suitable for any cluster.

For calculation of the initial-state corrections, labeled by a subscript i, we use the left-hand side of relation (6), while for the final-state corrections, labeled with a subscript f, we use the right-hand side of this relation.

The quantity Σ_i coincides exactly with the case of IES (see Eq. (4) and the comments following it), while Σ_f looks like the contribution due to final-state electron emission in IES except for expanded limits of integration over x.

It may be written in the following simple form:

$$\Sigma_f = \frac{\alpha}{2\pi} \int_{\rho_2^2}^{\rho_4^2} \frac{dz}{z^2} \left\{ -\frac{1}{2} + \int_{0}^{1} \left[1 - x + \frac{1+x^2}{1-x} L_1 \right] dx \right\} \;. \tag{8}$$

To find the additional (cluster-shape-dependent) contributions it is sufficient to use the simplified form of the differential cross section for single photon emission suitable for semicollinear kinematics. The additional contribution for the initial-state electron emission reads

$$\Sigma_i^c = \frac{\alpha}{2\pi} \int_{0}^{x_c} \frac{1+x^2}{1-x} dx \int \frac{dz}{z^2} \int dz_1 \Psi\Phi(z_1, z; \lambda, x) \;,\quad \lambda = \delta/\theta_1 \;. \tag{9}$$

The quantity Φ specifies the limits of integration

$$\Psi = [a^2, a_0^2](x^2 z_+, x^2) + [b^2, a^2](x^2 z_+, x^2 z_-) + [b_0^2, b^2](x^2\rho_3^2, x^2 z_-) \;,\quad z_\pm = (\sqrt{z} \pm \lambda(1-x))^2 \;,$$

where the pairs in the square brackets and parentheses give the upper and lower limits of integration over z and z_1, respectively. For wide–narrow angular acceptance

$$a_0 = \rho_2 \ , \quad b_0 = \rho_4 \ , \quad a = \max(\rho_2, 1 + \lambda(1-x)) \ , \quad b = \min(\rho_4, \rho_3 - \lambda(1-x)) \ .$$

The function Φ under the integral sign in the right-hand side of Eq. (9) is given by

$$\Phi = \frac{2}{\pi}\left(\frac{1}{z_1 - xz} + \frac{1}{z - z_1}\right)\arctan\left\{\frac{z - z_1}{(\sqrt{z} - \sqrt{z_1})^2}R\right\} \ , \tag{10}$$

$$R = \sqrt{\frac{\lambda^2 x^2 (1-x)^2 - (\sqrt{z_1} - x\sqrt{z})^2}{(\sqrt{z_1} + x\sqrt{z})^2 - \lambda^2 x^2 (1-x)^2}} \ .$$

The additional contribution due to final-state electron emission may be may be written as

$$\Sigma_f^c = \frac{\alpha}{2\pi}\int\limits_0^{x_c}\frac{1+x^2}{1-x}dx\left[\int\limits_{a_0^2}^{b^2}\frac{dz}{z^2}\left(\ln\left|\frac{x\rho_3^2 - z}{\rho_3^2 - z}\right| + l_+\right) + \int\limits_{a^2}^{b_0^2}\frac{dz}{z^2}\left(\ln\left|\frac{x - z}{1 - z}\right| + l_-\right) + \right.$$

$$\left. + \int\frac{dz}{z^2}\int dz_1 \Psi F(z_1, z; \lambda, x)\right] \ , \quad l_\pm = \ln\frac{\lambda[2\sqrt{z} \pm \lambda(1-x)]}{z \mp 2x\lambda\sqrt{z} - \lambda^2 x(1-x)} \ , \tag{11}$$

$$F = \frac{2}{\pi}\left(\frac{1}{z_1 - xz} - \frac{1}{z_1 - x^2 z}\right)\arctan\left\{\frac{(\sqrt{z_1} - x\sqrt{z})^2}{z_1 - x^2 z}R^{-1}\right\} \ .$$

As to the contribution due to the positron emission, the quantity $\tilde{\Sigma}_i$ is equal to the part in the first square brackets in Eq. (4) which is multiplied by the x-dependent θ functions. To obtain $\tilde{\Sigma}_f$ it is sufficient to expand the limits of integration over x to the interval 0 to 1 for the rest of this part of Eq. (4). The result is

$$\tilde{\Sigma}_f = \frac{\alpha}{2\pi}\int\limits_1^{\rho_3^2}\frac{dz}{z^2}\left[-\frac{1}{2}\Delta_{42} + \int\limits_0^1\left((1 - x + \frac{1+x^2}{1-x}\tilde{L}_1)\Delta_{42} + \frac{1+x^2}{1-x}(\bar{\theta}_4 - \theta_2)\tilde{L}_4\right)dx\right] \ . \tag{12}$$

The cluster-shape-dependent contribution due to initial positron emission coincides with the right-hand side of Eq. (9) except for the limits of integration over z and z_1 and can be derived by using $\tilde{\Psi}$ instead of Ψ:

$$\tilde{\Psi} = [\tilde{b}^2, \tilde{a}^2](x^2 z_+, x^2\rho_2^2) + [\tilde{c}^2, \tilde{b}^2](x^2 z_+, x^2 z_-) + [\tilde{d}^2, \tilde{c}^2](x^2\rho_4^2, x^2 z_-) \ , \tag{13}$$

where

$$\tilde{a} = \max(1, \rho_2 - \lambda(1-x)) \ , \ \tilde{b} = \rho_2 + \lambda(1-x) \ , \ \tilde{c} = \rho_4 - \lambda(1-x) \ , \ \tilde{d} = \min(\rho_4 + \lambda(1-x), \rho_3) \ .$$

Finally, the quantity $\tilde{\Sigma}_f^c$ may be written as

$$\tilde{\Sigma}_f^c = \frac{\alpha}{2\pi}\int\limits_0^{x_c}\frac{1+x^2}{1-x}dx\left[\int\limits_{\tilde{a}^2}^{\tilde{c}^2}\frac{dz}{z^2}\left(\ln\left|\frac{x\rho_4^2 - z}{\rho_4^2 - z}\right| + l_+\right) + \int\limits_{\tilde{b}^2}^{\tilde{d}^2}\frac{dz}{z^2}\left(\ln\left|\frac{x\rho_2^2 - z}{\rho_2^2 - z}\right| + l_-\right) + \right.$$

$$+\int_1^{\rho_3^2}\frac{dz}{z^2}[\theta(\tilde{a}^2-z)-\theta(z-\tilde{d}^2)]\widetilde{L}_4+\int\frac{dz}{z^2}\int dz_1\widetilde{\Psi}F(z_1,z;\lambda,x)\Bigg] \ . \quad (14)$$

For symmetrical angular acceptance one must suppose that $\rho_2 = 1$, $\rho_4 = \rho_3 = \rho$. In this case, of course, $\Sigma^\gamma = \Sigma_\gamma$.

3. The CALO2 event selection differs from CALO1 in the shape of the cluster (see [3]). Only the cluster-dependent contributions to Σ_1 will change in this case. The analytical formulas are very cumbersome, and we give the result only for the symmetrical wide–wide case ($\Sigma^\gamma = \Sigma_\gamma$):

$$\Sigma_i^c = \frac{\alpha}{2\pi}\int_0^{x_c}\frac{1+x^2}{1-x}dx\int\frac{dz}{z^2}\int dz_1\frac{2}{\pi}\left(\frac{1}{z_1-xz}+\frac{1}{z-z_1}\right)[\Psi_1\Phi_1+\Psi_2\Phi_2+\Psi_3\Phi_3] \ , \quad (15)$$

$$\Phi_1 = \arctan Q_i^{(-)} - \arctan\eta \ , \quad \Phi_2 = \arctan\eta^{-1} \ , \quad \Phi_3 = \arctan\frac{1}{Q_i^{(+)}} \ , \quad \eta = r_i\cot\frac{\Phi-\delta}{2} \ ,$$

$$r_i = \frac{(\sqrt{z}-\sqrt{z_1})^2}{z-z_1} \ , \quad Q_i^{\pm} = r_i\sqrt{\frac{x^2(\sqrt{z}+\sqrt{z_1})^2-(1-x)^2(\sqrt{z_1}\pm x\bar{\lambda})^2}{(1-x)^2(\sqrt{z_1}\pm x\bar{\lambda})^2-x^2(\sqrt{z}-\sqrt{z_1})^2}} \ ,$$

$$\Psi_1 = [z_3^{(-)},1](x^2J_+^2,x^2z_+)+[(\rho_3-(1-x)\bar{\lambda})^2,z_3^{(-)}](x^2\rho_3^2,x^2z_+) \ ,$$

$$\Psi_2 = [z_1^{(+)},1](x^2z_+,x^2)+[(\rho_3-(1-x)\bar{\lambda})^2,z_1^{(+)}](x^2z_+,x^2J_-^2)+$$
$$[\rho_3^2,(\rho_3-(1-x)\bar{\lambda})^2](x^2\rho_3^2,x^2J_-^2) \ ,$$

$$\Psi_3 = [z_1^{(+)},(1+(1-x)\bar{\lambda})^2](x^2J_+^2,x^2)+[\rho_3^2,(1+(1-x)\bar{\lambda})^2](x^2J_-^2,x^2z_-) \ . \quad (16)$$

The corresponding formula for the contribution due to the final-electron emission reads

$$\Sigma_f^c = \frac{\alpha}{2\pi}\int_0^{x_c}\frac{1+x^2}{1-x}dx\Bigg[\int\frac{dz}{z^2}\int dz_1\frac{2}{\pi}\left(\frac{1}{z_1-xz}-\frac{1}{z_1-x^2z}\right)[\Psi_1F_1+\overline{\Psi}_2F_2+\Psi_3F_3]+$$

$$+\int_1^{z_3^{(-)}}\frac{dz}{z^2}\ln\left|\frac{(x\rho_3^2-z)(J_+^2-z)}{(\rho_3^2-z)(xJ_+^2-z)}\right|+\int_{(1+(1-x)\bar{\lambda})^2}^{\rho_3^2}\frac{dz}{z^2}\left(\ln\left|\frac{x-z}{1-z}\right|+\bar{l}_-\right)\Bigg] \ ; \quad (17)$$

$$F_1 = \arctan\frac{1}{Q_f^{(-)}} \ , \quad F_2 = \arctan\zeta \ , \quad F_3 = \arctan Q_f^{(+)} \ , \quad \zeta = r_f\cot\frac{\Phi-\delta}{2} \ ,$$

$$r_f = \frac{(\sqrt{z_1}-x\sqrt{z})^2}{z_1-x^2z} \ , \quad \bar{l}_- = l_-(\lambda\to\bar{\lambda}) \ , \quad Q_f^{(\pm)} = \frac{r_f}{r_i}Q_i^{(\pm)} \ , \quad \sin\delta = \sqrt{\frac{z_1}{z}}\sin\Phi \ ,$$

$$\overline{\Psi}_2 = [z_1^{(+)},1](x^2J_+^2x^2)+[z_3^{(-)},z_1^{(+)}](x^2J_+^2,x^2J_-^2)+[\rho_3^2,z_3^{(-)}](x^2\rho_3^2,x^2J_-^2) \ . \quad (18)$$

The quantities Φ and $\bar{\lambda}$ in Eqs. (15)–(18) specify the shape and size of the CALO2 cluster, namely

$$\Phi = \frac{3\pi}{32} \ , \quad \bar{\lambda} = \frac{\theta_0}{\theta_1} \ , \quad \theta_0 = \frac{0.051}{16} \ .$$

Finally, the functions $J_\pm$ and $z_i^{(\pm)}$ are defined as follows:

$$J_{(\pm)} = \frac{1}{\beta}\left[\sqrt{z\beta - x^2(1-x)^2\bar{\lambda}^2\sin^2\Phi} \pm (1-x)\bar{\lambda}(1 - 2x\sin^2\frac{\Phi}{2})\right] ,$$

$$\beta = 1 - 4x(1-x)\sin^2\frac{\Phi}{2} , \quad z_i^{(\pm)} = (\rho_i \pm (1-x)\bar{\lambda})^2 - 4x(1-x)\rho_i(\rho_i \pm \bar{\lambda})\sin^2\frac{\Phi}{2} .$$

The results of calculations of the QED correction with the vacuum polarization switched off are shown in Table 1 for three different angular acceptances: symmetrical wide–wide and narrow–narrow and asymmetrical wide–narrow. For comparsion we give also the corresponding numbers obtained using the Monte Carlo program BHLUMI [3] for the symmetrical wide–wide case.

As one can see from Table 1, there is an approximately constant difference, at a level of 0.3 per thousand, between our analytical results and the MC results within the first-order correction. A possible cause of this effect is as follows. In our calculation we systematically ignore terms containing $\theta^2 \simeq |t|/s$ as compared with unity. But it is well known that terms of this kind have double-logarithmic asymptotic behavior and are parametrically equal to $(\alpha|t|/\pi s)\ln^2\frac{|t|}{s}$, which is 0.1 per thousand for the conditions at LEP1. We note that the MC program BHLUMI takes into account all the first-order contributions [5].

The SABS cross section at LEP1 with the first-order QED correction

x_c	BHLUMI ww	ww	nn	wn
		CALO1		
0.1	166.329	166.285	131.032	134.270
0.3	166.049	166.006	130.833	134.036
0.5	165.287	165.244	130.416	133.466
0.7	161.794	161.749	128.044	130.542
0.9	149.925	149.866	118.822	120.038
		CALO2		
0.1	131.032	130.997	94.666	98.354
0.3	130.739	130.705	94.491	98.127
0.5	130.176	130.141	94.177	97.720
0.7	127.528	127.491	92.981	95.874
0.9	117.541	117.491	86.303	87.696

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