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RESISTANCE OF SUPERCONDUCTOR–NORMAL METAL–SUPERCONDUCTOR JUNCTIONS

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It is shown that the conductance of a superconductor-normal metal-superconductor junction can exhibit a significant dependence on the phase of the superconducting order parameter in the situation when the size of the normal region of the junction is much larger than the normal metal coherence length and the critical current of the junction is already exponentially small. The period of the conductance oscillations as a function of the phase can be equal to π or 2π depending on parameters of the system.

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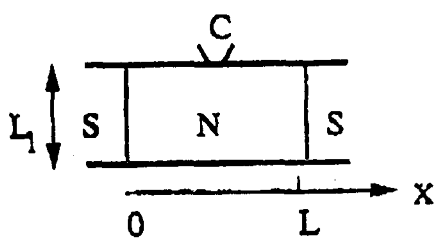
The critical current of the superconductor-normal metal-superconductor (SNS) junction $I_c = I_{c0}\exp(-\frac{L}{L_T})$ decays exponentially and can be neglected when $L >> L_T$ (See, for example, [1]). Here L is the length of the normal metal region of the junction shown in Figure, $L_T = \sqrt{\frac{D}{T}}$ is the coherence length of normal metal, T is the temperature, $D = \frac{lv_F^2}{3}$ is the electron diffusion coefficient, v_F is the Fermi velocity and l is the electron elastic mean free path. On the other hand, the χ dependent part of the conductance $\delta G(\chi)$ of the SNS junction can survive even in the case $L_T \ll L \ll L_{in}$. Here $\chi = \chi_1 - \chi_2$, $\chi_{1,2}$ are phases of the order parameters in superconductors composing the junction, $L_{in} = \sqrt{D\tau_{in}}$ and τ_{in} are inelastic diffusion length and inelastic mean free time respectively. The χ-dependence of δG originates from the fact that the amplitude of the Andreev reflection from the superconductor-normal metal (SN) boundary of an electron into a hole acquires an additional phase factor $\exp(i\chi_{1,2})$, while the amplitude of the reflection of a hole into an electron acquires the phase factor $\exp(-i\chi_{1,2})$. The weak localization contribution $\delta G_1(\chi)$ to $\delta G(\chi)$ has been considered long time ago [2]. It arises in the first order approximation in the parameter $\frac{\hbar}{p_F l} \ll 1$ and is connected with the interference of electrons traveling clockwise and counterclockwise along diffusive paths with close loops which contain Andreev reflections. Here p_F is the Fermi momentum. The value of δG_1 is insensitive to the ratio between L and L_T and the characteristic energy interval which gives the main contribution into δG_1 is $\epsilon \sim T$. The period of $\delta G_1(\chi)$ as a function of χ is π [2].

In this paper we consider two other contributions δG_2 and δG_3 to δG and show that the period of the oscillations of δG as a function of χ can be either π or 2π depending on parameters of the system and the way how the conductance is measured. δG_2 can be associated with the spatial coherence between electrons and holes arising due to Andreev reflection from the SN boundary [3]. It arises in zeroth order approximation in the parameter $\frac{\hbar}{p_F l} \ll 1$. The contribution from this mechanism to the resistance of SN junction was considered in [4–11]. It has been pointed out [10] that the electron-hole coherence is extended in the metal over the distance of order $L_\epsilon = \sqrt{\frac{D}{\epsilon}}$. Therefore, it is clear that the main contribution to δG_2 comes from the relatively small energy interval $\epsilon \sim E_c = \frac{D}{L^2} \ll T$ and at $L_T \ll L \ll L_{in}$, δG_2 decreases with L only as L^{-2}. The period of $\delta G_2(\chi)$ as a function of χ is 2π. The sum $\delta G_1 + \delta G_2$ gives the main contribution to δG, provided the voltage drop V between the two superconductors composing the junction is zero and consequently χ does not change in time. In this case, $\delta G(\chi)$ can be measured with the help of an additional lead "C" shown in Figure, while the phase difference χ on the junction can be determined by an additional Josephson junction. In the case where the resistance of the SNS junction is measured by applying the voltage V between the superconductors, there is a third contribution δG_3 to δG. The origin of δG_3 is similar to the Debye relaxation mechanism of microwave absorption in dielectrics. In this case, due to the Josephson relation, χ and, consequently, the quasiparticle density of states in the metal $\nu(\epsilon, x)$ are functions of time. In other words, at small V the quasiparticle energy levels move slowly. The electron populations of the energy levels follow adiabatically the motion of the levels themselves and, as a result, the electron distribution becomes nonequilibrium. Relaxation of the nonequilibrium distribution due to inelastic processes leads to the entropy production, to the absorption of the energy of the external field and therefore contributes to δG.



We start with the calculation of δG_2. In the zeroth order approximation in the parameter $\frac{\hbar}{p_F l}$ the most adequate theoretical description of the system is provided in the framework of the Keldysh Green's function technique elaborated for superconductivity in [12–14]. In the diffusive approximation and in the absence of electron-electron interaction in the normal metal region of the junction the linear response to the external electric field is described by the following set of equations:

$$j_n = eD\nu_0 \int_{-\infty}^{+\infty} \cosh^2\theta_2(\epsilon, x)\partial_x f_1(\epsilon, x) d\epsilon \quad (1)$$

$$\frac{D}{2}\partial_x^2\theta(\epsilon, x) + (i\epsilon - \frac{1}{\tau_{in}})\sin\theta(\epsilon, x) - \frac{1}{2}(\partial_x\chi(\epsilon, x))^2 \sin 2\theta(\epsilon, x) = 0$$
$$\partial_x(\sin^2(\theta(\epsilon, x)\partial_x\chi(\epsilon, x)) = 0 \quad (2)$$

$$D\partial_x\{\cosh^2\theta_2(\epsilon, x)\partial_x f_1(\epsilon, x)\} = 0 \quad (3)$$

Here j_n is the normal current density across the junction, ν_0 is the density of states in the bulk normal metal, Eq.2 is the Usadel equations for the retarded normal $g^R(\epsilon, x) = \cos\theta(\epsilon, x)$ and anomalous $F^R(\epsilon, x) = -i\exp(i\chi(\epsilon, x))\sin\theta(\epsilon, x)$ Green's functions ($\theta(\epsilon, x) = \theta_1(\epsilon, x) + i\theta_2(\epsilon, x)$ is a complex variable), Eq.3 is the diffusion equation for the distribution function of quasiparticles f_1, which describes the imbalance of populations of electron and hole branches of spectrum in metal. Inside the superconductor $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = 0$. The boundary conditions for Eq.2,3 have the form [15]

$$D\partial_x\theta(\epsilon, x) = t\cos(\theta(\epsilon, 0^+))\cos(\frac{\chi}{2} - \frac{\chi(\epsilon, 0^+)}{2})$$

$$D\sin(\theta(\epsilon, 0^+))\partial_x\chi(\epsilon, x = 0^+) = t\sin(\frac{\chi}{2} - \frac{\chi(\epsilon, 0^+)}{2}) \tag{4}$$

$$D\cosh\theta_2(\epsilon, 0^+)\partial_x f_1(\epsilon, 0^+) = t\{f_1(\epsilon, 0^+) - f_1(\epsilon, 0^-)\}\sin(\theta_1(\epsilon, 0^+))cos^{-1}(\frac{\chi}{2} - \frac{\chi(\epsilon, 0^+)}{2})\times$$

$$\times f_1(\epsilon, x = 0^-) = 0, f_1(\epsilon, x = L^+) = -eV\partial_\epsilon\tanh\frac{\epsilon}{2kT} \tag{5}$$

Here $0^+(0^-, L^+)$ represents the normal metal(superconductor) side of the SN boundary, $t = t_0 v_F$ and t_0 is dimensionless transmission coefficient through the SN boundary, and V is the voltage drop on the junction.

Using Eqs.(1)-(5) we get the expression for the resistance of SNS junction

$$G_{SNS} = G_N L\int_{-\infty}^{+\infty} d\epsilon\partial_\epsilon\tanh\frac{\epsilon}{kT}\{\frac{L_t}{\cosh\theta_2(\epsilon, 0^+)\sin\theta_1(\epsilon, 0^+)\cos(\frac{\chi}{2} - \frac{\chi(\epsilon, 0^+)}{2})} +$$

$$+ \int_0^x \frac{1}{\cosh\theta_2(\epsilon, x')}dx'\}^{-1} \tag{6}$$

Here $G_N = \sigma_D\frac{S}{L}$, $\sigma_D = e^2 D\nu_0$ and $S = L_1\times L_2$ are conductance of the normal metal part of the junction, Drude conductivity and the area of the junction respectively. The first and the second terms in Eq.(6) can be associated with the resistance of the SN boundary and the resistance of the normal region of the junction respectively. There are two major effects in metal, which are due to proximity of the superconductor: 1) The effective diffusion coefficient in Eq.3 is renormalized due to Andreev reflection and is governed by the parameter θ_2. The correction to the local conductivity of the metal from this effect leads to the second term in Eq.(6). 2) The local density of states $\nu(\epsilon, x) = \nu_0 Reg^R(\epsilon, x) = \nu_0\cos\theta_1(\epsilon, x)\cosh\theta_2(\epsilon, x)$ in the metal at small ϵ is suppressed due to Andreev reflection and is governed by the parameter θ_1. The contribution to the conductance of the SN boundary from this effect corresponds to the first term in Eq.(6). The χ dependence of G_{SNS} originates from the corresponding χ dependence of θ_1 and θ_2. It follows from Eqs.2,4 that near the SN boundary at small ϵ the value of $\theta(\epsilon, x)$ should be close to its value in the superconductor $\theta_S(\epsilon < \Delta) = \frac{\pi}{2}$. It approaches its metallic value $\theta_M = 0$ only after the distance L_ϵ. The main contribution to Eq.(6) comes from the energy interval $\epsilon \sim E_c \ll T$. As a result,

$$\delta G_2 = \alpha G_N\frac{E_c}{T}g(\chi) \tag{7}$$

Here $g(\chi)$ is a universal function of χ with the period 2π; $\alpha \sim 1$ at $L \gg L_t = \frac{D}{t}$ and $\alpha \sim (\frac{L}{L_t})^2$ at $L \ll L_t$.

Let us now discuss the contribution of the Debye relaxation mechanism which arises in the case where the voltage V is applied between superconductors in Figure and χ changes in time $\frac{d\chi}{dt} = \frac{2e}{\hbar}V$. Generaly speaking, in this case one has to solve a nonstationary version of Eqs.2,3. However, in the case when $eV \ll E_c$ one can use the adiabatic approximation where the time dependences of $\theta(\epsilon, x, \chi(t))$ and local density state $\nu(\epsilon, x, \chi(t))$ originate from the corresponding time dependence of $\chi(t)$. The standard expression for the power absorption due to the Debye relaxation has the form (see, for example, [16])

$$Q = v\nu_0^{-1} \int d\epsilon < \left(\int_{-\infty}^{\epsilon} \frac{d\bar{\nu}(\epsilon', \chi(t))}{dt} d\epsilon' \right)^2 > \frac{\tau_{in}(\epsilon)}{1 + (\omega \tau_{in}(\epsilon))^2} \partial_\epsilon f_0(\epsilon) \tag{8}$$

where $\bar{\nu}(\epsilon, \chi(t))$ is the local density of states averaged over the volume $v = LL_1L_2$ of the normal metal region and breackets $<>$ correspond to the averaging over the period of the oscillations $\frac{\hbar}{eV}$. Using Eqs.2,4, one can prove that in the absence of insulator barrier $\bar{\nu}(\epsilon, \chi) = \nu_0 \Pi_1(\frac{\epsilon}{E_c}, \chi)$. When $L \ll L_t$, $\bar{\nu}(\epsilon, \chi) = \nu_0 \frac{L^2}{L_t^2} \Pi_2(\frac{\epsilon}{E_c}, \chi)$. Here $\Pi_{1,2}(\mu, \chi)$ are universal dimensionless functions. $\Pi_1(\mu \gg 1, \chi) \sim \Pi_2(\mu \gg 1, \chi) = \cos\chi \exp(-\sqrt{\mu})$. Furthermore at $E_c \ll T$ one can neglect the ϵ-dependence of $\tau_{in}(\epsilon)$. The main contribution to Eq.8 comes from the energy interval $\mu \sim 1$ or $\epsilon \sim E_c$, where the quasiparticle density of states is significantly suppressed compared with ν_0 in the absence of insulator at the boundary. As a result, we have the expression for the contribution of this mechanism to the d.c. conductance of the junction $(Q = V^2 \delta G_3)$

$$\delta G_3 = \alpha^2 G_N \frac{E_c^2 \tau_{in}}{T\hbar} \tag{9}$$

which can be even larger than G_N. Eq.9 is valid when $\frac{eV\tau_{in}}{\hbar} \ll 1$. In this limit one can introduce $\delta G_3(\chi(t))$ which has the magnitude of order of Eq.9 and the period 2π. In the opposite limit $eV\frac{\tau_{in}}{\hbar} \gg 1$, Q saturates, which means that δG_3 decays as $(\frac{\hbar}{eV\tau_{in}})^2$.

We would like to mention that the contribution of the Debye mechanism to the d.c. resistance of a close metallic sample of the Aharonov-Bohm geometry has been discussed in [17]. In that case the time dependence of the electron density of states was induced by the change of a magnetic flux Φ through the ring. The important difference is that the average density of states in the normal metal is flux (and, consequently, time) independent. Therefore, the Debye absorption is nonzero only due to mesoscopic fluctuations of the density of states, whereas in the case of SNS junction, the average density of states can be time dependent. Using results obtained in [17] in the case when $\frac{\hbar}{\tau_{in}} \gg \delta_0$, $L_t \gg L$, $L_{in} \gg L, L_2, L_2$ we can estimate the contribution to δG due to the mesoscopic part of the Debye relaxation mechanism as $\delta G_3^m \sim \alpha_1 \frac{e^2}{\hbar^3} E_c \delta_0 \tau_{in}^2$. Here δ_0 is the mean spacing between energy levels in the normal metal, $\alpha_1 \sim 1$ when $L_t \ll L_{in}$ and $\alpha_1 \sim (\frac{L_{in}}{L_t})^4$ when $L_t \gg L_{in}$. We will neglect this contribution because, as we will see below, $\delta G_3^m \ll \delta G_1$.

Another contribution to δG which arises in the first order approximation in the parameter $\frac{\hbar}{p_F l}$ is the above mentioned weak localization correction δG_1. It reflects the fact that in cource of each Andreev reflection amplitude of diffusion electron paths aquire mentioned above additional phases $\pm\chi_{1,2}$, but does not take into account the spatial coherence between the electron and the hole which arises due to Andreev reflection. This is correct if $L_T \ll L$ or $E_c \ll T$. As a result [2],

$$\delta G_1 = -\alpha_1 \frac{e^2}{\hbar} g_3(\chi) \begin{cases} E_c \tau_{in} & \text{for } 0D \text{ case} \\ \frac{L_{in} L_1}{L^2} & \text{for } 1D \text{ case} \\ \ln \frac{L_{in}}{l} \frac{L_1 L_2}{L^2} & \text{for } 2D \text{ case} \end{cases} \tag{10}$$

Here $g_3(\chi)$ is a periodic function with the period π; $0D$ case corresponds to $L, L_1, L_2 \ll L_{in}$; $1D$ case corresponds to $L_1 \gg L_{in} \gg L_2, L$ and $2D$ case corresponds to $L_1, L_2 \gg L_{in} \gg L$.

Ratios between the three above considered contributions to δG depend on the parameters and the dimentionality of the system. For example, in $0D$ case at $0 < \frac{eV\tau_{in}}{\hbar} \ll 1$ we have

$$\frac{\delta G_1}{\delta G_2} \approx \frac{\alpha_1}{\alpha} \frac{e^2}{\hbar G_N} T\tau_{in}; \quad \frac{\delta G_1}{\delta G_3} \approx \frac{\alpha_1}{\alpha^2} \frac{e^2}{\hbar G_N} \frac{T}{E_c} \tag{11}$$

At large enough $eV \gg \sqrt{\frac{\hbar}{\tau_{in}} E_c \alpha}$ (but still smaller than E_c), δG_3 becomes much smaller than δG_2. In this case, χ dependent part of the resistance is determined by the sum $(\delta G_1 + \delta G_2)$. For example, in $0D$ case the ratio between δG_1 (with the period π) and δG_2 (which has the period 2π) is of the order of

$$\frac{\delta G_1}{\delta G_2} \approx \frac{\alpha_1}{\alpha} \frac{T}{V} \frac{e^2}{\hbar G_N} \tag{12}$$

If $V = 0$ and the conductance is measured with the help of the contact "C" in Figure, δG is the sum of δG_1 and δG_2, ratio of which is determined by the correponding terms in Eqs.11,12. These ratios can be both larger and smaller than unity, which means that the period of the oscillations of $\delta G(\chi)$ can be either π or 2π. This can explain why some experiments demonstrate π periodicity of δG [18,19] while the others demonstrate 2π period [20,21]. The reason why δG_1, which arises only in the first order approximation in the small parameter $\frac{\hbar}{p_F l}$, can be comparable with δG_2 is that δG_2 is determined by the small energy interval $\epsilon \sim E_c \ll T$ while δG_1 is determined by $\epsilon \sim T$.

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