

FERMIONIC ENTROPY OF THE VORTEX STATE IN *d*-WAVE SUPERCONDUCTORS

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In the *d*-wave superconductors the electronic entropy related to one isolated vortex logarithmically diverges with the size of the system even at low temperature. In the vortex array the entropy per one vortex per layer, S_V , is essentially larger than k_B and depends on the distribution of the velocity field v_s around the vortex. If there is a first order transition with the change of the velocity distribution, then there will be a big entropy jump $\Delta S_V \sim k_B$ at the transition. This entropy jump comes from the electronic degrees of freedom on the vortex background, which is modified by the vortex transition. This can explain the big jump in the entropy observed in the so called vortex melting transition [1], in which the vortex array and thus the velocity field are redistributed. The possibility of the Berezinskii-Kosterlitz-Thouless transition in the 3-dimensional *d*-wave superconductor due to the fermionic bound states in the vortex background is discussed.

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1. **Gap nodes and scaling.** The low energy properties of the superconductors with nodes in the energy gap are governed by the electronic excitations close to the gap nodes. The electronic density of states (DOS) in the homogeneous superconductor is (see eg the Review paper [2])

$$N(E) \sim N_F \left(\frac{E}{T_c} \right)^{2-D}, \quad E \ll T_c, \quad (1)$$

where D is the dimension of nodes and N_F is the DOS on the Fermi level in the normal metal. In the mixed state of superconductor the superflow around the vortex leads to the Doppler shift of the energy E to $E + v_s(\mathbf{r}) \cdot \mathbf{p}$, where $v_s(\mathbf{r})$ is the local superfluid velocity. This gives the finite DOS at zero energy

$$N(0) \sim N_F \left(\frac{p_F v_s}{T_c} \right)^{2-D}. \quad (2)$$

Here v_s is the characteristic value of the superfluid velocity in the vortex array $v_s \sim \hbar/m_3 R_V$, where $R_V \sim \xi(B_{c2}/B)^{1/2}$ is the intervortex distance, B is magnetic field and $\xi \sim v_F/T_c$ is the coherence length. Thus $p_F v_s/T_c \sim \sqrt{B/B_{c2}}$ and this gives the following electronic DOS at zero energy as a function of magnetic field

$$N(0) \sim N_F \left(\frac{B}{B_{c2}} \right)^{1-D/2}. \quad (3)$$

There are two different regimes of strong and weak fields with the crossover parameter

$$x \sim \frac{T}{k_F v_s} \sim \frac{T}{T_c} \left(\frac{B_{c2}}{B} \right)^{1/2}, \quad (4)$$

which separates the superflow dominating regime $x \ll 1$ from the temperature dominating regime $x \gg 1$ (see [4]). In general the thermodynamic functions depend on the parameter x . For example, the free energy of the excitations on the background of the vortex array is

$$F(T, B) = N_F T_c^2 \left(\frac{B}{B_{c2}} \right)^{2-D/2} \tilde{F}(x), \quad (5)$$

where $\tilde{F}(x)$ is the dimensionless function of the dimensionless parameter x (see also the recent paper by Simon and Lee [5]; their crossover parameter differs from our by the factor $\sqrt{T_c/E_F}$. This is because Simon and Lee used the linearized spectrum of the fermions in the very vicinity of the gap nodes, which can be justified only at rather low temperature, $T \ll T_c^2/E_F$). The normalization can be found from the low field asymptote $x \gg 1$, where the largest contribution comes from the bulk superconductor, while the effect of vortices is small. As follows from Eq.(1) the free energy of the homogeneous state is $\sim -N_F T_c^2 (T/T_{c2})^{4-D}$ and this gives the normalization of $F(T, B)$ and the low field asymptote

$$\tilde{F}(x) \sim -x^{4-D}, \quad x \gg 1. \quad (6)$$

2. Scaling for d -wave superconductor. *Free energy.* For the case of the nodal lines, ie for the dimension of nodes $D = 1$ one has the following estimation of the free energy of the excitations in the background of the vortex array

$$F(T, B) = N_F T_c^2 \left(\frac{B}{B_{c2}} \right)^{3/2} \tilde{F}(x). \quad (7)$$

Let us find the asymptotes of $\tilde{F}(x)$ at $x \gg 1$ and $x \ll 1$. Let us consider first the case of the weak field $x \gg 1$, ie of diluted vortex array. In addition to the largest asymptote $\tilde{F}(x) \sim -x^3$ from the bulk superconductor there is also the contribution from vortices. It comes from the modification of the normal component density due to excitations: $\rho_n(T) \sim \rho N(T)/N_F$. This leads to decrease of the kinetic energy of superflow around the vortex, thus the contribution to the energy of the vortex array with the vortex density $n = B/\Phi_0$, where Φ_0 is the flux quantum, is

$$F(B \rightarrow 0) = -\frac{1}{2} n \rho_n(T) \int_{R_V > r > \hbar v_F / T} d^2 r v_s^2 \sim -N_F T_c^2 \frac{B}{B_{c2}} \frac{T}{T_c} \ln \frac{R_V T}{\xi T_c}. \quad (8)$$

This corresponds to the term $-x \ln x$ in $\tilde{F}(x)$. So, the leading terms in the asymptote of $\tilde{F}(x)$ at $x \gg 1$ are

$$\tilde{F}(x) \sim -x \ln x - x^3, \quad x \gg 1. \quad (9)$$

The asymptote of $\tilde{F}(x)$ at $x \ll 1$ is

$$\tilde{F}(x) \sim -1 - x^2, \quad x \ll 1, \quad (10)$$

Here both terms are from the vortices. The first one is temperature independent and does not contribute the entropy and specific heat, but contributes the vortex magnetization. It comes from the nonzero normal component density at $T=0$ due to the superfluid velocity: $\rho_n(T=0) \sim \rho N(E=0)/N_F \sim \rho(p_F v_s/T_c)^{2-D}$ (see [6-9])

$$F(T \rightarrow 0) = -\frac{1}{2} n \rho \int_{R_V > r > \xi} d^2 r v_s^2 \left(\frac{p_F v_s}{T_c} \right)^{2-D} \sim -N_F T_c^2 \left(\frac{B}{B_{c2}} \right)^{3/2} \quad (11)$$

The second (quadratic in x) term in Eq.(10) gives the linear in temperature term in the specific heat: $C(T, B) \propto T \sqrt{B}$ [3].

Vortex entropy. For the entropy density one has

$$S(T, B) = -\frac{\partial F}{\partial T} = N_F \frac{B}{B_{c2}} T_c \tilde{S}(x) \quad , \quad (12)$$

$$\tilde{S}(x) = -\frac{\partial \tilde{F}}{\partial x} \quad . \quad (13)$$

The asymptotes of $\tilde{S}(x)$ at $x \gg 1$ and $x \ll 1$ are

$$\tilde{S}(x) \sim \ln x + x^2 \quad , \quad x \gg 1 \quad (14)$$

$$\tilde{S}(x) \sim x \quad , \quad x \ll 1 \quad . \quad (15)$$

The vortex part of the normalized entropy $\tilde{S}(x)$, ie without the bulk term x^2 in Eq.(14), can be written using the interpolating formula

$$\tilde{S}(x) \sim \ln(x+1) \quad , \quad (16)$$

which gives both the logarithmic term in Eq.(14) at large x and linear term in Eq.(15) at low x .

It is instructive to write the vortex entropy per one vortex per one layer

$$\frac{S_V(T, B)}{k_B} \sim \frac{E_F}{T_c} \ln(x+1), \quad (17)$$

where E_F is the Fermi energy. Note that the logarithmic vortex entropy in Eq.(14) follows also from the $1/E$ behavior of the vortex DOS found in [4]. Using the result of [4] one can find an exact equation for the vortex entropy at large x using an axial distribution of the superfluid velocity around the vortex, $v_s = (\hbar/2m_3 r) \hat{\phi}$:

$$\frac{S_V(T, B)}{k_B} = 2 \ln 2 \frac{v_{FPF}}{\Delta'} \ln x \quad , \quad x \gg 1. \quad (18)$$

Here Δ' is the angle derivative of the gap at the node.

Heat capacity. For the heat capacity one has

$$C(T, B) = T \frac{\partial S}{\partial T} = N_F T_c \frac{B}{B_{c2}} \tilde{C}(x), \quad (19)$$

$$\tilde{C}(x) = x \frac{\partial \tilde{S}}{\partial x}. \quad (20)$$

Using the interpolating Eq.(17) for the entropy one obtains the interpolating equations for the vortex part of the heat capacity

$$C(T, B) \sim N_F T_c \frac{B}{B_{c2}} \frac{x}{1+x}, \quad (21)$$

which gives both asymptotes found in [4]: $\tilde{C}(x) \sim x$ for $x \ll 1$ and $\tilde{C}(x) \sim 1$ for $x \gg 1$.

3. Discussion. The electronic entropy per one vortex per one layer S_V in Eq.(17) is essentially larger than k_B even at $T \ll T_c$. For an isolated vortex this entropy diverges as the logarithm of the dimension R of the system: $S_V \sim k_B (E_F/T_c) \ln R$ (actually R is limited by the penetration length) and is at least by factor $E_F/T_c \gg 1$ larger than the configurational entropy of the vortex in the 2-dimensional system, $S_{conf} \sim k_B \ln R$. The logarithmic behavior of S_V , with R limited by the intervortex distance R_V , persists till $T \sim T_c \sqrt{B/B_{c2}}$ (or $x \sim 1$). Due to the large factor $E_F/T_c \gg 1$ the entropy per one vortex per one layer can be of order k_B even at $T < T_c \sqrt{B/B_{c2}}$ (or $x < 1$), but it finally disappears in the high field limit ($T \ll T_c \sqrt{B/B_{c2}}$ or $x \ll 1$).

It is important that S_V depends on the distribution of the velocity field v_s around the vortex. If there is a first order transition with the change of the velocity distribution, one can expect a big entropy jump, $\Delta S \sim 1k_B$. This entropy jump comes from the electronic degrees of freedom in the vortex background, which is modified by the vortex transition. This can explain the latent heat $L \sim 0.45k_B T/\text{vortex}/\text{layer}$ observed on the so called vortex melting line in detwinned Y-123 crystal [10] and $L \sim 0.6 \pm 0.1 k_B T/\text{vortex}/\text{layer}$ in twinned sample of Y-123 [1]. Even higher values of the entropy jump was deduced from the magnetization measurements [11]. Such entropy jump can occur both in the vortex melting transition and in the 1-st order transition at which the structure of the vortex lattice changes, say, from the hexagonal lattice close to T_c to the distorted tetragonal lattice far from T_c . The latter structure has been observed in [12] and discussed in [13].

Note that the fermionic entropy of the 3-dimensional vortex loop of the length R is $\propto R \ln R$, as distinct from the configurational entropy of the loop $S_{conf} \propto R$. This $R \ln R$ behavior of the vortex loop entropy, together with the large prefactor, can in principle cause the Berezinskii-Kosterlitz-Thouless transition in the 3-dimensional system. This is supported by the following observation. It appears that if one uses the quantum mechanical approach to the vortex DOS, by calculating the discrete bound states of the fermions on the background of the inhomogeneous distribution of the superflow around the vortex, one obtains the two times larger magnitude for the vortex DOS than that obtained using the classical consideration for the fermions in terms of the Doppler shifted energy $E + v_s(\mathbf{r}) \cdot \mathbf{p}$ [4].

The classical approach gives the conventional expression for the energy of isolated vortex in terms of the superfluid density $\rho_s(T) = \rho - \rho_n(T)$

$$E_V = \frac{\pi \hbar^2}{4m_s^2} (\rho - \rho_n(T)) \ln \frac{R}{\xi} \quad (22)$$

At low T , where $\rho_n(T)/\rho \sim T/T_c$, the second term corresponds to the logarithmic entropy of the vortex. The quantum mechanical approach in terms of the fermionic bound states in the vortex background suggests the larger contribution

of the fermions to the vortex entropy.* This can be written using the enhancement factor $1 + \alpha(T)$

$$F_V = \frac{\pi \hbar^2}{4m_e^2} [\rho - (1 + \alpha(T))\rho_n(T)] \ln \frac{R}{\xi} . \quad (23)$$

According to [4] one has $\alpha(0) = 1$. If this value of α persists to higher temperatures, the energy of the isolated vortex becomes zero at some temperature $T_V < T_c$, where $\rho_n(T_V) = \frac{1}{2}\rho$. Thus at T_V one would have the Berezinskii-Kosterlitz-Thouless transition in the 3-dimensional system, occurring due to the essential contribution of the fermionic bound states to the vortex entropy. However it is more natural to expect that $\alpha(T)$ continuously decreases with temperature approaching zero value at T_c , since the effect of the bound states is to be negligible in the Ginzburg-Landau region. So, the possibility of the Berezinskii-Kosterlitz-Thouless transition in this 3D-system depends on the details of the behavior of $\alpha(T)$.

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