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NUCLEAR FUSION IN $d\mu^3\text{He}$ MESIC MOLECULE

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A new scheme of physical processes leading to the nuclear fusion reaction $d(^3\text{He}, ^4\text{He})p$ catalyzed by a negatively charged muon (μ^-) is presented. It is shown that the observable rate and yield of the nuclear reaction depend on a chain of ion-molecular reactions in which the $d\mu^3\text{He}$ -molecule participates. New calculations of the nuclear fusion rates in the $d\mu^3\text{He}$ -molecule are presented.

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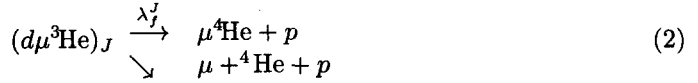
1. The nuclear fusion reaction



is of interest due to many reasons: it is involved in the primordial nucleosynthesis of the light elements in the early universe [1], as a discussed perspective source of thermonuclear energy [2], as a mirror reaction of the important process $d(t, ^4\text{He})n$ [3], etc. In all these cases it is especially important to know the cross sections of reaction (1) at low collision energies $E \lesssim 10$ keV, i.e., in the region where direct measurements in the beam experiments are complicated. For these reasons, any alternative way to measure this value is interesting.

The phenomenon of muon catalysis gives the possibility to study this reaction (as well as many other fusion reactions [4]) at practically zero collision energy from the mesic molecular state $(d\mu^3\text{He})^{++}$. (In what follows we will put $(d\mu^3\text{He})^{++} \equiv d\mu^3\text{He}$.) In recent

years the rates λ_f^J of nuclear reactions

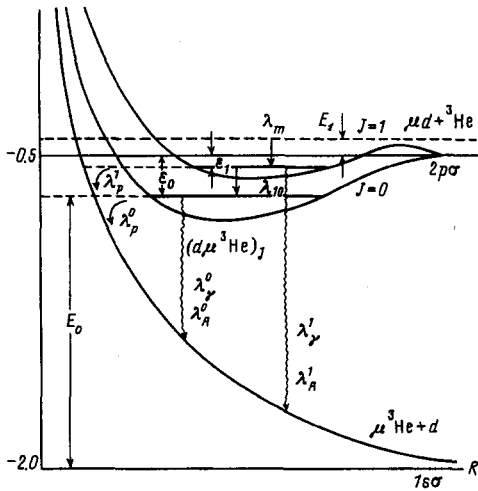
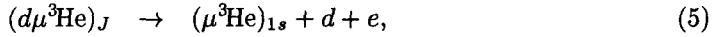
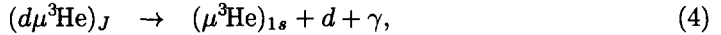


from the states $(d\mu^3\text{He})_J$ with total angular momentum J were calculated many times [5-9], however, results of these calculations differ by several orders of magnitude. The most recent experimental upper limit for this rate is [10]

$$\lambda_f < 1.3 \cdot 10^6 \text{ s}^{-1}. \quad (3)$$

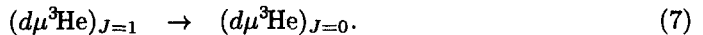
In this paper new results of λ_f^J calculations [11, 12] and a new scheme of kinetics of ion-molecular reactions preceding fusion in the $d\mu^3\text{He}$ mesic molecule [13] are used.

2. The scheme of the processes of $(d\mu^3\text{He})_J$ mesic molecule formation and decay is presented in Figure: in collisions of slow $d\mu$ -atoms with ^3He atoms mesic molecules $(d\mu^3\text{He})_J$ are formed [14]; states $(d\mu^3\text{He})_J$ are quasistationary due to decay to $(\mu^3\text{He})_{1s} + d$ by γ -emission (4), Auger transitions (5) and predissociation (6), with rates λ_γ^J , λ_A^J and λ_p^J , respectively:



Scheme of formation and decay of the $d\mu^3\text{He}$ molecule. The muonic molecule $d\mu^3\text{He}$ is formed in $d\mu + ^3\text{He}$ collisions in the bound state $(d\mu^3\text{He})_{J=1}$ which decays into system $\mu^3\text{He} + d$ with the rates λ_γ^1 , λ_p^1 and λ_A^1 ; the transition $(d\mu^3\text{He})_{J=1} \rightarrow (d\mu^3\text{He})_{J=0}$ with the rate λ_{10} competes with these decays; the binding energies of the states $(d\mu^3\text{He})_J$ equal ε_J ; the collision energies in the states $d\mu + ^3\text{He}$ and $\mu^3\text{He} + d$ equal E_1 and E_0 correspondingly; the fusion rates from the states $J = 0$ and $J = 1$ are λ_f^0 and λ_f^1

Besides that, in collisions of $[(d\mu\text{He})e]^+$ with D_2 and He in the chain of ion-molecular reactions [13], transitions $(J = 1) \rightarrow (J = 0)$ with the rate λ_{10} are possible



The yield N_f of nuclear fusion per one stopped μ^- is determined by fusion rates λ_f^J from the states J and populations w_J of these states which depend on the rates of processes (4)-(7) and kinetics of ion-molecular reactions in which $d\mu^3\text{He}$ molecule participates.

3. The rates λ_f^J are determined by the relation [4]

$$\lambda_f^J = \sum_L A_L G_L^J. \quad (8)$$

Here, A_L are reaction constants for the nuclear states with orbital angular momentum L , determined by extrapolation of the cross sections of reaction (1) to zero collision energy, and the quantities G_0^J and G_1^J are calculated with formulae:

$$G_0^J = \int d\mathbf{r} |\Psi^J(\mathbf{r}, \mathbf{R} = 0)|^2, \quad (9)$$

$$G_1^J = \int d\mathbf{r} |\nabla_{\mathbf{R}} \Psi^J(\mathbf{r}, \mathbf{R})|_{\mathbf{R}=0}^2, \quad (10)$$

where $\Psi^J(\mathbf{r}, \mathbf{R})$ is the wave function of the $(d\mu^3\text{He})_J$ mesic molecule (\mathbf{R} is the internuclear distance, \mathbf{r} is the muon coordinate with respect to the center of mass of the nuclei). Since $A_0 \gg A_1$, in the following we are interested only in values G_0^J .

To calculate values G_0^J two independent methods have been used and two high accuracy numerical algorithms were developed [11, 15], which gave results in a reasonable agreement. Reaction (2) of muon catalysis, being compared with the mirror reaction $d\mu t \rightarrow \mu^4\text{He} + n$, $\mu + ^4\text{He} + n$ [16], has some $(d\mu^3\text{He})_J$. Specifically, contrary to the $(d\mu t)_J$ mesic molecule, where the bound states are predominantly localized in the potential $W_{1s\sigma}(R)$ formed by muon motion in the state with quantum numbers $(Nlm) = (100)$ of the system $(t\mu)_{1s} + d$, the states of the $(d\mu^3\text{He})_J$ molecule are localized in the potential $W_{2p\sigma}(R)$ with quantum numbers $(Nlm) = (210)$ of the system $(d\mu)_{1s} + ^3\text{He}$.

Unlike the case of $d\mu t$, due to the strong coupling between channels $1s\sigma$ and $2p\sigma$, the wave function $\Psi^J(\mathbf{r}, \mathbf{R})$ [11, 15] contains all the components $\psi^L(\mathbf{R})$ representing the relative motion of nuclei in $d\mu^3\text{He}$ -molecule with different L .

In the limit $R \rightarrow 0$ it has the form [17]

$$\Psi_m^J(\mathbf{r}, \mathbf{R}) \underset{\mathbf{R} \rightarrow 0}{\approx} \sum_j \phi_j(\mathbf{r}; R) \psi_j^L(\mathbf{R}) = \sum_{Nl} \phi_{Nlm}(\mathbf{r}; R) \psi_{Nlm}^L(\mathbf{R}), \quad (11)$$

where $\phi_j(\mathbf{r}; R)$ are orthonormalized adiabatic basis functions and functions $\psi_j^L(\mathbf{R})$ represent the relative motion of nuclei with angular momentum $L = |J - l|, \dots, |J + l|$ in the potential $W_j(\mathbf{R})$, formed by the muon motion in the quantum state $j = (Nlm)$ at fixed distance R between nuclei [18]. Thus, for states $J = 0$ both combinations ($l = 0, L = 0$) and ($l = 1, L = 1$), are essential

$$\Psi^{J=0}(\mathbf{r}; \mathbf{R}) \underset{\mathbf{R} \rightarrow 0}{\approx} \sum_N \phi_{N00}(\mathbf{r}; R) \psi_{N00}^{L=0}(\mathbf{R}) + \sum_N \phi_{N10}(\mathbf{r}; R) \psi_{N10}^{L=1}(\mathbf{R}). \quad (12)$$

For $J = 1$ the analogous expansion has the form [17]:

$$\Psi_m^{J=1}(\mathbf{r}, \mathbf{R}) \underset{\mathbf{R} \rightarrow 0}{\approx} \sum_N \delta_{0m} \phi_{N00}(\mathbf{r}; R) \psi_{N00}^{L=1}(\mathbf{R}) + \sum_N \phi_{N1m}(\mathbf{r}; R) \psi_{N1m}^{L=0}(\mathbf{R}). \quad (13)$$

It follows from definitions (9), (12), (13) that

$$G_0^0 = \sum_N |\psi_{N00}^{L=0}(0)|^2, \quad (14)$$

$$G_0^1 = \sum_{N,m} |\psi_{N1m}^{L=0}(0)|^2. \quad (15)$$

Functions $\psi_{Nlm}^J(\mathbf{R})$ have been recently calculated in paper [11] by complex coordinate rotation method, expanding variational function $\Psi^J(\mathbf{r}; \mathbf{R})$ over the adiabatic basis.

The calculated G_0^J -factors are equal to:

$$G_0^0 = 0.63 \cdot 10^{-12} a_\mu^{-3} = 3.8 \cdot 10^{19} \text{ cm}^{-3} \quad (16)$$

$$G_0^1 = 0.86 \cdot 10^{-15} a_\mu^{-3} = 5.1 \cdot 10^{16} \text{ cm}^{-3} \quad (17)$$

($a_\mu = 2.56 \cdot 10^{-11} \text{ cm}$ is a mesic atomic length unit). In the other approach [12, 15] exploiting expansion of the function $\Psi^J(\mathbf{r}, \mathbf{R})$ over the adiabatic hyperspherical basis [19] the finite width of the quasistationary $d\mu^3\text{He}$ state was explicitly taken into account. The result obtained

$$G_0^0 = 0.75 \cdot 10^{-12} a_\mu^{-3} = 4.4 \cdot 10^{19} \text{ cm}^{-3} \quad (18)$$

is in a reasonable agreement with (16).

The reaction constant for unpolarized nuclei equals $A_0 = 0.34 \cdot 10^{-14} \text{ cm}^3 \cdot \text{s}^{-1}$ [3].

The low energy cross section of reaction (1) is dominated by the $J^P = 3/2^+$ nearthreshold resonance. Due to this fact the nuclear fusion rates λ_f^J are equal to [20]

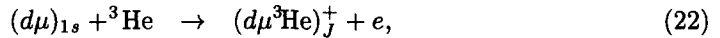
$$\lambda_f^0 = 3/2 \cdot A_0 G_0^0 = 1.9 \cdot 10^5 \text{ s}^{-1}, \quad (19)$$

$$\lambda_f^1 = 0.65 \cdot 10^3 \text{ s}^{-1}. \quad (20)$$

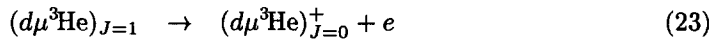
At G_0^0 from (18) the rate is

$$\lambda_f^0 = 2.3 \cdot 10^5 \text{ s}^{-1}. \quad (21)$$

4. Mesic molecules $d\mu^3\text{He}$ are formed in the reaction



predominantly in the state with total angular momentum $J = 1$ by dipole $E1$ -transition [21]. Since $\lambda_f^1 \ll \lambda_f^0$ the fusion reaction in the $d\mu^3\text{He}$ mesic molecule can be observed only in the state $J = 0$. It is possible if the rate λ_{10} of reaction (7) is comparable to the decay rates $\lambda_\gamma^1, \lambda_p^1$ and λ_A^1 of the state $J = 1$. The internal Auger transition



is forbidden, since the difference between energies of states $J = 1$ and $J = 0$, $\epsilon_1 - \epsilon_0 = 22.8 \text{ eV}$, is less than the ionization energy of a helium atom (24.6 eV). Hence transition (7) can occur only in collisions of $(d\mu^3\text{He})_J$ with atoms of the medium. The whole set of ion-molecular reactions, leading to transition (7) has been considered in [13]. These reactions ($i = 1 \div 9$) and their rates λ_i are listed below, where notation $M_J = (d\mu^3\text{He})_J$ is introduced (φ is the mixture density, C_{He} is the helium concentration and $X = D_2, \text{He}$).

- | | | |
|-----|----------------------------------------------------------------------|---------------------------------------------------------------------------|
| (1) | $(M_1e)^+ + D_2 + X \rightarrow (M_1eD_2)^+ + X,$ | $\lambda_1 \approx 3 \cdot 10^{13} \varphi \text{ s}^{-1};$ |
| (2) | $(M_1e)^+ + \text{He} + X \rightarrow (M_1e\text{He})^+ + X,$ | $\lambda_2 \approx 3 \cdot 10^{13} \varphi C_{\text{He}} \text{ s}^{-1};$ |
| (3) | $(M_1eD_2)^+ + \text{He} \rightarrow (M_1e\text{He})^+ + D_2,$ | $\lambda_3 \approx 3 \cdot 10^{13} \varphi C_{\text{He}} \text{ s}^{-1};$ |
| (4) | $(M_1e\text{He})^+ + \text{He} \rightarrow (M_1ee) + \text{He}_2^+,$ | $\lambda_4 \approx 10^{13} \varphi C_{\text{He}} \text{ s}^{-1};$ |
| (5) | $(M_1eD_2)^+ \rightarrow (M_0e)^+ + D_2^+ + e,$ | $\lambda_5 \approx 5 \cdot 10^{11} \text{ s}^{-1};$ |
| (6) | $(M_1e\text{He})^+ + D_2 \rightarrow (M_0e\text{He})^+ + D_2^+ + e,$ | $\lambda_6 \lesssim 10^9 \varphi \text{ s}^{-1};$ |
| (7) | $(M_1ee) + D_2 \rightarrow (M_0ee) + D_2^+ + e,$ | $\lambda_7 \approx 4 \cdot 10^9 \varphi \text{ s}^{-1};$ |
| (8) | $(M_1e)^+ + \text{He} \rightarrow (M_1ee) + \text{He}^+,$ | $\lambda_8 \approx 3 \cdot 10^{12} \varphi C_{\text{He}} \text{ s}^{-1};$ |
| (9) | $(M_1e)^+ + D_2 \rightarrow (M_0e)^+ + D_2^+ + e,$ | $\lambda_9 \approx 10^7 \varphi \text{ s}^{-1}.$ |

The total rates $\lambda_{dec}^J = \lambda_\gamma^J + \lambda_p^J + \lambda_A^J$ of quasistationary states $(d\mu^3\text{He})_J$ decay to channels (4) – (6) equal, correspondingly (see Table):

$$\lambda_{dec}^0 \simeq 0.9 \cdot 10^{12} \text{ s}^{-1} \quad (25)$$

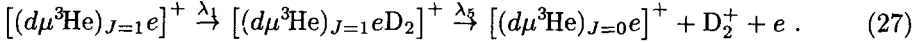
$$\lambda_{dec}^1 \simeq 1 \cdot 10^{12} \text{ s}^{-1} \quad (26)$$

Table

Rates of the main processes in the $(d\mu^3\text{He})$ muonic molecule

Reaction	λ	J	$\lambda, \text{ s}^{-1}$	Ref.
$d\mu + {}^3\text{He} \rightarrow (d\mu^3\text{He})_{J\nu}$	λ_m^J	1	$1.4 \cdot 10^8$	[21]
		0	$\sim 2 \cdot 10^6$	[13]
$(d\mu^3\text{He})_J \rightarrow \mu^4\text{He} + p$	λ_f^J	1	$0.65 \cdot 10^3$	[11]
		0	$1.9 \cdot 10^5$	
$(d\mu^3\text{He})_J \rightarrow \mu^3\text{He} + d + \gamma$	λ_γ^J	1	$1.6 \cdot 10^{11}$	[24]
		0	$1.8 \cdot 10^{11}$	
$(d\mu^3\text{He})_J \rightarrow \mu^3\text{He} + d$	λ_p^J	1	$0.8 \cdot 10^{12}$	[25]
		0	$0.7 \cdot 10^{12}$	
$(d\mu^3\text{He})_J \rightarrow \mu^3\text{He} + d + e$	λ_A^J	1	$0.41 \cdot 10^{11}$	[26]
		0	$0.47 \cdot 10^{11}$	
$(d\mu^3\text{He})_{J=1} \rightarrow (d\mu^3\text{He})_{J=0}$	λ_{10}	$1 \rightarrow 0$	$0.5 \cdot 10^{12}$	[13]

It follows from the comparison of rates λ_i that at $\varphi \sim 0.1$ and $C_{\text{He}} \leq 0.1$ the dominating channel in the chain of ion-molecular reactions, leading to a change of the mesic molecule angular momentum $(J = 1) \rightarrow (J = 0)$, is related with the formation of the complex $[(d\mu^3\text{He})_{J=1}eD_2]^+$ and its subsequent decay with conversion of an electron of the D_2 molecule, namely:



In comparison with this process the rate $\lambda_7 \lesssim 10^9 \varphi \text{ s}^{-1}$ of reaction $(J = 1) \rightarrow (J = 0)$ by the external Auger transition (reaction (7) of (24)) is negligibly small, in contrast to the statement of Ref.[22].

At $\varphi \simeq 0.1$ and $C_{\text{He}} \lesssim 0.1$, $\lambda_1 \simeq 3\lambda_{dec}$ and $\lambda_5 \sim \lambda_3$, i.e., a noticeable fraction (~ 0.2) of mesic molecules $(d\mu^3\text{He})_{J=1}$ reaches the state $J = 0$, where the fusion (2) can be observed.

5. The detailed analysis of the kinetics of processes in $D_2 + {}^3\text{He}$ mixture is yet to be done, but even simple estimates allow to obtain rather reliable information about the expected yield N_f of fusion reactions (2) per muon stop. According to these estimates:

$$\begin{aligned} N_f &\simeq \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_8 + \lambda_{dec}^1} \frac{\lambda_5}{\lambda_5 + \lambda_3 + \lambda_{dec}^1} \frac{\lambda_f^0}{\lambda_f^0 + \lambda_{dec}^0} n_f \approx \quad (28) \\ &\approx n_f \frac{\lambda_f^0}{\lambda_{dec}^0} (1 + 0.03\varphi^{-1})^{-1} (2.8 + 60\varphi C_{\text{He}})^{-1}, \end{aligned}$$

where coefficient $n_f = n_f(C_d)$ is the statistical weight of the mesic molecule $(d\mu^3\text{He})_J$ states with total nuclear spin $S = 3/2$ calculated by taking into account the kinetics of its formation in collisions $(d\mu)_F + \text{He}$ and spin-flip processes $(d\mu)_F + d \rightarrow (d\mu)_F + d$ [20, 23]. At $\varphi = 0.075$, $C_{\text{He}} = 0.05$, $n_f \simeq 0.5$

$$N_f \simeq 0.12 \cdot \lambda_f^0 / \lambda_{dec}^0 \approx 3 \cdot 10^{-8} / \mu^-. \quad (29)$$

6. The understanding and quantitative description of the process of nuclear fusion catalysis in the $d\mu^3\text{He}$ mesic molecule required the development of new theoretical methods and the consideration of new physical processes. Experiment R-94-05.1, planned at PSI, will allow to check the correctness and self-consistency of these methods and adequacy of the considered processes. In particular, the observation of the φ -dependence of the yield N_f (28) would be the confirmation of the ion-molecular mechanism of the transition $(d\mu^3\text{He})_{J=1} \rightarrow (d\mu^3\text{He})_{J=0}$ via formation of clusters $[(d\mu^3\text{He})eD_2]^+$. The comparison of the fusion rate λ_f^0 , extracted from the measurements of N_f , with its theoretical values will test the validity of the sophisticated calculations of the Coulomb three-body problem performed recently.

The methods and details of theoretical calculations of rates λ_f^J and λ_i will be published elsewhere [11 – 13, 15, 20]. The preliminary version of this paper was published in [27].

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