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WEAK ANTILOCALIZATION IN A 2D ELECTRON GAS WITH THE CHIRAL SPLITTING OF THE SPECTRUM

M.A.Skvortsov

L.D.Landau Institute for Theoretical Physics RAS

117940 Moscow, Russia

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Motivated by the recent observation of the metal-insulator transition in Si-MOSFETs we consider the quantum interference correction to the conductivity in the presence of the Bychkov–Rashba spin splitting. For a small splitting, a crossover from the localizing to antilocalizing regime is obtained. The antilocalization correction vanishes however in the limit of a large separation between the chiral branches. The relevance of the chiral splitting for the 2D electron gas in Si-MOSFETs is discussed.

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Since the appearance of the scaling theory of localization [1] in 1979, it was a common belief that there can be no metal-insulator transition (MIT) in 2D electron systems since all the states are localized at arbitrary weak disorder. Recent experiments on high-mobility Si-MOSFETs by Kravchenko et al [2] showed however an evidence for a MIT at zero magnetic field which is controlled by the density n_s of 2D carriers. For small densities $n_s < n_c \simeq 10^{11}\text{cm}^{-2}$ the system is insulating with exponentially diverging resistivity in the limit $T \to 0$, whereas for n_s higher than the critical density a strong drop in resistivity (by one order of magnitude) is observed for $T < 2\,\text{K}$.

The origin of the new metallic phase has not been understood yet. Nevertheless it is evident that the electron-electron interaction plays an important role as the critical density, n_c, is quite low so that the Coulomb interaction dominates the kinetic energy. Their ratio is $r_s \simeq 10$ at the transition point and decreases $\propto n_s^{-1/2}$ deep into metallic phase. Several theoretical approaches to the treatment of the strong Coulomb interaction such as p-wave [3], triplet [4] or anyon [5] superconductivity and superconductivity resulting from a negative dielectric function [6] were suggested during the last year.

Besides a strong Coulomb interaction Si-MOS structures are characterized by a spin-orbit splitting of the spectrum [7]. It originates from a strong asymmetry of the confining potential $V(z)$ of the quantum well. The corresponding term in the Hamiltonian of 2D EG, the so-called Bychkov–Rashba term, is given by [8]

$$H_{so} = \alpha[\hat{\vec{\sigma}} \times \hat{\mathbf{p}}]. \qquad (1)$$

Here $\hat{\vec{\sigma}}$ is the vector of the Pauli matrices, $\hat{\mathbf{p}}$ is the 2D momentum operator, α is a constant of the spin-orbit symmetry breaking measured in the units of velocity, and $[\cdot \times \cdot]$ stands for the z-component of the vector product. This term lifts the spin-degeneracy at zero magnetic field and results in the splitting of the spectrum into two chiral branches:

$$\epsilon_{\pm}^{(0)}(p) = \frac{p^2}{2m} \pm \alpha p, \qquad (2)$$

with the splitting growing linearly with p.

For a Si-MOSFET, the minimum of the spectrum (2), $-\epsilon_0 = -m\alpha^2/2$, is estimated as 1 K [7, 9] while the Fermi energy is $\epsilon_F \simeq 6$ K at the transition. Then the ratio of the concentrations of left- and right-chiral fermions is $n_+/n_- = (\sqrt{\epsilon_F + \epsilon_0} + \sqrt{\epsilon_0})^2/(\sqrt{\epsilon_F + \epsilon_0} - \sqrt{\epsilon_0})^2 \simeq 5$. Thus we conclude that the spin splitting results in a drastic change of the internal properties of the system even without allowing for the Coulomb interaction. This observation may question the remark by Belitz and Kirkpatrick [4] that the spin-orbit scattering is irrelevant due to a long-ranged Coulomb interaction. The latter should be strongly modified by the predominance of one type of chirality.

The relevance of the spin correlations was also demonstrated in magnetic measurements [10]. Magnetic field applied in the 2D plane was shown to suppress the metallic state leading to a huge increase in resistivity. The measurements in a perpendicular magnetic field show a large positive magnetoresistance at high densities $n_s > 2n_c$ also indicating the spin-related origin of the conducting phase.

We argue that the understanding of the new conducting phase as well as the MIT itself can hardly be obtained without taking the strong chiral splitting into account. Thus the theory of the metallic state should be the theory of the Coulomb interacting chiral fermions. The necessary first step then is to consider the noninteracting particles with the chiral splitting of the spectrum.

In this letter we study the first quantum correction to the conductivity for the noninteracting particles in the presence of the Bychkov–Rashba term (1) and obtain it as a function of the spin-orbit splitting. There are three energy scales in the problem: the first is the Fermi energy ϵ_F, the second is the chiral splitting $\Delta = 2\alpha p_F$ between the two branches (2) at the Fermi level, and the third is the inverse elastic mean free time τ^{-1} introduced by disorder. We will assume ϵ_F to be the largest energy scale:

$$\epsilon_F \gg \frac{1}{\tau}, \quad \epsilon_F \gg \Delta. \tag{3}$$

The relationship between Δ and τ^{-1} is not specified so that the variable

$$x = \Delta\tau \tag{4}$$

that controls the strength of the chiral splitting may vary from 0 to ∞ provided that the relations (3) are fulfilled. At the critical density, the ratio Δ/ϵ_F is of the order of 1 but decreases as n_s^{-1} into the metallic phase. The experimental value of the parameter x slightly depends on the density, varying from 5 to 10 when n_s varies from 10^{11}cm^{-2} to $3 \times 10^{12}\text{cm}^{-2}$.

The spin-orbit scattering *at random potential* is known to drive the system into the symplectic ensemble resulting in an antilocalizing correction to the conductivity $\Delta\sigma_{symp} = (e^2/\pi h)\ln(l_\varphi/l)$ [11], where l is the mean free path, $l_\varphi = (D\tau_\varphi)^{1/2}$ is the phase-breaking length associated with the phase relaxation time τ_φ, D is the diffusion coefficient. In the case of the Bychkov–Rashba term, SU(2) symmetry is broken on the level of *the regular Hamiltonian* while the potential scattering may be considered as spin independent. From the symmetry consideration one might expect that the symplectic correction $\Delta\sigma_{symp}$ should be recovered in the limit of a large spin splitting. We will see however that the correction becomes antilocalizing at $x = (l_\varphi/l)^{1/3} \ll 1$, nearly approaches $\Delta\sigma_{symp}$ for $x \leq 1$ but *vanishes* for $x \gg 1$. Such a peculiar behavior is due to the presence of the two chiral branches that are well separated in the limit $\Delta \gg \tau^{-1}$.

Weak localization effects in the presence of different types of spin-orbit splittings, including the Bychkov–Rashba one, were studied extensively in Refs. [12]. However the authors were interested mainly in the behavior of magnetoresistance while the quantum correction at zero magnetic field and for $x \geq 1$ when H_{so} cannot be treated as a small perturbation had not been investigated.

We consider a 2D noninteracting electron gas with the Bychkov–Rashba term in the Hamiltonian:

$$H = \frac{\hat{\mathbf{p}}^2}{2m} + \alpha \hat{p}_y \sigma_x - \alpha \hat{p}_x \sigma_y + U(\mathbf{r}), \tag{5}$$

where $U(\mathbf{r})$ is a random spin-independent impurities' potential, which for the sake of simplicity is assumed to be Gaussian δ-correlated: $\langle U(\mathbf{r})U(\mathbf{r}')\rangle = \delta(\mathbf{r}-\mathbf{r}')/2\pi\nu\tau$. Here $\nu = m/2\pi$ is the density of states for the free Hamiltonian $\mathbf{p}^2/2m$.

The classical conductivity can easily be shown to be independent on x and given by the Drude formula $\sigma_0 = ne^2\tau/m$ provided that the random potential is δ-correlated. The first quantum correction to the conductivity [13] is given by the expression [14]

$$\Delta\sigma = -\frac{e^2}{h}\frac{v_F^2}{2}\int \frac{d^2\mathbf{p}}{(2\pi)^2}\langle G^R(\mathbf{p})\rangle^{\rho\alpha}\langle G^R(\mathbf{p})\rangle^{\lambda\sigma}\langle G^A(\mathbf{p})\rangle^{\sigma\beta}\langle G^A(\mathbf{p})\rangle^{\mu\rho}\int_{1/l_\varphi}^{1/l}\frac{d^2\mathbf{q}}{(2\pi)^2}C_{\beta\mu}^{\alpha\lambda}(\mathbf{q}), \tag{6}$$

where $\langle G^{R,A}\rangle$ are disorder-averaged retarded (advanced) Green functions which for our problem are nondiagonal in the spin space and the static Cooperon $C(\mathbf{q})$ is determined by the ladder equation

$$C_{\beta\mu}^{\alpha\lambda}(\mathbf{q}) = \frac{\delta^{\alpha\lambda}\delta^{\beta\mu}}{2\pi\nu\tau} + \frac{1}{2\pi\nu\tau}\int \frac{d^2\mathbf{p}}{(2\pi)^2}\langle G^R(\mathbf{p}+\frac{\mathbf{q}}{2})\rangle^{\alpha\alpha'}\langle G^A(-\mathbf{p}+\frac{\mathbf{q}}{2})\rangle^{\beta\beta'}C_{\beta'\mu}^{\alpha'\lambda}(\mathbf{q}). \tag{7}$$

The averaged Green function obeys the Dyson equation $\langle G(\mathbf{p})\rangle^{-1} = G^{(0)}(\mathbf{p})^{-1} - \Sigma$, where $G^{(0)}(\mathbf{p})$ is the Green function of the unperturbed Hamiltonian. In the quasiclassical limit, $\epsilon_F\tau \gg 1$, only diagrams without intersections of impurity lines are important and the self-energy function

$$\Sigma_{R,A}^{\alpha\beta} = \frac{1}{2\pi\nu\tau}\int \frac{d^2\mathbf{p}}{(2\pi)^2}\langle G^{R,A}(\mathbf{p})\rangle^{\alpha\beta}.$$

On solving the Dyson equation we obtain the Green function that can be written near the poles as ($\mathbf{n} = \mathbf{p}/p$)

$$\langle G^{R,A}(\mathbf{p})\rangle = \frac{-\xi(p) \pm \frac{i}{2\tau} + \Delta(n_y\sigma_x - n_x\sigma_y)/2}{\left(-\xi(p) - \frac{\Delta}{2} \pm \frac{i}{2\tau}\right)\left(-\xi(p) + \frac{\Delta}{2} \pm \frac{i}{2\tau}\right)}. \tag{8}$$

Here we have taken an advantage of $\Delta \ll \epsilon_F$ and substituted αp by $\Delta/2$. The relaxation times for the two chiral branches appear to be equal to each other and coincide with the mean free time τ. This is a consequence of the model with δ-correlated disorder. For a more realistic model with finite correlation length the lifetimes will be different for the two chiralities but the difference will be small in the limit $\Delta \ll \epsilon_F$.

The crucial quantity that determines the spin structure of the Cooperon is the integral of the retarded and advanced Green functions,

$$I_{\beta\beta'}^{\alpha\alpha'}(\mathbf{q}) = \frac{1}{2\pi\nu\tau}\int \frac{d^2\mathbf{p}}{(2\pi)^2}\langle G^R(\mathbf{p}+\frac{\mathbf{q}}{2})\rangle^{\alpha\alpha'}\langle G^A(-\mathbf{p}+\frac{\mathbf{q}}{2})\rangle^{\beta\beta'}. \tag{9}$$

Calculating this integral as a function of x, expanding to the second order in q, and substituting into Eq. (7) we get

$$\hat{C}(\mathbf{q}) = \frac{\hat{A}^{-1}(\mathbf{q})}{2\pi\nu\tau}, \tag{10}$$

where the operator $\hat{A}(\mathbf{q}) = \hat{\mathbf{1}} - \hat{I}(\mathbf{q})$ expressed in terms of the total Cooperon spin $\hat{\mathbf{S}} = \frac{1}{2}(\hat{\sigma}^R + \hat{\sigma}^A)$ reads

$$\hat{A}(\mathbf{q}) = \frac{1}{2}q^2l^2 + x^2\left(\frac{1}{2(1+x^2)} - \frac{6+3x^2+x^4}{8(1+x^2)^3}q^2l^2\right)(\hat{\mathbf{S}}^2 - \hat{S}_z^2) -$$

$$-\frac{x^2(6+3x^2+x^4)}{4(1+x^2)^3}(\mathbf{q}\times\hat{\mathbf{S}})^2l^2 - \frac{x}{(1+x^2)^2}(\mathbf{q}\times\hat{\mathbf{S}})l. \tag{11}$$

The next step is to invert the matrix $\hat{A}$ and to obtain the Cooperon. According to Eq. (11), the singlet mode is gapless while the triplet sector acquires a gap proportional to x. To study the lifting of the triplet sector consider first the case of small $x \ll 1$. Then, for $ql \gg x$, the spin structure of $\hat{A}$ may be neglected so that $\hat{A}^{-1} = (2/q^2l^2)\hat{\mathbf{1}}$. For $ql \leq x$, the triplet sector of the inverse operator $\hat{A}^{-1}$ becomes complicated, with different triplet modes having different gaps because of the low symmetry of Eq. (11), but this region does not contribute to the logarithmic integral over q. So we may write

$$\hat{A}^{-1} \simeq \frac{2}{q^2l^2}\left(1 - \frac{\hat{\mathbf{S}}^2}{2}\right) + \frac{2}{q^2l^2+x^2}\frac{\hat{\mathbf{S}}^2}{2}. \tag{12}$$

This is not an exact formula but it captures correctly log-large terms in q-integration.

Inserting (12) to Eq. (10) and performing the integration, we obtain the expression for the Cooperon integral:

$$\int_{1/l_\varphi}^{1/l} \frac{d^2\mathbf{q}}{(2\pi)^2} C^{\alpha\lambda}_{\beta\mu}(\mathbf{q}) = \frac{1}{8\pi^2\nu v_F^2\tau^3}\left\{\left(\ln\frac{l_\varphi}{l} + 3f\right)\delta^{\alpha\lambda}\delta^{\beta\mu} - \left(\ln\frac{l_\varphi}{l} - f\right)\sum_{i=1}^{3}\sigma_i^{\alpha\lambda}\sigma_i^{\beta\mu}\right\}, \tag{13}$$

where the contribution of the triplet sector,

$$f\left(x, \frac{l_\varphi}{l}\right) = \begin{cases} \ln\frac{l_\varphi}{l} & \text{for } x \ll \frac{l}{l_\varphi}; \\ \ln\frac{1}{x} & \text{for } \frac{l}{l_\varphi} \ll x \ll 1; \\ O(1) & \text{for } x \gg 1. \end{cases} \tag{14}$$

The last thing to do it to compute the integral of four Green functions in Eq. (6):

$$\int \frac{d^2\mathbf{p}}{(2\pi)^2}\langle G^A(\mathbf{p})\rangle^{\mu\rho}\langle G^R(\mathbf{p})\rangle^{\rho\alpha}\langle G^R(\mathbf{p})\rangle^{\lambda\sigma}\langle G^A(\mathbf{p})\rangle^{\sigma\beta} =$$

$$= \frac{4\pi\nu\tau^3}{1+x^2}\left[\left(1+\frac{x^2}{2}\right)\delta^{\mu\alpha}\delta^{\lambda\beta} + \frac{x^2}{4}\left(\sigma_x^{\mu\alpha}\sigma_x^{\lambda\beta} + \sigma_y^{\mu\alpha}\sigma_y^{\lambda\beta}\right)\right]. \tag{15}$$

This integral is diagonal in the spin space for small $x \ll 1$ but has a more complex structure for $x \gg 1$ when the chiral branches are well separated.

Finally, we combine all together. Substituting (13) and (15) into Eq. (6), after some arythmetics with the Pauli matrices we obtain the final expression

$$\Delta\sigma = \frac{2e^2}{\pi h}\frac{1}{1+x^2}\left[\frac{1}{2}\ln\frac{l_\varphi}{l} - \left(\frac{3}{2}+x^2\right) f\left(x, \frac{l_\varphi}{l}\right)\right]. \tag{16}$$

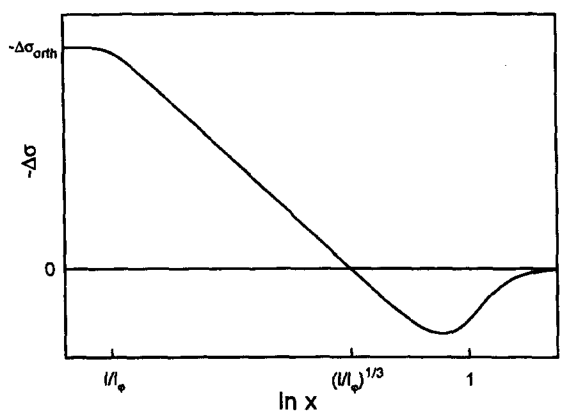
Let us study $\Delta\sigma$ as a function of x for a given $l_\varphi \gg l$. For $x \ll l/l_\varphi$, the spin splitting can be neglected and we obtain the orthogonal universality class correction $\Delta\sigma_{orth}/$ which can be interpreted as a sum of a localizing contribution from the triplet sector and an antilocalizing contribution from the singlet sector. Then, for $l/l_\varphi \ll x \ll 1$, the triplet modes acquire a gap that reduces their contribution and the total correction changes its sign and becomes antilocalizing at

$$x_* = \left(\frac{l}{l_\varphi}\right)^{1/3}. \tag{17}$$

For $x_* \ll x \leq 1$, the antilocalization becomes more pronounced, nearly approaching $\Delta\sigma_{symp}$. However, for $x \geq 1$ it rapidly (as x^{-2}) goes down to zero. Summarizing, we present the behavior of $\Delta\sigma$ in the form

$$\Delta\sigma = \frac{2}{\pi}\frac{e^2}{h}\begin{cases} -\ln\frac{l_\varphi}{l} & \text{for } x \ll \frac{l}{l_\varphi}; \\ \frac{1}{2}\ln\left(\frac{l_\varphi}{l}x^3\right) & \text{for } \frac{l}{l_\varphi} \ll x \ll 1; \\ \frac{1}{2x^2}\ln\frac{l_\varphi}{l} & \text{for } x \gg 1. \end{cases} \tag{18}$$

The crossover from the orthogonal to the symplectic corrections obtained for $x \ll 1$ is related to the appearance of the gap in the triplet sector of the Cooperon. On the other hand, the reduction of $\Delta\sigma$ for $x \gg 1$ must be attributed to the spin structure of the integral (15) that annihilates the singlet Cooperon mode in the limit of a large splitting between the chiral branches. In other words, the result obtained means the absence of the first quantum correction to the conductivity in the system of 2D chiral fermions with only one sort of chirality. The other example where a certain type of the spin-orbit coupling leads to the absence of the first interference correction was considered in [15]. The behavior of $\Delta\sigma$ as a function of x is sketched in Fig.



A sketch of $\Delta\sigma$ vs. the strength x of the chiral splitting; $\Delta\sigma_{orth} = -(2e^2/\pi h)\ln(l_\varphi/l)$

The large-x asymptotics can be traced up to $x \sim \sqrt{\ln(l_\varphi/l)}$. In order to find $\Delta\sigma$ for even larger x, one has to go beyond diffusion approximation to calculate $f(x)$ that competes with the vanishing term $\ln(l_\varphi/l)/x^2$.

In conclusion, we considered the quantum interference correction to the conductivity of the noninteracting fermions in the presence of the Bychkov–Rashba spin-orbit interaction. At small chiral splittings, $x < 1$, the correction changes the sign and becomes antilocalizing. It vanishes for $x \gg 1$ when the scattering between the different chiralities is strongly suppressed. The present theory may be considered as the step toward the understanding of the conducting phase in Si-MOSFETs that are likely made of the Coulomb-interacting chiral fermions. It might also explain a low temperature $\log T$ behavior of the resistivity obtained for some samples below 300 mK deep in the metallic phase. The correction is antilocalizing, $\Delta\sigma \simeq -C(e^2/h)\ln(T/T_0)$, with very small $C \sim 10^{-2}$ [9] that is consistent with our formula for the experimental values of $x \gg 1$.

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