

CRITICAL VELOCITY AND EVENT HORIZON IN PAIR-CORRELATED SYSTEMS WITH "RELATIVISTIC" FERMIONIC QUASIPARTICLES

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The condition for the appearance of event horizon is considered in such paircorrelated systems (superfluids and superconductors) where the fermionic quasiparticles obey "relativistic" equations. In these systems, the Landau critical velocity of superflow corresponds to the speed of light. In conventional systems, such as *s*-wave superconductors, the superflow remains stable even above the Landau threshold. We show, however, that, in the "relativistic" systems, the quantum vacuum becomes unstable and the superflow collapses after the "speed of light" is reached, so that the horizon cannot appear. Thus an equilibrium dissipationless superflow state and the horizon are incompatible due to quantum effects. This negative result is consistent with the quantum Hawking radiation from the horizon, which would lead to a dissipation of the flow.

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1. It is known that some aspects of the problem of black holes can be modelled in condensed matter physics [1 – 5]. This comes from the fact that acoustic waves propagating in a moving classical liquid [1 – 3] and fermions propagating in a texture of superfluid $^3\text{He-A}$ [4, 6, 7] obey relativistic-type equations in curved space, whose metric is produced by both the flow field and the texture (in $^3\text{He-A}$). In both systems the corresponding velocity of light can be exceeded, which leads to a possibility to investigate the event horizon problem.

The $^3\text{He-A}$, as well as in other pair-correlated systems (including *d*-wave superconductors, which also contains the relativistic fermions, see eg [8]), are better models for simulations of the event horizon than a classical liquid, since these are the quantum systems, whose ground states are in many respects similar to the quantum vacuum of high-energy physics. That is why they can be used for investigating the quantum effects related with the event horizon, such as Hawking radiation [9] and statistical entropy [10].

Here we address the stability problem of a quantum vacuum in the presence of the event horizon: whether a nondissipative flow of superfluid is possible in the presence of horizon, or the horizon always leads to a vacuum reconstruction into a state with dissipation. This can be considered using an example of a superflow with velocity exceeding the Landau critical velocity v_L .

Let the superfluid move at $T = 0$ with a superfluid velocity \mathbf{v}_s , while the container walls, i.e. the preferable reference frame, move with a normal velocity \mathbf{v}_n . The physical properties of the vacuum state depend on the relative (counterflow) velocity $\mathbf{w} = \mathbf{v}_s - \mathbf{v}_n$. In a subcritical regime, $w < v_L$, the order parameter is independent of w : the observer moving with \mathbf{v}_s does not see any difference in the liquid as compared to the case when

$\mathbf{v}_s = \mathbf{v}_n$. The system thus retains some kind of Galilean invariance even in the presence of the container wall. For example the density of superfluid component in the expression for the mass current $\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$, which represents the vacuum component of the liquid, does not depend on w and coincides with the total density: $\rho_s(T = 0, w < v_L) = \rho$, while the density of the normal component, which represents the matter, is always zero: $\rho_n(T = 0, w < v_L) = \rho - \rho_s(T = 0, w < v_L) = 0$.

The observer starts to see a dependence on the velocity with respect to the reference frame if w exceeds the Landau critical velocity v_L , at which the negative energy levels appear, ie the states with the negative Doppler shifted energy $E_{\mathbf{p}} + \mathbf{p} \cdot \mathbf{w} < 0$. If the system is fermionic, the typical situation in a supercritical flow regime above v_L is as follows: The negative energy levels become finally occupied and, after such vacuum reconstruction, the system comes again into a new equilibrium state with a frictionless superflow. However all the physical quantities do now depend on w , as seen by a comoving observer. The vacuum state becomes anisotropic, the superfluid density does depend on w and becomes smaller than the total mass density, $\rho_s(T = 0, w > v_L) < \rho$. The other part of the liquid with the so-called normal density $\rho_n = \rho - \rho_s$ comprises the normal component (matter) and consists of the trapped fermions with the negative energy. In equilibrium, this component is at rest with respect to the container reference frame, ie it moves with the velocity \mathbf{v}_n . The previous "Galilean" symmetry is thus broken by the created matter.

There is another critical velocity, v_c , at which the superfluid vacuum is exhausted, ie the superfluid density completely disappears, $\rho_s(w = v_c) = 0$, and thus the nondissipative superflow does not exist anymore. Typically, $v_c > v_L$ and thus the violation of the Galilean symmetry occurs earlier than the superfluidity collapses. This happens for example in conventional s -wave superconductors, where v_L and v_c are of the same order with $v_c > v_L$, and also in superfluid $^3\text{He-A}$, where $v_L = 0$, while v_c is finite (see eg [11–13]).

Existence of a region with the stable supercritical superflow, $v_L < w < v_c$, allows us to rise a question concerning the quantum effects of the event horizon. Let us consider the system, where quasiparticles are described by effective relativistic equations, such that the Landau velocity corresponds to the "speed of light" c . In this case a supercritical velocity $w > v_L$ corresponds to a superluminal one and thus the event horizon can be constructed (see below). The problem appears if one takes into account the quantum effects: On one hand the frictionless superflow is stable in the regime $v_L < w < v_c$, and this stability cannot be violated by the presence of horizon. On the other hand, the Hawking radiation from the horizon means that such a superflow is dissipative in the presence of horizon.

Thus we have a dilemma: (i) either one should doubt the fundamentality of the Hawking radiation from the event horizon in the supercritical regime, (ii) or in the relativistic-type systems the "superluminal" regime of superflow is prohibited, ie $v_L = v_c = c$. Here we consider the pair-correlated fermionic system with the superconducting/superfluid state of the polar-type, which has "relativistic" Bogoliubov fermionic quasiparticles. We find that, in this system, the second alternative occurs: a non-dissipative superflow collapses at $w = v_L = c$, which means that the horizon never appears in the stationary nondissipative superflow: it can only exist in a dissipative flow state.

2. The energy spectrum of the pair-correlated system and its vacuum state are determined by the self-consistent equations for the gap function $\Delta_{\mathbf{p}}$ which determines the

quasiparticle spectrum and thus the "speed of light" c :

$$\Delta_{\mathbf{p}} = \sum_{\mathbf{p}'} V_{\mathbf{p},\mathbf{p}'} \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - n_{\mathbf{p}'} - n_{-\mathbf{p}'}). \quad (1)$$

Here $V_{\mathbf{p},\mathbf{p}'}$ is the pairing potential, $E_{\mathbf{p}}$ is the quasiparticle energy in the pair-correlated state and $n_{\mathbf{p}}$ is their thermal distribution

$$E_{\mathbf{p}} = \sqrt{\Delta_{\mathbf{p}}^2 + \epsilon_{\mathbf{p}}^2}, \quad n_{\mathbf{p}} = \frac{1}{1 + \exp[(E_{\mathbf{p}} + \mathbf{p} \cdot \mathbf{w})/T]}, \quad (2)$$

$\epsilon_{\mathbf{p}} = (p^2 - p_F^2)/2m$ is the fermion energy in the absence of the pair correlation. If the superflow velocity \mathbf{v}_s deviates from the container reference frame velocity \mathbf{v}_n , the distribution function is Doppler shifted; further we assume that $\mathbf{v}_n = 0$ and thus $\mathbf{w} = \mathbf{v}_s$.

Let us consider how the vacuum state ($T = 0$) is disturbed by the counterflow w in the supercritical regime. We are interested in the case when the spectrum of quasiparticles is "relativistic", so that the horizon problem can arise. For this reason we consider the two-dimensional case, spin-triplet pairing with the orbital momentum $L = 1$ for which the pairing potential $V_{\mathbf{p},\mathbf{p}'} = 2(V_1/p_F^2)\mathbf{p} \cdot \mathbf{p}'$, and the gap function corresponding to the polar phase [11]:

$$\Delta_{\mathbf{p}} = cp_x, \quad (3)$$

where the factor c plays the part of the speed of light along axis x .

Let the velocity of the superflow be along the same axis x , ie $\mathbf{v}_s = w\hat{x}$, thus the superflow does not violate the symmetry of the polar state and the Eq.(3) remains the solution even in the presence of the superflow. The Doppler shifted energy of the fermions in this pair-correlated state is

$$E(p_x, \epsilon) = E_{\mathbf{p}} + \mathbf{p} \cdot \mathbf{v}_s = \sqrt{\epsilon^2 + c^2 p_x^2} + wp_x \quad (4)$$

or

$$(E - wp_x)^2 = \epsilon^2 + c^2 p_x^2 \quad (5)$$

This corresponds to the relativistic 1D particle with the mass ϵ moving in the Lorentzian metric:

$$g^{00} = -1, \quad g^{01} = w, \quad g^{11} = c^2 - w^2. \quad (6)$$

If the counterflow velocity w can exceed c , one can construct an inhomogeneous flow state with such coordinate dependence of $w(x)$ and $c(x)$, that $w(x)$ crosses $c(x)$. In this case the metric element $g^{11}(x) = c^2 - w^2$ crosses zero at the points $x = x_h$ where $w(x_h) = c(x_h)$ and thus the event horizon appears at these points.

From Eq.(1) one obtains that there are no quasiparticles for $w < c$ and $T = 0$: $n_{\mathbf{p}} = 0$, ie the vacuum remains intact. The system is effectively Galilean invariant and the speed of light is independent of w .

The problem is whether the velocity of the flow w can exceed the Landau velocity v_L which is now the "speed of light" c . If the answer is yes, the horizon can be constructed. Let us consider the gap equation Eq.(1) in the case when $w > c$. We assume that the speed of light is small compared to the Fermi velocity, $c \ll v_F = p_F/m$, which is typical

for a weakly interacting Fermi-liquid. In this case, the momentum is concentrated near the Fermi momentum, $\mathbf{p} = (p_F \sin \phi, p_F \cos \phi)$, and one can write

$$\sum_{\mathbf{p}} = \int \frac{d^2 p}{(2\pi)^2} = \frac{m}{2\pi} \int d\epsilon \int \frac{d\phi}{2\pi}. \quad (7)$$

In principle, one can expect that at $w > c$ the speed of light becomes dependent on w for $w > c$. Thus let us introduce the bare speed of light $c_0 = c(w = 0)$ and the current speed of light $c(w)$ if $w > c(w)$. As we have seen, $c(w < c) = c_0$, and this solution persists until w reaches the Landau velocity c_0 . Thus the first branch of $c(w)$ is

$$c_1(w) = c_0, \quad w \leq c_0. \quad (8)$$

3. If $w > c$, the Galilean invariance becomes broken due to fermions filling the negative levels of the energy in Eq.(4). The number of particles on the negative energy levels is the fermionic step function of the energy

$$n_{\mathbf{p}} = \Theta(-E(p_x, \epsilon)). \quad (9)$$

From Eq.(1) one obtains the following equation for the factor $c(w)$ in the gap function Eq.(3):

$$\begin{aligned} \int_0^\infty d\epsilon \int_0^{2\pi} \frac{d\phi}{2\pi} \left(\frac{\sin^2 \phi}{\sqrt{\epsilon^2 + c^2 \sin^2 \phi}} - \frac{\sin^2 \phi}{\sqrt{\epsilon^2 + c_0^2 \sin^2 \phi}} \right) = \\ = 2 \int_0^\pi \frac{d\phi}{2\pi} \sin^2 \phi \int_0^{\sin \phi \sqrt{w^2 - c^2}} \frac{d\epsilon}{\sqrt{\epsilon^2 + c^2 \sin^2 \phi}}. \end{aligned} \quad (10)$$

Note that the details of pair interaction are concealed in the bare speed of light c_0 , determined by this interaction. From Eq.(10) one has

$$\ln \frac{c_0}{c} = \text{arsh} \left(\sqrt{\frac{w^2}{c^2} - 1} \right) \quad (11)$$

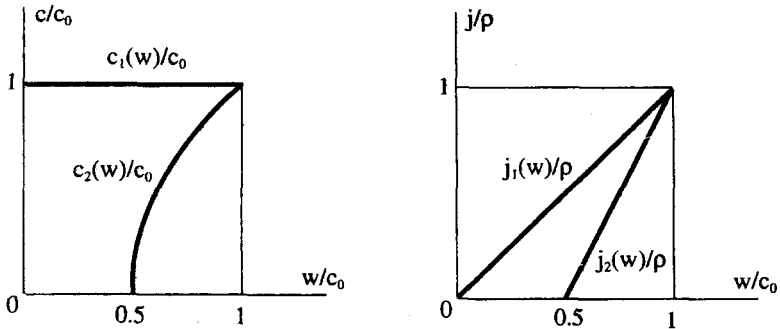
which gives the solution for the "speed of light" c in the superluminal regime $w > c(w)$:

$$c_2(w) = c_0 \sqrt{\frac{2w}{c_0} - 1}, \quad \frac{1}{2}c_0 < w < c_0. \quad (12)$$

It follows that no solution exists above the Landau velocity, ie at $w > c_0$, which means that the Landau velocity coincides with the velocity of the superflow collapse and thus with the bare speed of light: $v_L = v_c = c_0$.

Below the Landau velocity, one has two branches, $c_1(w) = c_0$ and $c_2(w)$ (see Figure). Both can be obtained as extrema of the superfluid vacuum energy in the presence of the mass current

$$\begin{aligned} \Omega_S(s, w) - \Omega_N = -jw + \frac{1}{2}\rho w^2 - \frac{1}{4}\rho c^2 - \frac{1}{2}\rho c^2 \ln \frac{c_0}{c} + \\ + \frac{1}{2}\rho c^2 \left\{ \ln \left[\frac{w}{c} + \sqrt{\frac{w^2}{c^2} - 1} \right] - \frac{w}{c} \sqrt{\frac{w^2}{c^2} - 1} \right\} \Theta(w - c). \end{aligned} \quad (13)$$



Two states of the supereflow. In the subluminal flow state the speed of light $c_1(w) = c_0$ does not depend on flow velocity w with respect to the container wall. The superluminal state with $c_2(w) < w$ is locally unstable. The mass current in these two states is presented on the right

Here Ω_N is the free energy of the normal state, ie at $c = 0$; the mass density in this 2D model is $\rho = mp_F^2/2\pi\hbar^2$; the first term $-jw$ means that the free energy is to be extremized at the given mass current $\mathbf{j} = \rho\mathbf{v}_s + \sum_{\mathbf{k}} \mathbf{k}n_{\mathbf{k}}$. The current in a given state (see Figure) can be obtained from the extremum of the vacuum energy with respect to w : $\partial\Omega_S/\partial w = 0$. This gives the general expression for the mass current density

$$j(w, c) = \rho \left(w - \Theta(w - c) \sqrt{w^2 - c^2} \right). \quad (14)$$

The second branch, corresponding to the superluminal flow $c_2(w) < w$, represents the saddle point solution of the vacuum energy and thus is unstable towards the formation of the regular branch, corresponding to the subluminal flow, $c_1(w) = c_0 < w$. This second branch with similar behavior has been also found for the $^3\text{He-B}$ under the superflow [14].

In conclusion, we found that, in a superfluid analog of the relativistic system, the stable superflow with velocity exceeding the corresponding "speed of light", $w > c$, does not exist and thus the dissipationless state with a horizon does not appear. The collapse of the superfluid quantum vacuum in the superluminal regime is compatible with the Hawking radiation, which leads to the dissipation in the presence of horizon and thus cannot exist in the stable superflow. Horizon can appear only if the flow state is dissipative. This can happen if the external body or the order parameter texture moves in the superfluid with the supercritical velocity, as was discussed in [4] for the case of moving topological soliton in $^3\text{He-A}$. The Hawking radiation gives rise to the additional dissipation during the motion of the object.

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