Supplemental material to the article

High-temperature Aharonov-Bohm effect in transport through a single-channel quantum ring

In this supplemental material we apply the approach based on the picture of resonant tunnelling through pairs of levels in the ring outlined in Sec. 2 of the main text to the case of a ring with impurities (addressed in Sec. 4 within a different formalism). We focus on the antiresonances at $\phi = 1/2$ and disregard backscattering off the contacts inside the ring [see discussion below Eq. (11) of the main text].

In the general case, the amplitude g_n entering the transmission coefficient $\mathcal{T}(\phi)$ in Eq. (10) of the main text reads:

$$g_n(\epsilon, x) = \frac{\Psi_n^{+*}(0)\Psi_n^{+}(x)}{\epsilon - E_n^{+} + i\Gamma/2} + \frac{\Psi_{n+1}^{-*}(0)\Psi_{n+1}^{-}(x)}{\epsilon - E_{n+1}^{-} + i\Gamma/2}. \qquad \text{(A1)}$$

Here $\Psi_n^{\pm}$ and $E_n^{\pm}$ are wave-functions and energy levels of an isolated ring, see Fig. 1. In a clean ring, $\Psi_n^{\pm}(x) = \psi_n^{\pm}(x) = L^{-1/2}\exp(\pm 2\pi i n x/L)$ and $E_n^{\pm} = \epsilon_n^{\mu} = (n \pm \phi)\Delta$, which yields Eq. (11) of the main text. The interference terms involving amplitudes of tunneling through two distinct levels are comparable with classical contributions (describing tunneling through a single level) only if $|E_n^{\mu} - E_{n'}^{\mu'}| \lesssim \Gamma$. Therefore the dips in the $\mathcal{T}(\phi)$ are related to scattering on pairs of close levels and the interference terms containing amplitudes from different pairs, can be neglected. This allows us to restrict our consideration to a single pair of almost resonant levels.

Let us now consider a ring with sufficiently weak impurities (see corresponding criterion below). The impurity potential

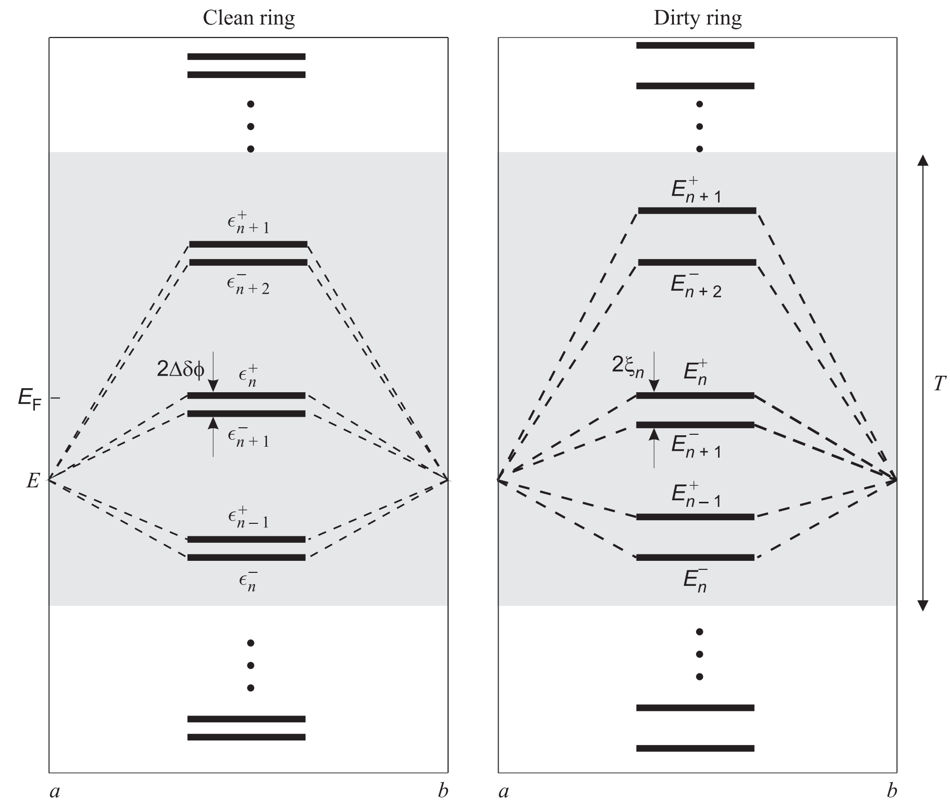

Figure 1: Schematics of energy levels in clean (left) and disordered (right) rings and tunneling of an electron with energy E through pairs of close levels in the ring. In a clean ring the distance between levels in all pairs is the same, $2\Delta\delta\phi$, whereas disorder leads to level repulsion: $E^+_n - E^-_{n+1} = 2\xi_n = 2\sqrt{\Delta^2\delta\phi^2 + |V_n|^2}$.

$V(x) = \sum_{\nu=1}^{N} U(x - x_\nu)$ can be accounted for by using two-level approximation for two close levels. Within this approximation the energies and wave functions of potential-disturbed states are given by

$$E_n^+ - E_{n+1}^- = 2\xi_n, \tag{A2}$$

$$\xi_n = \sqrt{\Delta^2 \delta\phi^2 + |V_n|^2}, \tag{A3}$$

$$\Psi_n^+ = \frac{\psi_n^+ + \psi_{n+1}^- V_n / W_n}{\sqrt{1 + |V_n|^2 / W_n^2}}, \tag{A4}$$

$$\Psi_{n+1}^- = \frac{\psi_{n+1}^- - \psi_n^+ V_n^* / W_n}{\sqrt{1 + |V_n|^2 / W_n^2}}, \tag{A5}$$

where $W_n = \Delta\delta\phi + \xi_n$, and

$$V_n = \int dx \psi_{n+1}^{-*}(x) V(x) \psi_n^+(x) = \frac{ir\Delta}{2\pi} \sum_\nu e^{2\pi i(2n+1)x_\nu/L}. \tag{A6}$$

is the matrix element of the impurity potential expressed in terms of reflection amplitude r, and $\psi_n^\pm$ are the wavefunctions of electrons in a clean ring.

The two-level approximation is valid provided $|V_n| \ll \Delta$. For randomly distributed impurities, the matrix element of the potential is estimated as $|V_n| \sim |r| \Delta \sqrt{N}$. Hence, the above approach is justified for $|r| \sqrt{N} \ll 1$, which is the same inequality that was used in Sec. 4.2 of the main text.

Due to the level repulsion caused by the impurity potential the minimal distance between levels in any pair is given by $|V_n|$, see Fig. 1 (right panel). When $|V_n| \ll \Gamma$, i.e. $\sqrt{N}|r| \ll \gamma$, impurities do not have any significant impact: as in the case of a clean ring, there exists a dip in $\mathcal{T}(\phi)$ which arises due to the interference term, while the "classical" term is featureless at $\phi = 1/2$.

In contrast, in the opposite case, $|V_n| \gg \Gamma$ (i.e. for sufficiently strong reflection $|r| \gg \gamma/\sqrt{N}$) the energy distance between levels in any pair is always much larger than Γ, and, consequently, the contribution of the interference terms to the transmission coefficient is small compared to "classical" ones:

$$\mathcal{T}(\phi) \simeq \mathcal{T}_{cl}(\phi) = 4\gamma^2 v^2 \left\langle \sum_{n,\mu=\pm} \frac{|\Psi_n^\mu(0)|^2 |\Psi_n^\mu(x)|}{(\epsilon - E_n^\mu)^2 + \Gamma^2/4} \right\rangle_\epsilon , \qquad (A7)$$

Remarkably, this does not destroy the dip in the transmission coefficient: it turns out that in the dirty ring the "classical" terms acquire sharp dependence on ϕ.

Indeed, as seen from Eqs. (A3)-(A5), for $|\Delta\delta\phi| \gg |V_n|$ the wave functions in the $n-$th pair are simply given by clockwise- and counterclockwise-moving waves ψ_n^+ and ψ_{n+1}^-. The "classical" contribution to the transmission coefficient from each of these levels, say level $(n,+)$, is proportional to $|\psi_n^+(0)|^2 |\psi_n^+(L/2)|^2 = 1/L^2$. In the opposite case, $|\Delta\delta\phi| \ll |V_n|$, disorder potential strongly mixes clockwise- and counterclockwise-propagating waves. Consider, for simplicity, the case $\delta\phi = 0$. From Eqs. (A3)-(A5) we get $\Psi_n^+ = (\psi_n^+ + e^{i\varphi_n}\psi_{n+1}^-)/\sqrt{2}$ and $\Psi_{n+1}^- = (\psi_n^+ - e^{-i\varphi_n}\psi_{n+1}^-)/\sqrt{2}$ with $e^{i\varphi_n} = V_n/|V_n|$. For many weak impurities, the product $|\Psi_n^+(0)|^2 |\Psi_n^+(L/2)|^2$ self-averages over the random phase φ_n which yields a value $1/2L^2$. This implies that the classical contribution yields a dip of width $\delta\phi \sim |V_n|/\Delta \sim \sqrt{N}|r|$ in the transmission coefficient, where $\mathcal{T}$ decreases by a factor of 2.

The physical picture discussed above, in particular, transition from the interference to "classical" mechanism of formation of the dip can be illustrated by an example of the ring with a single impurity. As seen from Eq. (29) of the main text, in this case the transmission coefficient is given by the Lorentz-shape antiresonance:

$$\mathcal{T} \approx 2\gamma \frac{\pi^2 \delta\phi^2 + |r|^2/8}{\pi^2 \delta\phi^2 + \gamma^2 + |r|^2/4}. \tag{A8}$$

We see that the transmission coefficient at $\phi = 1/2$ is no longer equal to zero and the antiresonance broadens so that its width becomes $\sqrt{\gamma^2 + |r|^2/4}$. We also find that the depth of the dip changes from 2γ to γ with increasing $|r|$. In other words, in contrast to the antiresonance width, its depth remains parametrically the same. The "classical" and interference contributions to Eq. (A8) read

$$\mathcal{T}_{cl}(\phi) = \frac{2\gamma(\pi^2 \delta\phi^2 + |r|^2/8)}{\pi^2 \delta\phi^2 + |r|^2/4}, \tag{A9}$$

$$\mathcal{T}_{int}(\phi) = -\frac{2\gamma^3(\pi^2 \delta\phi^2 + |r|^2/8)}{(\pi^2 \delta\phi^2 + |r|^2/4)(\pi^2 \delta\phi^2 + \gamma^2 + |r|^2/4)}. \tag{A10}$$

As seen, the interference contribution leads to formation of the dip at $\gamma \gg |r|$ and can be neglected for $\gamma \ll |r|$.