Supplemental material to the article

Peculiarities of Faraday rotation in Cs atomic vapor with use of a cell with thickness less than light wavelength?

We consider a steady-state interaction of a linearly polarized laser field with a vapor of Cs Λ-atoms confined in an extremely thin cell of thickness L, which is less than the wavelength λ of the atomic resonant transition. The medium is subjected to an external longitudinal magnetic field B, directed along the laser beam propagation. The atomic number density is assumed to be small enough so that the time of free flight of the atoms with a mean thermal velocity $\langle v\rangle$ largely exceeds the time $\tau = L/\langle v\rangle$ of the atomic flight between two walls of the cell. It is also assumed that the atoms experience inelastic collisions with the cell walls where they lose both the optical excitation and atomic coherence between the ground states. The incident laser beam diameter largely exceeding the cell thickness, thus mainly atoms flying parallel to windows, which do not experience frequent collisions with the windows, contribute to formation of the Faraday rotation (FR) signal. With these two assumptions, we can avoid using a phenomenological constant responsible for the collisional relaxation of atomic excitation and coherences and take these effects into account exactly by solving the temporal equations for the atomic density matrix with proper boundary conditions for each atom (with a given velocity v) separately. Then, the polarization is found in all orders of the laser field by averaging the total contribution of the atoms over the atomic velocity distribution, as described below.

The atomic system is presented in Fig. 1, where the levels $|1\rangle$ and $|2\rangle$ are the ground states with the magnetic quantum numbers $m = -1$ and $+1$, respectively, and $|3\rangle$ is the excited state having zero magnetic quantum number $m = 0$. The atoms are excited under the normal incidence by a linearly polarized optical field $E_{\rm in}\exp(ikz - i\omega t)$, the frequency ω of which is detuned from the atomic transitions by $\Delta = \omega_0 - \omega + kv$, where ω_0 is the energy of the excited state, kv is the Doppler shift for the atoms with velocity $v = v_z$ along the field propagation direction (z-axis), $k = \omega/c = 2\pi/\lambda$.

The total Hamiltonian of the interaction corresponding to this system reads

$$H = \hbar(\Delta+\delta)|1\rangle\langle 1| + \hbar(\Delta-\delta)|2\rangle\langle 2| - \hbar\delta|3\rangle\langle 3| - \hbar\Omega_-|3\rangle\langle 1| - \hbar\Omega_+|3\rangle\langle 2| + \text{H.c.}, \quad (1)$$

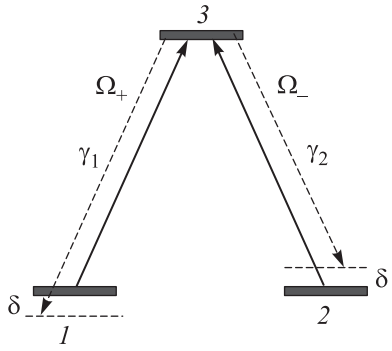

Figure 1: The atomic system, the levels $|1\rangle$ and $|2\rangle$ are the ground states with the magnetic quantum numbers $m = -1$ and $+1$, respectively, and $|3\rangle$ is excited state having $m = 0$, $\gamma_{1,2}$ are the radiative decay of the upper level to the ground states 1 and 2, as well as the optical pumping to the other ground hyperfine levels. In the presence of a longitudinal magnetic field, the Zeeman sublevels of the ground state are shifted in energy by δ. This leads to a difference in resonance frequencies for left- (σ^+) and right- (σ^-) circularly polarized light, having the Rabi frequencies Ω_+ and Ω_-, respectively

where $\delta = g\mu_0 B/\hbar$ is the Larmor frequency, g is the gyromagnetic ratio of the ground state, μ_0 is the Bohr magneton. The Rabi frequencies of the circularly polarized components of the field, acting on the transitions $|1\rangle \to |3\rangle$ and $|2\rangle \to |3\rangle$ with $\Delta m = +1$ and -1, respectively, are defined as $\Omega_\pm(z) = E_\pm(z)d/\hbar$. Here the dipole matrix elements $|\mu_{13}| = |\mu_{23}| = d$ are the same for both transitions due to the symmetry of the system.

At a distance z in the medium the electric field amplitude of initially x-polarized optical field becomes

$$E(z) = \frac{1}{\sqrt{2}} E_0(z) \left\{ \hat{e}_+ [1 + \eta_+(z)] + \hat{e}_- [1 + \eta_-(z)] \right\} e^{ikz - i\omega t}, \tag{2}$$

where $\hat{e}_\pm = \frac{1}{\sqrt{2}}(\hat{x} \pm \widehat{iy})$ are the basic unit vectors, and $E_0(z)$ is the field amplitude in transparent atomic medium (or in an empty cavity), which with taking into account the Fabry-Perot (FP) effect is connected to the incident field $E_{\rm in}$ by $E_0(z) = \frac{E_{\rm in} t_1}{F}[1 - r_2 e^{2ik(L-z)}$. Here $F = 1 - r_1 r_2 e^{2ikz}$, while the transmission and reflection coefficients of two dielectric windows of the cavity with the refractive indices $n_{1,2}$ are given by $t_1 = 2n_i/(1 + n_i)$ and $r_i = (1 - n_i)/(1 + n_i)$, $i = 1, 2$, respectively. In Eq. (2), $\eta_\pm = \varepsilon_\pm(z)/E_0(z)$ take into account the resonant contributions of the medium $\varepsilon_\pm(z)$(z) imposed to the right- and left-polarization components of the field

only in the first order using the fact that the dilute character of the vapor implies $|\varepsilon_\pm(z)| \ll |E_0(z)|$.

The output amplitudes of circular components can be presented in the form $E_\pm^{(\mathrm{out})} = (1 - r_2)E_\pm(L)$, where $E_\pm(L) = \frac{1}{\sqrt{2}}[E_0(L) + \varepsilon_\pm(L)] \approx \frac{1}{\sqrt{2}}E_0(L)e^{-\alpha L}\exp[i\Phi_\pm(L)]$ with the phases $\Phi_\pm(L) = \mathrm{Im}[\eta_\pm(L)]$ and the absorption $\alpha = \mathrm{Re}[\eta_\pm(L)]$. The total atomic responses $\varepsilon_\pm(z)$ induced by the field $E_0(z)$ on the corresponding transitions are expressed as a combination of the forward $\varepsilon_\pm^f(z)$ and backward $\varepsilon_\pm^b(z)$ signals. Since in the FP cavity the latter is reflected by the first window and then propagates in the forward direction, we have $\varepsilon_\pm(L) = \frac{1}{F}[\varepsilon^f(L) - r_1\varepsilon_\pm^b(0)]$. The fields $\varepsilon^f(z)$ and $\varepsilon^b(z)$ are calculated according to the formulae (T. A. Vartanyan et al., PR **A51**, 1959 (1995))

$$\varepsilon_\pm^f(L) = 2\pi ik\int_0^L P_\pm(z)dz, \quad \varepsilon_\pm^b(L) = 2\pi ik\int_0^L P_\pm(z)e^{2ikz}dz,$$

where $P_\pm(z)$ are the medium polarizations induced by the electric field on the transitions $|1\rangle \to |3\rangle$ and $|2\rangle \to |3\rangle$, respectively. Obviously, these two groups of atoms experience opposite Doppler shifts $\Delta^\pm = \omega_0 - \omega \pm \omega\frac{v}{c}$. The density matrix equations obtained with the Hamiltonian (1) read

$$\frac{d}{dt} = -\frac{i}{\hbar}[H, \rho] + \Lambda\rho,$$

where Λ is the relaxation matrix comprising the radiative decay of the upper level to the ground states 1 and 2, as well as the optical pumping to the other ground hyperfine levels. These equations are solved numerically for each group of atoms with the corresponding initial conditions. We take into account the atomic motion which is assumed to obey the Maxwell distribution $W(v) = (u\sqrt{\pi})^{-1}\exp(-v^2/u^2)$ with u being the most probable velocity. Thus, for polarization rotation angle Φ averaged over the atomic velocity distribution we obtain

$$\langle\Phi\rangle = A\int_0^\infty vdv\exp(-v^2/u^2)[J_2(x) - J_1(v)]$$

with

$$J_1(v) = \int_0^\infty dt\mathrm{Re}[\rho_{31}^+(t, \Delta^+, E_0(v_z t))(1 - r_1r_2e^{2ikv_z t}) +$$

$$+ \rho_{31}^{-}(t, \Delta^{-}, E_0(L - v_z t))(1 - r_1 r_2 e^{2ik(L - v_z t)})]$$

and

$$J_2(v) = J_1(v, \rho_{31} \to \rho_{32}),$$

where the coefficient A is defined as $A = \dfrac{4N\sqrt{\pi}\omega\mu}{cuE_{\text{in}}(1 - r_2)}$.

Note that the laser Rabi frequency is estimated by the formula $\Omega/2\pi = a\gamma(I/8)^{1/2}$ [see, for example, A. Krmpot et al., Opt. Express **13**, 1448 (2005)], where I is the laser intensity in mW/cm^2, γ is the decay rate of the excited state (4.5 MHz), and a is a fit parameter (for our case a is of ~ 0.1). For $P_L = 5\,\mu W$ and laser beam diameter of ≈ 1.2 mm, the intensity is $I = 0.44$ mW/cm^2, and Rabi frequency $\Omega/2\pi = 0.1$ MHz. For $P_L = 8.4$ mW, we have $\Omega/2\pi = 1.3$ MHz.