Supplemental material to the article

Quantum Hall effect in a system with reservoir of electrons

In the Supplemental Material, we consider a model of a field-effect transistor with an electron channel produced by two two-dimensional electron systems (2DES) (Fig. 1). The systems are located in two different

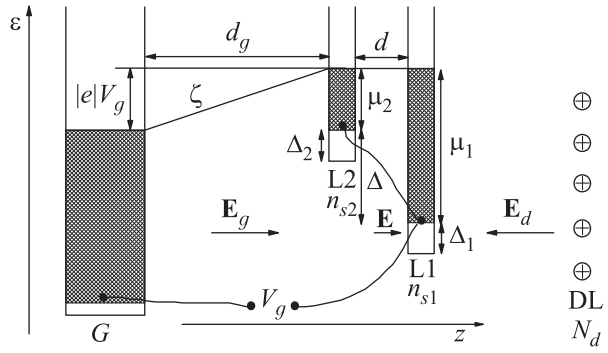

Figure 1: Schematic of a field-effect transistor with an electron channel containing two 2DES located in narrow quantum wells L1 and L2 which are separated by distance d. The electron systems in the quantum wells are electrically connected and, therefore, have the same value of electrochemical potential ζ. A gate of the transistor is denoted by G. Electrons get in the quantum wells from donor layer DL and from a source of the gate voltage V_g. Electrical fields are equal to E_g between the gate and well L2, E between the wells, and E_d between well L1 and the donor layer. Chemical potentials μ_1 and μ_2 of electrons in wells L1 and L2 are measured from subband bottoms Δ_1 and Δ_2 respectively. Energy ranges occupied by electrons at $B = 0$ and temperature $T = 0$ are shaded

quantum wells of a fixed profile with negligible tunneling between them. The wells are electrically connected, so that the electrochemical potential of electrons ζ is equal in the systems and electrons can move between the wells. Similar model was considered in Refs. [10, 16], and such structures can be easily produced. Due to difference of electron densities in the two 2DES, the well known magneto-oscillations of the chemical potential occur in these systems asynchronously. This leads to magneto-oscillations of the electrical potential difference between the wells, which are necessary to keep there equal values of the electrochemical potential. Since chemical potentials of 2DES and electric field between the wells are related to an electron density in 2DES, variation of all these quantities occur self-consistently.

The well width is much less than d and d_g. The gate is considered as an ideal metal, so that electric field exists only in a space between the gate and donor layer DL. Areal density of charged donors N_d is assumed to be constant. Difference of the electrochemical potentials between the electron system and gate is equal to $|e|V_g$. Electric fields are determined by areal electron densities. Then it is easy to obtain the following relations between electron densities n_{s1} and n_{s2} and chemical potentials μ_1 and μ_2 of 2DES:

$$|e|(V_g - V_{g0}) = \Lambda\left[(n_{s1} + n_{s2})d_g + n_{s1}d\right] + \mu_1(n_{s1}, B), \tag{1}$$

$$|e|(V_g - V_{g0}) = \Lambda(n_{s1} + n_{s2})d_g + \mu_2(n_{s2}, B) + \Lambda N_d d + \Delta_2 - \Delta_1. \tag{2}$$

Here, V_{g0} is the gate voltage at which the electron channel gets depleted ($n_{s1} + n_{s2} = 0$) at zero magnetic field, $\Lambda = 4\pi e^2/\chi$, where χ is the dielectric constant. V_{g0} is determined by several unknown parameters of the system one of which is the work function of the gate material. $\Delta_1 \neq \Delta_2$ for wells of different width.

From Eqs. (1) and (2) one can get the following equation for quantum capacitance per unit area $C_0 = |e|dn_s/dV_g$:

$$C_0 = \frac{\chi}{4\pi d_g}\left[1 + \frac{d/d_g + 1/\Lambda d_g D_1^{(th)}}{1 + \Lambda d D_2^{(th)} + D_2^{(th)}/D_1^{(th)}}\right]^{-1}. \quad (3)$$

Here, $D_l^{(th)} = \partial n_{sl}/\partial\mu_l$ ($l = 1, 2$) is the thermodynamic density of states of electrons in well l. In the model under consideration, the electron-electron interaction is included through the mean electric field between the wells only. Then, at $T = 0$, the electron densities are determined as

$$n_{sl} = \int_0^{\mu_l} D(\varepsilon, B)d\varepsilon. \quad (4)$$

Here and below, energy ε is measured from a subband bottom. To calculate the $D_l^{(th)}(n_{sl}, B)$ dependencies we chose a model form of the density of sates in a magnetic field introduced earlier (see, for example, Ref. [19]):

$$D(\varepsilon, B) = 2\alpha N_0/\hbar\omega_c + [(1-\alpha)N_0/\sqrt{\pi}\Gamma]\sum_{\pm,n}\exp\left(-\left[\varepsilon - \hbar\omega_c(n - 1/2) \mp g\mu_B B/2\right]^2/\Gamma^2\right). \quad (5)$$

Here, $N_0 = |e|B/hc$ is the Landau level degeneracy per spin, $\omega_c = |e|B/m^*c$ is the cyclotron frequency (m^* is the electron effective mass), μ_B is the Bohr magneton, and g is the g-factor. In this equation, the Landau level broadening by a disorder is described by a Gaussian of width Γ depending on magnetic field: $\Gamma = \hbar\sqrt{\omega_c/\pi\tau}$, where τ is the quantum lifetime. The first term in Eq. (5) describes a small ($\alpha \ll 1$) finite density of states between Landau levels normally observed in experiment. It also removes the singularities in the $\mu(B)$ dependence in the case of well separated ($\Gamma \ll \hbar\omega_c$) levels.

Dependence of capacitance on magnetic field for different V_g can be calculated from Eqs. (1)–(5). However, because the terms proportional to d_g dominate in the right-hand side of Eqs. (1) and (2), we have found the following procedure to be much more convenient. (i) Analytical equations for $d\mu_l/dB$ were obtained from Eqs. (1)–(5). (ii) For given values of $V_g - V_{g0}$ and $B = B_{up}$, electron densities n_{sl} and chemical potentials μ_l were numerically calculated from Eqs.(1)-(5). The B_{up} value was chosen to correspond to a non-integer filling factor ν, usually we took $\nu = 3/2$. (iii) By integrating $d\mu_l/dB$ over magnetic field in interval (B, B_{up}), dependencies $\mu_l(B)$ and $n_{sl}(B)$ for $B \in (B_{down}, B_{up})$ were calculated. (iv) Initial values of $\mu_l(B_{up})$ were slightly corrected (usually the correction did not exceed 1 %) to get agreement with experimental results in all (B_{down}, B_{up}) range (see Fig. 3 in the main text). The main agreement criterion was coincidence of positions of the capacitance minima (except the $\nu = 2$ minima which will be discussed below).

Parameters in Eqs. (1)–(5) were chosen as following. $d_g = 850$ nm, as in the heterostructure studied experimentally. $\chi = 10.8$ was taken to fit the mean measured capacitance value small corrections to which were the main subject of the study. $\alpha = 0.1$ and $\tau = 2.8\cdot10^{-10}$ s provided the Landau energy spectrum in form of narrow peaks on a small pedestal and did not essentially affect a width of the wide capacitance minima observed in the regime of two occupied subbands. In such a regime, the width of the minima was determined by relation between an energy gap in which the electrochemical potential was locked and variation of the electric potential difference between the wells which is proportional to distance d. The cyclotron energy was taken for electrons with effective mass $m^* = 0.067m_e$. Value of $d = 9$ nm was chosen to adequately describe both the width and depth of the minima as is explained below. These values of the parameters were used in calculations for *all* gate voltages. $g = 2.4$ was found appropriate to describe width of all wide minima except of that at $\nu_2 = 1$, $V_g = 0.6$ V (see. Fig. 3 in the main text). To fit the width of the latter minimum an essentially larger value of the g-factor is necessary. In Fig. 3 in the main text, results for $g = 2.4$ and $g = 9.6$ are shown. It should be emphasized that very large enhancement of the spin-splitting in comparison with $|g| = 0.4$ for electrons in GaAs is a well known fact for 2DES in GaAs/AlGaAs heterostructures when the chemical potential lies in the Zeeman gap. It is explained [20] by the exchange interaction.

To get a better insight of the chemical potential locking, consider magnetic field range (B_1, B_2) in Fig. 4 in the main text (range II). From energy spectrum in Fig. 4a the following relation for these fields can be easily obtained:

$$B_2 - B_1 \approx B_1 \frac{|e|\hbar/m^* c}{2|e|\Lambda d/hc + g\mu_{\rm B}}. \tag{6}$$

Variation of the electron density in layer L1 is given as

$$n_{s1}(B_2) - n_{s1}(B_1) \approx 2|e|(B_2 - B_1)/hc. \tag{7}$$

Eqs. (6) and (7) are exact when $\alpha = 0$ and $\Gamma \to 0$. Note that width $B_2 - B_1$ decreases with increasing d and becomes inversely proportional to d when $2|e|\Lambda d/hc \gg g\mu_{\rm B}$. Similarly, it can be shown that width of region I also decreases with increasing d. On the other hand, a depth of the capacitance minima increases with d. Indeed, if $\alpha = 0$ and $\Gamma \to 0$, and filling factor in layer L1 is not an integer (i.e., the thermodynamic density of states $D_1^{(th)}$ is large), it follows from Eq. (3) that the depth of the capacitance minima at integer ν_2 is given as

$$C_0(D_2^{(th)} \to \infty) - C_0(D_2^{(th)} = 0, D_1^{(th)} \to \infty) \approx \frac{\chi}{4\pi d_g} \frac{d}{d_g}. \tag{8}$$

Hence, at small value of pedestal α and narrow Landau levels, parameter d determines depth of the capacitance minima and strongly affects their width. The chosen value of $d = 9$ nm describes width of the minima and gives reasonable value for their depth. Moreover, it correlates with total width $d_{\rm QW} = 60$ nm of the studied quantum well containing two electron layers.

Our model describes redistribution of electrons between the subbands, which occurs when the magnetic field is varied. This effect determines the large width of the capacitance minima and existence of particular quantum Hall effect states. However, it is clear that a number of important effects are not captured by the model. First of all, this is the dependence of the spin gap on the position of the chemical potential relative to the Zeeman sublevels [20], which still is not completely studied. Next, in the model, QHE states at $\nu = 2$ do not appear while they are present in the experiment. Absence of these states in the model is due to the fact that, at $\nu = 2$, the electrochemical potential is pinned to the two coinciding levels $2{-}1^-$ and $1{-}1^+$ and an energy gap necessary for QHE states is absent (see Fig. 4a in the main text). This disagreement may be eliminated within complete self-consistent calculations taking into account dependence on magnetic field of both, form of the quantum well and Landau wave functions, as, for example, has been done pertubationally in Ref. [11].