Supplemental material to the article "Superconducting Properties of Long TiN Wires"

It is known that the temperature dependence of the resistance of superconducting films in the range $T_c < T \ll \Theta$ (Θ is the Debye temperature), where the dependence $R(T)$ associated with scattering by phonons can be neglected, is determined by quantum effects: weak localization of electrons (WL) and electron–electron interaction in the diffusion (ID) and Cooper channels. The latter contributions are referred to as superconducting fluctuations (SF) and are commonly divided into three distinct types: (i) the Aslamazov–Larkin contribution (AL) caused by the addition to the conductivity from fluctuating Cooper pairs, (ii) the contribution to the density of states (DOS) reflecting a change in the density of states of normal electrons as a result of fluctuating Cooper pairing, and (iii) the Maki–Thompson contribution (MT) corresponding to coherent scattering of electrons constituting a fluctuating Cooper pair by impurities. The total conductivity of the disordered system is thus the sum of all the quantum contributions to conductivity added to the bare Drude conductivity G_0

$$\frac{\Delta G(T)}{G_{00}} = \frac{\Delta G^{ID}(T)}{G_{00}} + \frac{\Delta G^{WL}(T)}{G_{00}} + \frac{\Delta G^{MT}(T)}{G_{00}} + \frac{\Delta G^{DOS}(T)}{G_{00}} + \frac{\Delta G^{AL}(T)}{G_{00}}, \quad (1)$$

$$G_{00} = e^2/(2\pi^2\hbar). \quad (2)$$

Since the thickness of samples under study in the temperature interval under consideration is smaller than the thermal coherence length l_T (and the width is bigger than l_T), the samples are quasi-two-dimensional with respect to the effects under discussion. For this reason, for the description of measured $R(T)$, we use theoretical expressions corresponding to the quasi-two-dimensional case.

The contributions WL and ID have the same logarithmic temperature dependence [17,19] (see Fig. 1):

$$\frac{\Delta G^{ID}(T)}{G_{00}} + \frac{\Delta G^{WL}(T)}{G_{00}} = A \ln\left(\frac{kT\tau}{\hbar}\right), \quad (3)$$

where τ is mean free time, A is a numerical constant

$$A = ap + A^{ID}, \quad (4)$$

where the constant A^{ID} is of the order of unity [17]. The constant $a = 1$ in the case of weak spin-orbit interaction ($\tau_\varphi \ll \tau_{so}$ where τ_φ is the dephasing time and τ_{so} is the spin-orbit scattering time) and $a = -1/2$ in the case of fast relaxation of the spin ($\tau_\varphi \gg \tau_{so}$), and p is the exponent in the temperature dependence of the dephasing time $\tau_\varphi \propto T^{-p}$. At low temperatures, when the electron-electron scattering dominates,

$$\tau_\varphi^{-1} = \frac{\pi k_B T}{\hbar} \frac{e^2 R}{2\pi^2 \hbar} \ln \frac{k_B T \tau_\varphi}{\hbar}. \tag{5}$$

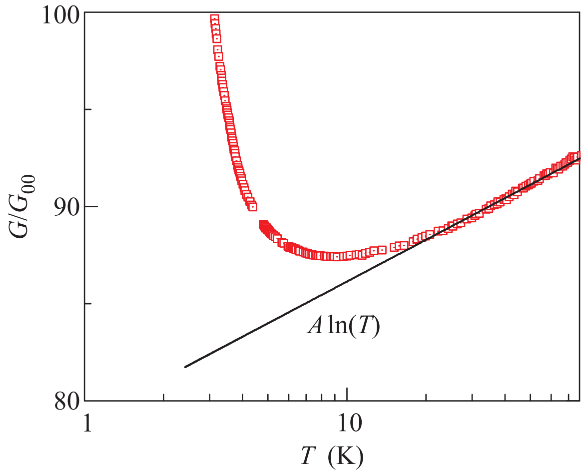

Figure 1: Temperature dependence of the resistance of wire $w = 100$ nm replotted as the dimensionless conductance G/G_{00}. The semilogarithmic scale representation reveals logarithmic decrease of the conductance with temperature owing to WL and ID effects. $A = 3.1 \pm 0.05$

The contributions from superconducting fluctuations (DOS [18] and AL [20]) depend only on T_c. Thus, T_c can be determined by juxtaposing the measured $R(T)$ with the results of the theory of superconducting fluctuations (SF) in the region $T > T_c$, as it enters the listed equations as a fitting parameter,

$$\frac{\Delta G^{DOS}(T)}{G_{00}} = -\ln\left(\ln\left(\frac{T_c}{T}\right) / \ln(T_c\tau)\right), \tag{6}$$

$$\Delta G^{AL} = \frac{e^2}{16\hbar} \frac{1}{\ln(T/T_c)}, \tag{7}$$

Table 1. Sample characteristics. w – width; T_c is the critical temperature determined from the quantum contribution fits together with the pair-breaking parameter δ Eq. (9) and the coefficient A (Eq. (4))

w [μm]	T_c [K]	δ	A
50	2.44 ± 0.02	0.035	3.1 ± 0.05
0.1	2.42 ± 0.02	0.032	3.1 ± 0.05

The Maki-Tompson contribution $\Delta G^{MT}(T)$ depends also on τ_φ through the pair-breaking parameter [21–23]:

$$\frac{\Delta G^{MT}(T, B=0)}{G_{00}} = \beta(T, \tau_\varphi) \ln\left(\frac{\ln(T/T_c)}{\delta}\right), \tag{8}$$

$$\delta = \pi\hbar/(8k_B T\tau_\varphi), \tag{9}$$

$\beta(T, \delta)$ is the function from work [22]:

$$\beta(T, \delta) = \frac{\pi^2}{4}\sum_m (-1)^m \Gamma(|m|) - \sum_{n\geq 0} \Gamma''(2n+1), \tag{10}$$

where m is an integer $m = 0, \pm 1, \pm 2, ...$, and

$$\Gamma(|m|)^{-1} = \ln\frac{T}{T_c} + \psi\left(\frac{1+|m|}{2}\right) - \psi\left(\frac{1}{2}\right) - \psi'\left(\frac{1+|m|}{2}\right)\frac{2\delta}{\pi^2}. \tag{11}$$