Supplemental material to the article

"Phase Transitions in Hybrid SFS Structures with thin Superconducting Layers"

In the previous theoretical analysis of the phase transitions in the superconductor-ferromagnet-superconductor (SFS) trilayers with transparent SF barriers, the order parameter Δ was assumed to be a nearly constant over the thin S layers. In addition, we neglected a dependence $T_c^S(d_s)$ of the superconducting transition temperature T_c^S on the S film thickness d_s, assuming that T_c^S simply coincides with the critical temperature of the bulk superconductor T_{c0} ($T_c^S \equiv T_{c0}$). However, in the experiment a monotonic growth of $T_c(d_{Nb})$ with increasing niobium film thickness d_{Nb} was observed (see Fig. 2 of the paper). This growth seems to be attributed to the proximity effect in the presence of a normal ("dead") Nb layer where superconductivity has been destroyed. Here we discuss in short the properties of the phase transitions in SFS trilayers taking into account the typical dependence $T_c^S(d_s)$ due to the proximity of normal and superconducting materials.

For simplicity, we consider the hybrid trilayers which consist of two thin dirty superconductor–normal-metal bilayers (BL) of total thickness $d_{sn} = d_s + d_n$ as superconducting electrodes (d_n is the thickness of the normal layer), and an intermediate F layer of thickness d_f (see Fig. 1). The critical temperature of the bilayers T_c^{BL} is

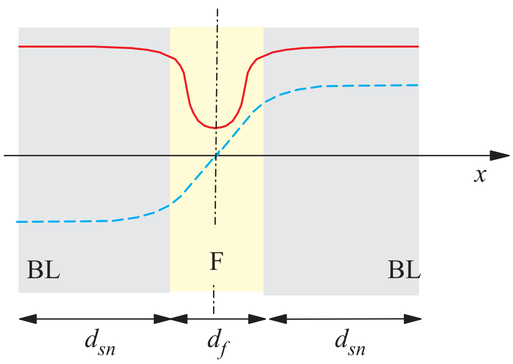

Figure 1: (Color online) The schematic behavior of the pair wave function $F(x) = F_{s,f}$ inside the BL/F/BL structure: the red solid line (blue dashed line) represents sketchy the behavior of the pair wave function in 0-state (π-state)

assumed to be described by the relation

$$\sqrt{T_c^{BL}/T_{c0}}\,\ln(T_{c0}/T_c^{BL}) = d_n/d_s, \tag{1}$$

which is the typical for SN structures [26]. The critical temperature $T_c^{0,\pi}$ of the second-order phase transition of BL/F/BL structure into 0 or π states can be estimated using the expressions (2), (3) of the paper by replacing the bulk critical temperature T_{c0} by the value T_c^{BL}:

$$\ln\left(T_c^{0,\pi}/T_c^{BL}\right) = \Psi\left(1/2\right) - \mathrm{Re}\left[\Psi\left(1/2 + \Omega_{0,\pi}\right)\right]. \tag{2}$$

where Ψ is the digamma function. The depairing parameters $\Omega_{0,\pi}$ are assumed to be the same as for SFS trilayers [20]:

$$\Omega_{0,\pi} = \frac{\varepsilon}{2}\frac{T_c^{BL}}{T_c^{0,\pi}}\begin{cases} k\,\tanh(ks_f), & \text{0-phase,} \\ k\,\coth(ks_f), & \pi\text{-phase,}\end{cases} \tag{3}$$

with $\varepsilon = (\sigma_f/\sigma_s)(\xi_s^2/d_{sn}\xi_f)$. Certainly, this assumption is correct if the FS interfaces were not modified due to presence of a normal layer. Figure 2 shows a typical dependency of the critical temperature $T_c^{0,\pi}$ on the bilayers thickness d_{sn}, obtained

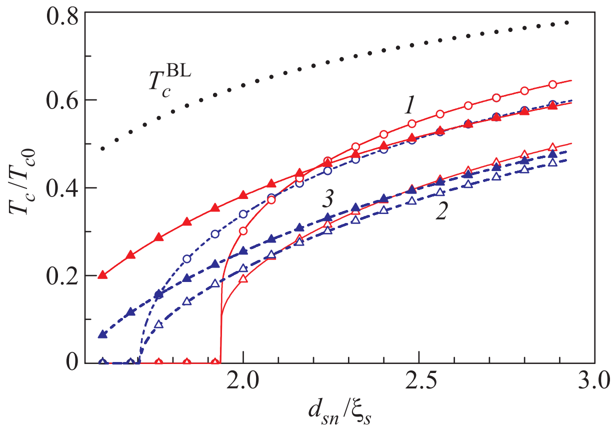

Figure 2: (Color online) The typical dependence of the critical temperature $T_c^{0,\pi}$ on the thickness of SN bilayer d_{sn} for 0-phase (solid red line) and for π-phase (dashed blue line): *1* – $d_n = 0$, $h = 10\,T_{c0}$; *2* – $d_n = 0.5\,\xi_s$, $h = 10\,T_{c0}$; *3* – $d_n = 0.5\,\xi_s$, $h = 7\,T_{c0}$. The dependence $T_c^{BL}(d_{sn})$ for $d_n = 0.5\,\xi_s$ is shown by the dotted line. Here we choose: $d_f = 2\xi_f$; $\sigma_f/\sigma_s = 0.12$; $\xi_s/\xi_f = 3$; $\tau_s = 0.7/T_{c0}$

from Eqs. (1)–(3) for different values of the normal layer thickness d_n and the exchange field h. An expected decrease of the superconducting second-order transition temperature of the hybrid structure taken as a whole occurs due to the presence of a normal layer (d_n parameter) in the superconducting electrodes. Note, that the crossing of the curves $T_c^0(d_{sn})$ and $T_c^\pi(d_{sn})$ ($T_c^0(d_{sn}^*) = T_c^\pi(d_{sn}^*)$) is shifted negligibly: d_{sn}^* is reasonably constant for different values of d_n. This means that the position of the switch between 0 and π ($d_{sn} = d_{sn}^*$) states appears to be rather insensitive to a thickness dependence of the critical temperature of the superconducting electrodes $T_c^S(d_s)$. At the same time, the degradation of the exchange field (h parameter) plays a stronger role: a shift of the position of the $0-\pi$ transition is considerable. As a result, there exists no crossing of the curves $T_c^0(d_{sn})$, and $T_c^\pi(d_{sn})$ at the phase diagram, and the $0-\pi$ transition disappears for chosen parameters. Certainly, both compositional and structural modifications of the sample can be attributed to the effect of ageing [21].